**Differential Equations with Linear Algebra, Math 3280**
**Lab #3**
**Qualitiative, Numerical, and Analytic solutions to the Logistic Differential Equation.**

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**Directions:** The following tasks are intended to supplement the homework problems in sections 1.3 (slope fields), 1.4 (separable D.E.s), 2.1 (population models), and 2.2 (equilibrium solutions and stability) in Edwards and Penny. Grading: Goals (G) 2, Procedures (Pro) 2, *Mathematica* and hand tasks (Ma) 14 (details below), Compare (6pts), Conclusions (C) 2, Presentation and Organization (P+O) 2, Total 28.

Turn in a written lab report for the following tasks. Adhere to the "Lab Procedures and Guidelines."

Consider the initial value problem:

$$\frac{dy}{dt} = y(3-y), \quad y(0) = y_0$$

Method 1: (4pts) Since this differential equation is autonomous ($y'$ depends only on $y$, not on $t$) it is appropriate to sketch a phase line. Do it (by hand). Identify any equilibrium solutions (critical points), and indicate the stability of each. Using only the information on your phase line, sketch graph in the $(t, y)$ plane which *could* be solutions corresponding to initial conditions $y_0 = 2, 3.5, -0.5$.

Method 2: (2pts) Sketch from slope field/Streamplot. Using *Mathematica*, obtain a plot of the slope field using VectorPlot, and of several solution curves using the StreamPlot commmand. (Recall how to enter the vector field corresponding to the given differential equation from Lab 2.) Plot both together using the Show command. Print out the result. By hand, on the printout, sketch and/or highlight the three solutions corresponding to the three initial conditions $y_0 = 2, 3.5, -0.5$.

Method 3: (2pts) Computer graph of numerical solution. Compute the solution corresponding to the initial condition $y_0 = 2$. Hint: Modify the example NDSolve command below:
sln1 = NDSolve[{y'[t] == y[t] (3-y[t]), y[0]==1},y,{t,-5,5}]
(This NDSolve command computes a numerical solution to $y' = y(3-y)$ with initial condition $y(0) = 1$. It is saved (in sln1) as an interpolating function which can only be evaluated for t between -5 and 5.)
Plot[y[t]/.sln1,{t,-3,4}] (This plots y[t], substituting in the numerical solution saved in sln1. The plot range (-3,4) should be inside the interval on which the solution was computed (-5,5).)

Method 4: (6pts) Find an analytic (formula) for the general solution, then graph specific solutions. The given differential equation is separable. Find a formula for the general solution using one of the following three strategies (indicate which one you are using):

(a) Solve completely with Mathematica using the Separation of Variables Template provided.
(b) Organize by hand but do some tasks with Mathematica.
(c) Solve completely by hand.

Find the specific formula solution for the initial conditions: $y_0 = 2$. As a check on your calculations, the solutions should all be equivalent to

$$y(t) = \frac{3y_0}{y_0 + (3-y_0)e^{-3t}}.$$

*Mathematica* Caution: When *Mathematica* integrates $\frac{1}{3y-y^2}$ it obtains an expression with Log's, but it is the "complex-valued" Log function, which allows negative (as well as complex) numbers as an argument. If you ignore this, you will see that once the solution is simplified, the resulting function is real (that is, no complex numbers appear). If you are unable to obtain the solution, either using Mathematica or by hand, continue on with the problem using the above formula solution.

**Check your solution.** (1pt) Either using *Mathematica* or by hand, plug your solution (or the above formula for $y(t)$, if you weren't able to obtain a solution) back into the original differential equation to verify that it is, in fact, a solution to the initial value problem (the differential equation *and* initial condition). **Plot your analytic solutions.** (1pt) Use *Mathematica* to plot the solution corresponding to the initial condition $y_0 = 2$.

Compare Methods 1 – 4 (6pts). Are the graphs obtained from all four methods consistent with each other? Discuss both the accuracy and the amount of work necessary in obtaining a graph with each method. Which method allows you to most easily evaluate a) $\phi(1)$ b) $\lim_{t\to\infty} \phi(t)$, where $\phi(t)$ is the solution to the differential equation above with initial condition $y_0 = 2$?

Extra Credit (+3pts): Compute and plot the solution for $y(0) = 3.5$ using methods 3 and 4. Compare with the solutions already plotted from methods 1 and 2. Explain the "extra branch" when you plot the analytic solution. Hand in on a separate sheet to Prof. Peckham