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Diff. Equations and Lin. Alg.
Math 3280
Quiz 1, Spring 2020
B. Peckham

1. (3 pts) Verify that the function $\phi(x) = 2e^{3x} - 4e^{2x}$ is a solution to the differential equation $y''(x) - 5y'(x) + 6y(x) = 0$. Show your work.

$\phi'(x) = 6e^{3x} - 8e^{2x}$, $\phi''(x) = 18e^{3x} - 16e^{2x}$

Sub ϕ for y: $y'' - 5y' + 6y = (18e^{3x} - 16e^{2x}) - 5(6e^{3x} - 8e^{2x}) + 6(2e^{3x} - 4e^{2x})$

$= e^{3x}(18 - 30 + 12) + e^{2x}(-16 + 40 - 24) = 0e^{3x} + 0e^{2x} = 0$ ✓

$\therefore \phi$ is a soln.

2. Consider the differential equation $\frac{dy}{dt} = y^3 t$.

(a) (3 pts) Find the general solution. Show your work.

$\frac{y'}{y^3} = t \Rightarrow y^{-3}dy = t\,dt \Rightarrow \boxed{\frac{y^{-2}}{-2} = \frac{t^2}{2} + c}$ implicit soln.

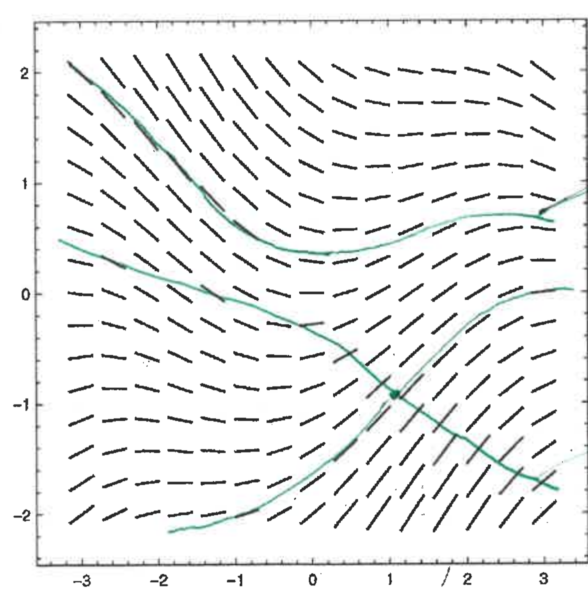
(b) (1 pt) What is the particular solution corresponding to $y(0) = 1$?

$y(0) = 1 \Rightarrow \frac{1^{-2}}{-2} = \frac{0^2}{2} + c \Rightarrow c = -\frac{1}{2}$. $\therefore \boxed{\frac{y(t)^{-2}}{-2} = \frac{t^2}{2} - \frac{1}{2}}$ (implicit soln)

(c) (+1 pt Extra Credit) What is the domain of the solution corresponding to the initial condition $y(0) = 1$? Solve for y: $\frac{1}{y^2} = -t^2 + 1 \Rightarrow y^2 = \frac{1}{1-t^2} \Rightarrow y = \pm\sqrt{\frac{1}{1-t^2}}$

$y(0) = 1 \Rightarrow +$ branch, $y = +\sqrt{1/(1-t^2)} \Rightarrow -1 < t < 1$.

3. (3pts) The slope field provided below is for the differential equation $dy/dx = f(x, y)$, where the formula for $f(x, y)$ is not given. Condider the initial value problem given by $dy/dx = f(x, y)$ and $y(1) = -1$. Sketch and label three functions on the graph that satisfy: (a) the differential equation, but not the initial condition, (b) the initial condition, but not the differential equation, (c) both the differential equation and the initial condition



a (or any function that lines up with hash marks, but does not go through (1, -1).)

c

b (or any curve (function) through (1, -1) that is not the same as c.)