Name A.K.

Diff. Equations and Lin. Alg.
Math 3280, B. Peckham
Quiz 6EC, Fall 2020

1. Let $S = \left\{ \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix} \right\}$. Show directly from the definition of linearly independent and span (not just computing a determinant) that

Solve $c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ (*)

(a) (1pts) S is a linearly independent set

Equivalently, $\begin{bmatrix}1&2\\1&1\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$. Row reduction: $\left[\begin{array}{cc|c}1&2&0\\1&1&0\end{array}\right] \xrightarrow{R2-R1} \left[\begin{array}{cc|c}1&2&0\\0&-1&0\end{array}\right]$ (row echelon form)

Backsolve: 2^{nd} eqn is $0c_1 - 1c_2 = 0 \Rightarrow c_2 = 0$

1^{st} eqn is $1c_1 + 2c_2 = 0 \Rightarrow 1\cdot c_1 + 2\cdot 0 = 0 \Rightarrow c_1 = 0$

Since the only solution to (*) is $c_1 = 0, c_2 = 0$, then S is a linearly independent set. //

(b) (1pts) the span of S is all of $\Re^2$. Solve $c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$, where x, y are arbitrary real #'s.

Equivalently: $\begin{bmatrix}1&2\\1&1\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$ Row reduction: $\left[\begin{array}{cc|c}1&2&x\\1&1&y\end{array}\right] \xrightarrow{R2-R1} \left[\begin{array}{cc|c}1&2&x\\0&-1&y-x\end{array}\right]$ (row echelon form)

Backsolve: $0c_1 - 1c_2 = y - x \Rightarrow \underline{c_2 = x - y}$

$1\cdot c_1 + 2\cdot c_2 = x \Rightarrow c_1 + 2(x-y) = x \Rightarrow \underline{c_1 = x - 2x + 2y = 2y - x}$

Since we can solve for ~~c_1~~ and c_2 for any x and y, span(S) $= \mathbb{R}^2$. //

2. (2 pts) Find a basis for the set of solutions to the following system. (Hint: first write down the solution to this system and put in vector form.) $\begin{bmatrix}1&2&3&0&5\\0&1&4&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$.

Augmented matrix: $\left[\begin{array}{ccccc|c}1&2&3&0&5&0\\0&1&4&0&1&0\end{array}\right]$ Already in row echelon form, so read off answer.

Free variables: $x_3 = t$, $x_4 = s$, $x_5 = r$,

Backsolve for leading variables: $0x_1 + 1x_2 + 4x_3 + 0x_4 + 1x_5 = 0$

$\Rightarrow x_2 = -4x_3 - x_5 = -4t - r$

$1x_1 + 2x_2 + 3x_3 + 0x_4 + 5x_5 = 0 \Rightarrow x_1 = -2x_2 - 3x_3 - 5x_5 = -2(-4t - r) - 3t - 5r = 5t - 3r$

$\Rightarrow$ Sln is $\vec{x} = (x_1, x_2, x_3, x_4, x_5) = (5t - 3r, -4t - r, t, s, r) = t(5, -4, 1, 0, 0) + s(0, 0, 0, 1, 0) + r(-3, -1, 0, 0, 1)$

So basis $\mathcal{B} = \left\{ \begin{bmatrix}5\\-4\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}-3\\-1\\0\\0\\1\end{bmatrix} \right\}$

3. (1 pt) Let $W = \{(a, b, c, d) \in \Re^4 : a + b + d = 1\}$. Define a matrix A, and a vector $\vec{b}$ such that $W = \{\vec{x} \in \Re^4 : A\vec{x} = \vec{b}\}$.

$\begin{bmatrix}1&1&0&1\end{bmatrix}\begin{bmatrix}a\\b\\c\\d\end{bmatrix} = [1]$, so $\boxed{A = [1\ 1\ 0\ 1],\ \vec{b} = [1]}$