Name A.K.

Diff. Equations and Lin. Alg.
Math 3280
Quiz 2, Spring 2020
B. Peckham

1. (2 pts) Consider the following differential equations. For each equation, state whether it is separable, linear, both or neither? (Do not solve.)

$y' = ye^x$ Separable and linear

$y' = xe^y$ Separable, but not linear

2. (4 pts) Use the first order linear differential equations technique (using an integrating factor) to obtain the general solution to the following differential equation and the solution that satisfies the initial value problem. Show your work and clearly indicate your final answer. $\frac{dy}{dx} = 3y + e^{4x}, y(2) = 1.$

Rewrite: $y' - 3y = e^{4x}$ int. factor $e^{\int -3\,dx} = e^{-3x}$

Multiply through: $e^{-3x}y' - 3ye^{-3x} = e^{4x}e^{-3x}$

ie, $(e^{-3x}y)' = e^x$

Integrate $\int (e^{-3x}y)'\,dx = \int e^x\,dx$

ie, $e^{-3x}y = e^x + C$

$y(x) = e^{4x} + Ce^{3x}$

$y(2) = 1 \Rightarrow y(2) = 1 = e^8 + Ce^6$

$\Rightarrow \frac{1-e^8}{e^6} = C$, so

$y(x) = e^{4x} + \frac{1-e^8}{e^6}e^{3x}$

3. (3 pts) Consider the differential equation $\frac{dP}{dt} = (4t - P)^3$. Make the substitution $u = 4t - P$ to eliminate the variable P. Write the differential equation in the new variable u; keep the independent variable as t. Show your work. Do not solve the differential equation. Extra Credit (+1 pt) Is this substitution useful in solving the differential equation? Why?

$u = 4t - P \Rightarrow \frac{du}{dt} = 4 - \frac{dP}{dt} \Leftrightarrow \frac{dP}{dt} = 4 - \frac{du}{dt}$

Sub into d.e.: $4 - \frac{du}{dt} = u^3$

Optional simplification: $\frac{du}{dt} = 4 - u^3$

E.C. This is separable, and the original d.e. was not separable or linear, so yes this sub is useful.

4. (1 pt) Write a differential equation for the temperature inside your apartment while you are at UMD going to class, and the heat is off. Assume the rate of change of the temperature inside is proportional to the difference between the indoor temperature and the outdoor temperature. Assume the outdoor temperature is constant. Define your variables and constants. Do not solve the differential equation.

Let $T(t)$ be the indoor temp at time t

T_0 be the (constant) outdoor temp.

$\frac{dT}{dt} = k(T - T_0)$ (k is a proportionality constant.)