Name $A.K.$

Diff. Equations and Lin. Alg.
Math 3280
Quiz 8, Fall 2018
B. Peckham

1. (3pts) Use the Laplace transform method to solve the following initial value problem:
$y' = 3y, y(0) = 2.$

Transform: $sY(s) - y(0) = 3Y(s)$

Solve: $(s-3)Y(s) = y(0)$

$Y(s) = \frac{y(0)}{s-3}$

Inverse Transform: $y(t) = y(0)e^{3t} = 2e^{3t}$

2. (3 pts) Transform the following differential equation using the Laplace transform. Solve the transformed equation. (That is, solve for $Y(s)$; do not "undo" the transform to find $y(t)$.)
$y'' - 2y' + 5y = 0, y(0) = 2, y'(0) = 1.$

Transform $\left(s^2 Y(s) - s\,y(0) - y'(0)\right) - 2\left(s\,Y(s) - y(0)\right) + 5Y(s) = 0$

Solve: $(s^2 - 2s + 5)Y(s) = s\,y(0) + y'(0) - 2y(0) = 2s + 1 - 2\cdot 2$
$= 2s - 3$

$\Rightarrow Y(s) = \frac{2s-3}{s^2-2s+5}$

3. (4 pts) Show directly from the definition of Laplace transform that the Laplace transform of $e^{3t}f(t)$ is $F(s-3)$, where $F(s)$ is the Laplace transform of $f(t)$.

$\mathcal{L}\{e^{3t}f(t)\}(s) = \int_0^\infty e^{-st}e^{3t}f(t)\,dt = \int_0^\infty e^{-(s-3)t}f(t)\,dt = F(s-3).$

Since $F(s) = \int_0^\infty e^{-st}f(t)\,dt$

4. (2pts EC) If $F(s) = \frac{3s+1}{(s-2)^2+9}$, what is $f(t)$, the inverse Laplace transform of $F(s)$.

$F(s) = \frac{3(s-2) + \frac{7}{3}\cdot 3}{(s-2)^2 + 3^2} \Rightarrow f(t) = 3e^{2t}\cos 3t + \frac{7}{3}e^{2t}\sin 3t$

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~~inverse Laplace transforms as in problems 23-32 in Sec. 10.2 of the Textbook.~~

1. (3pts) Use the Laplace transform method to solve the following initial value problem:
   $y' = 4y, y(0) = 3$.

Transform: $sY(s) - y(0) = 4Y(s)$

Solve: $(s-4)Y(s) = y(0) = 3$

$\Rightarrow Y(s) = 3/s-4$

Inverse Transf. $\Rightarrow y(t) = 3e^{4t}$

2. (3 pts) Transform the following differential equation using the Laplace transform. Solve the transformed equation. (That is, solve for $Y(s)$; do not "undo" the transform to find $y(t)$.)
   $y'' - 2y' + 4y = 0, y(0) = 1, y'(0) = 2$.

Transform: $(s^2 Y(s) - sy(0) - y'(0)) - 2(sY(s) - y(0)) + 4Y(s) = 0$

Solve: $Y(s)(s^2 - 2s + 4) = sy(0) + y'(0) - 2y(0) = 1\cdot s + 2 - 2\cdot 1 = s + 0 = s$

$$Y(s) = \frac{s}{s^2 - 2s + 4}$$

3. (4 pts) Show directly from the definition of Laplace transform that the Laplace transform of $e^{2t}f(t)$ is $F(s-2)$, where $F(s)$ is the Laplace transform of $f(t)$.

$$\mathcal{L}\{e^{2t}f(t)\}(s) = \int_0^\infty e^{-st} e^{2t} f(t)\,dt = \int_0^\infty e^{-(s-2)t} f(t)\,dt = F(s-2)$$

Since $F(s) = \int_0^\infty e^{-st} f(t)\,dt$

4. (2pts EC) If $F(s) = \frac{3s-12}{(s-2)^2+9}$, what is $f(t)$, the inverse Laplace transform of $F(s)$.

$$F(s) = \frac{3(s-2) - 3\cdot 2}{(s-2)^2 + 3^2}$$

$\Rightarrow f(t) = 3e^{2t}\cos 3t - 2e^{2t}\sin 3t$