Name A. K.

Diff. Equations and Lin. Alg.
Math 3280
Quiz 5, Spring 2020
B. Peckham

1. (3 pts) Use USING GAUSSIAN ELIMINATION (row reduction operations) on the "augmented matrix" to convert it to row echelon form to find all solutions to the following system. Write your answer in vector form.

$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 2 & 4 \\ 2 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R2-2R1} \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 2 & 4 \\ 0 & 0 & -3 & -7 \end{array}\right]$$

So x_2 is free: Let $x_2 = t$. 2nd eqn $\Rightarrow -3x_3 = -7 \Rightarrow x_3 = \frac{-7}{-3} = \frac{7}{3}$

1st eqn $\Rightarrow 1 \cdot x_1 + \frac{1}{2}x_2 + 2x_3 = 4$, or $x_1 = 4 - \frac{1}{2}x_2 - 2x_3 = 4 - \frac{1}{2}t - 2\frac{7}{3} = \frac{12-14}{3} - \frac{1}{2}t = \underline{\frac{-2}{3} - \frac{1}{2}t}$

Vector form: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2/3 \\ 0 \\ 7/3 \end{bmatrix} + t\begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}$

2. Consider the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 2 \end{bmatrix}$.

(a) (2pts) Compute $\det(A)$. Show your work.

Expand along 1st col:

$$\det(A) = 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 0 & 2 \\ 2 & 2 \end{vmatrix} + 0\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix}$$

$$= 1 \cdot (2-4) + 0 + 0 = \underline{\underline{-2}}$$

(b) (3pts) **Use row reduction** to compute A^{-1}.

$$[A : I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R3-2R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & -2 & 1 \end{array}\right] \xrightarrow{R1+1R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R2+1R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -2 & 0 & -2 & 1 \end{array}\right] \xrightarrow{R3\left(-\frac{1}{2}\right)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -\frac{1}{2} \end{bmatrix}$$

(c) (2pts) Use your answer for (b) to solve $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Do not solve using row reduction (other than computing A^{-1} in (b).)

$$\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} \end{bmatrix} \qquad \left(\text{Check: } A\begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \checkmark\right)$$

3. (2 pts extra credit) Write down a system of (linear) equations that has 3 unknowns and whose solution has 2 free variables.

$x_1 + 0x_2 + 0x_3 = 0$ or many other choices of 1 equation, 3 unknowns.