Math 3280, Differential Equations with Linear Algebra

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Brief Course Summary

Differential Equations:

1. Analytic Solutions

Order	Dim	Type	Solution Technique
1	1	Simplest nontrivial: $y' = ay$ Separable (nonlinear) $y' = f(x)g(y)$ linear: $y' + p(x)y = q(x)$ Nonlinear, not sep.: $y' = f(x, y)$	Every technique in the course!! Separation of Variables integrating factor No general technique; maybe substitutions
2	1	Linear, const coeff, homogeneous $y'' + ay' + cy = 0$ or $L[y] = 0$ ———————— Linear, const coeff, nonhomogeneous: $y'' + ay' + by = g$, that is, $L[y] = g$	Try e^{rx}; 3 root cases Laplace transforms; 3 root cases Convert to 1st order system; know 2 cases: distinct real, complex (not repeated) ————————- $y = c_1y_1 + c_2y_2 + y_p$ y_p: -Lucky guess/undet. coeffs/Annihilators -Variation of parameters: $y_p = v_1y_1 + v_2y_2$ -Laplace transforms - esp. for g discts.
k	1	Linear, const coeff, homog Linear, const coeff, nonhomog	Extend techniques for $k = 2$ Extend techniques for $k = 2$
1	n	Linear const coeff homog systems $\vec{x}\,' = A\vec{x}$	Eigenvalues/eigenvectors: use $e^{\lambda t}\vec{v}$; 2 cases for 2D (dbl roots not covered)
2	1	Linear nonconst coeff, nonhomogeneous: $y'' + a(x)y' + b(x)y = g(x)$	No general technique (but $y_1 \to y_2 = vy_1$) and y_p from var of pars
k	1	Nonlinear: $y^{(k)} = f(y^{(k-1)}, ..., y', y, x)$	No general technique
1	n	Nonlinear systems: $\vec{x}\,' = \vec{f}(\vec{x}, t)$	No general technique
k	n	Nonlinear higher-order systems	Convert to first order system

2. Qualitative Solutions

   (a) 1D Automomous only ($y' = f(y)$): Equilibria, phase line, vector field; sketch solutions consistent with phase line

   (b) 1D ANY ($y' = f(y, x)$): Slope field

   (c) 2D Automomous only ($\vec{x}\,' = \vec{f}(\vec{x})$): equilibria, phase plane, vector field; sketch $x_1(t)$ and/or $x_2(t)$ from curve in phase plane

3. Numerical Solutions

   (a) ANY!!!!: Euler's method (not covered: Runge-Kutta, ...)

   (b) ANY!! In Mathematica: NDSolve, Streamplot

4. Models/applications - construct given verbal information (for example, "X is proportional to Y")

   (a) Exponential growth (population), decay (radioactive decay)

   (b) Heating/Cooling

   (c) Falling object: $mv' = F_{gravity} + F_{friction}$

   (d) Mixing x' =rate in - rate out.

   (e) Logistic population growth: $y' = ay - by^2$

   (f) Spring/mass system - horizontal or vertical: comes from $F = ma = my''$.

   (g) Population models (predator-prey systems)

(other side for Linear Algebra)

Linear Algebra

1. Solve $A\vec{x} = \vec{b}$ (Row reduction, echelon forms, (0, 1, ∞: free params.))
2. For $n \times n$: Det(A), A^{-1} (if Det$(A) \neq 0$), eigenvalues, eigenvectors ($A\vec{x} = \lambda\vec{x}$)
3. Vector Space/subspace, basis, linearly independent, span, dimension
4. Linear transformation - "kernel" or "null space"
   Examples: D, integration, L (for lhs of linear differential equation), Laplace transform, multiply by matrix A, Annihilators
5. Theorems:
   (a) The following are vector subspaces (of a known vector space):
      i. Solutions to $A\vec{x} = \vec{0}$ (Dimension is number of free variables after row reduction.)
      ii. Solutions to $L[y] = 0$ (dimension depends on order of L.)
      iii. The set of eigenvectors for a specific eigenvalue of a matrix A (dimension is often one, never bigger than the eigenvalue multiplicity, never zero)
      iv. The span of any set of vectors
   (b) Differences of solutions to $A\vec{x} = \vec{b}$ are solutions to $A\vec{x} = \vec{0}$.
   (c) Differences of solutions to $L[y] = g$ are solutions to $L[y] = 0$.