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Diff. Equations and Lin. Alg.
Math 3280, B. Peckham
Quiz 7, Spring 2020

1. (3pts) Find the general solution to $y'' - 2y' - 15y = 0$. Hint: try functions of the form e^{rx}.

$y = e^{rx} \Rightarrow y' = re^{rx}, y'' = r^2e^{rx}$. Plug in: $r^2e^{rx} - 2re^{rx} - 15e^{rx} = e^{rx}(r^2-2r-15)$
$= e^{rx}(r-5)(r+3) = 0$ if $r = 5, -3$

$\therefore y(x) = c_1e^{5x} + c_2e^{-3x}$

2. (2pts) Find the general solution to $y'' - 2y' - 15y = 3e^{2x}$. Hint: one solution is $-\frac{e^{2x}}{5}$. You may use your solution from problem 1.

$$y(x) = y_c(x) + y_p(x) = \underbrace{c_1e^{5x} + c_2e^{-3x}}_{y_c \text{ from } \#1} \underbrace{- \frac{e^{2x}}{5}}_{y_p \text{ given}}$$

3. (2pts) Suppose that it is known that the general solution to some second order differential equation is $y(x) = c_1e^x + c_2e^{5x}$. Assume you are now given initial conditions: $y(0) = 1, y'(0) = 2$. Determine a matrix A and a vector $\vec{b}$ so that the solution to $A\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \vec{b}$ gives the constants c_1 and c_2. Do not solve for c_1 or c_2.

$y(x) = c_1e^x + c_2e^{5x} \Rightarrow y'(x) = c_1e^x + 5c_2e^{5x}$

So $y(0) = c_1 + c_2 = 1$ and $y'(0) = c_1 + 5c_2 = 2$

ie, $\begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ So $A = \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

4. (3pts) Find a basis for the set of solutions to $y'' - 8y' + 16y = 0$.

Try $y = e^{rx} \Rightarrow y' = re^{rx}, y'' = r^2e^{rx}$. Plug in and factor out e^{rx}: $e^{rx}(r^2-8r+16) = 0$

So $r^2 - 8r + 16 = (r-4)^2 = 0 \Rightarrow r = 4$. Double root $\Rightarrow e^{4x}, xe^{4x}$ are two (indep.) slns.

So a basis is $\{e^{4x}, xe^{4x}\}$

5. (EC 2pts) Show the following set of functions (defined on $\Re$) is linearly independent: $\{e^{2x}, e^{3x}\}$. Work directly from the definition of linear independence, not just a determinant. Hint: Set a linear combination of these two functions equal to 0. Differentiate both sides to get a second equation. Evaluate both equations at $x = 0$. Solve for the constants.

Assume $c_1e^{2x} + c_2e^{3x} = 0$
Differentiate: $2c_1e^{2x} + 3c_2e^{3x} = 0$

@ $x=0$: $\begin{cases} c_1 + c_2 = 0 \\ 2c_1 + 3c_2 = 0 \end{cases}$

Many ways to solve. I will use row reduc.

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 3 & 0 \end{array}\right] \xrightarrow{R2 - 2R1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right]$$

2nd eq $\Rightarrow c_2 = 0$. Then 1st eq. $\Rightarrow 1c_1 + 1\cdot 0 = 0$
$\Rightarrow c_1 = 0$

Since c_1 and c_2 must both be zero, $\{e^{2x}, e^{3x}\}$ is lin. indep.