Name $AK.$

Diff. Equations and Lin. Alg.
Math 3280
Quiz 4, Spring 2020
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Directions: Do all problems. Label axes and scale for all graphs. Use full notation for all general solutions and particular solutions (that is, write y(x)= ... rather than just y=... for solutions to y'=f(x,y)).

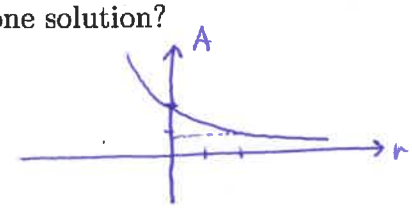
1. Consider the initial value problem $\frac{dA}{dr} = -1/2A$, $A(0) = 2$.

(a) Find an analytic solution "by inspection." Use your solution to determine $A(2)$. Give an exact and a decimal approximation (using a calculator).

$A(r) = Ce^{-\frac{1}{2}r}$ $\quad A(0) = 2 \Rightarrow 2 = Ce^{-\frac{1}{2}\cdot 0} \Rightarrow C = 2 \Rightarrow A(r) = 2e^{-\frac{1}{2}r} \Rightarrow$ $\boxed{A(2) = 2e^{-\frac{1}{2}\cdot 2} \approx .736}$

(b) Find a sketch of a solution by first drawing a phase line, then sketching a solution which is "consistent with the phase line." Estimate $A(2)$ from your solution sketch. What is the fate of this one solution?

$\Rightarrow$



$\boxed{A(2) \approx .9}$

or almost any answer between 0 and 2.

2. Consider initial value problem $\frac{dP}{ds} = -Ps^2/2$, $P(0) = 2$. Find or estimate $P(1)$ using the follwing techniques INDEPENDENTLY. For the analytic solutions, give an exact answer for $P(1)$ and a numerical approximation (using a calculator). Implicit solutions are acceptable if labelled.

(a) Find an analytic solution by separation of variables. What is $P(1)$?

$\int \frac{1}{P}\,dP = \int -\frac{s^2}{2}\,ds \Rightarrow \ln|P| = -\frac{s^3}{6} + C \Rightarrow |P| = e^{-\frac{s^3}{6} + C} = ke^{-\frac{s^3}{6}},\ k>0$

$P(0) = 2 > 0 \Rightarrow |P(s)| = P(s) \therefore P(s) = ke^{-\frac{s^3}{6}}$. $P(0) = 2 \Rightarrow k = 2$, so $\boxed{P(s) = 2e^{-s^3/6}}$ and $\boxed{P(1) = 2e^{-\frac{1}{6}}}$

(b) Find an analytic solution using the first order linear technique. What is $P(1)$?

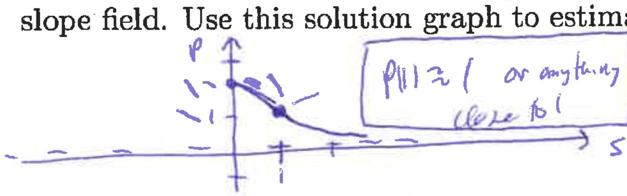
Rewrite $\frac{dP}{ds} + \frac{s^2}{2}P = 0 \Rightarrow \rho(s) = e^{\int \frac{s^2}{2}ds} = e^{\frac{s^3}{6}}$

$e^{s^3/6}\,dP/ds + e^{s^3/6}\cdot s^2/2 \cdot P = 0\,e^{s^3/6}$, ie $(e^{\frac{s^3}{6}}\cdot P)' = 0 \Rightarrow e^{\frac{s^3}{6}}P = C$ $\;$ or $P(s) = Ce^{-s^3/6}$ as in (a)

$\boxed{P(s) = 2e^{-\frac{s^3}{6}}}$ $\boxed{\text{and } P(1) = 2e^{-\frac{1}{6}} \approx 1.69}$

(c) Sketch a slope field and graph the solution corresponding to $P(0) = 2$ on the slope field. Use this solution graph to estimate $P(1)$. Label axes and scale.

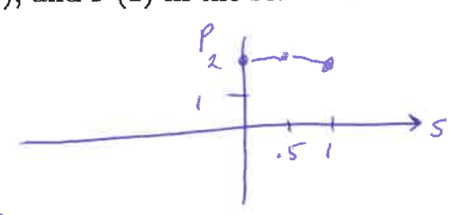
$-\frac{Ps^2}{2} = 0$ if $s = 0$ or $P = 0$



$\boxed{P(1) \approx 1 \text{ (or anything close to 1)}}$

s	P	dP/ds
± 1	1	$-\frac{1}{2}$
± 1	2	-1
$\pm\frac{1}{2}$	2	$-1/8$

(d) Use Euler's (numerical) method with a step size of $h = 0.5$ to estimate $P(1)$. Use $P(0) = 2$. Plot the Euler estimates for $P(0), P(.5)$, and $P(1)$ in the solution space.



n	s_n	P_n
0	0	2
1	.5	$2 + (.5)\left(-2\cdot\frac{0^2}{2}\right) = 2 + 0 = \underline{2}$
2	1	$2 + .5\left(-2\cdot\frac{(.5)^2}{2}\right) = 2 + \left(-\frac{1}{8}\right) = 1\frac{7}{8} = \underline{\frac{15}{8}} \approx P(1) \approx 1.875$

s