Separation of variables Template

Math 3280 B. Peckham

Goal: Find an analytic solution to a separable DE: f(y)dy = g(t)dt

Example: y'=2y, y(0)=3. Solve this IVP and determine how long it takes for the initial amount to double.

Clear variables

```
y =.
t =.
```

Separate by hand first: 1/y dy = 2 dt

```
f = 1 / y
```

$$\frac{1}{y}$$

```
g = 2
```

2

Integrate both sides:

```
LHS = Integrate[f, y]
```

Log[y]

```
RHS = Integrate [g, t] + C
```

C + 2 t

LHS = RHS would be an implicit solution. Solve for y to obtain an explicit solution:

```
gensln = Solve[RHS == LHS, y]
```

$$\left\{\left\{y \to e^{C+2t}\right\}\right\}$$

Trick to eliminate both sets of braces from gensln:

```
gensln = y /. gensln[[1]][[1]]
```

$$e^{C+2t}$$

Specify initial conditions:

```
t0 = 0
```

0

```
y0 = 3
```

3

Use the initial conditions to olve for the arbitrary constant.

```
IC = Solve[y0 == gensln /. t → t0, C]
```

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. More…
```

```
{{C → Log[3]}}
```

Use the value of the constant in the general solution to obtain the particular solution.

```
sln = gensln /. IC[[1]]
```

$$3\, e^{2\,t}$$

```
Simplify[sln]
```

$$3\, e^{2\,t}$$

```
doubletime = Solve[sln == 2 * y0, t, Reals]
```

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. More…
```

$$\left\{\left\{t \to \frac{\text{Log}[2]}{2}\right\}\right\}$$

Find the numerical value of log(2)/2:

```
N[doubletime]
```

```
{{t → 0.346574}}
```