**Differential Equations and Linear Algebra**
**Math 3280**
**Lab #2: Introduction to Differential Equations**
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**Directions:** Turn in a written lab report dealing with the tasks below. Your report should include goals, description of the procedures you used in the lab, *Mathematica* output with comments, and conclusions, as indicated from the "Lab Procedures and Guidelines" file. The writeup may be typed as *Mathematica* text, added neatly by hand to *Mathematica* output, or done with a word processor. Grading: Goals (G) 1, Procedures (Pro) 1, *Mathematica* tasks (Ma) 14 (details below), Conclusions (C) 2, Presentation and Organization (P+O) 2, Total 20.

Before starting the lab, reread the Mathematica Notes/Hints available from the link accompanying Lab 1. Do the following tasks with the help of *Mathematica.*

1. (8 pts) Slope field and qualitative graphical solution with varying amounts of help from *Mathematica*. A sheet will be provided in lab for parts (a), (b), (c), or you can download and print this sheet from the Course homepage.

   (a) On the sheet provided, sketch a slope field *by hand* for the differential equation $\frac{dx}{dt} = 1-0.5t$, and sketch *by hand* the solution to the initial value problem corresponding to the initial conditions $x(0) = 1$.

   (b) On the sheet provided, on the slope field printed using the VectorPlot command, *by hand*, sketch the solution to the differential equation corresponding to the initial condition $x(0) = 1$.

   (c) On the sheet provided, on the StreamPlot, by hand, sketch or highlight the solution corresponding to the initial condition $x(0) = 1$.

   (d) Create the slope field for $\frac{dx}{dt} = 1 - 0.5t$ with the following *Mathematica* command: VectorPlot[{1, 1-0.5 t}, {t,-2,2}, {x,-2,2}]. Why does the vector field {1,1-0.5t} correspond to the slope field for the differential equation? Add the option VectorStyle → "Segment" to plot the slope marks without vector arrows.

   (e) Display the slope field and stream plot graphs simultaneously using the Show command. (Hint: name each plot first.)

   (f) Comment on the effort and accuracy of each of the above methods for obtaining a plot of the particular solution. Assume the provided sheet had not been given to you.

   (g) Experiment with parameters and the Manipulate command: use the VectorPlot command to graph a slope field for $\frac{dx}{dt} = 1 - at$, where $a$ is "manipulated." Allow $a$ to vary from at least $-2$ to 2. Add another manipulated variable to manipulate the size of the plot window. Show the output for one value of $a$ and one value of the windowsize.

2. (3pts) Calc II Template. A template for solving Calc II type initial value problems is provided with the Mathematica Notes provided with Lab 1. Modify this template to find a formula (analytic) solution to the initial value problem:

$$\frac{dx}{dt} = 1 - 0.5t, \quad x(0) = 1.$$

   Plot your solution. Compare it with the plot you obtained for the this same initial value problem using the slope field in part 1 of this lab.

3. (3pts) Verifying solutions to initial value problems. Consider the function:

$$f(x) = 3e^{-x/2}\cos(3x) + 2$$

(a) Define and Plot $f(x)$ over the interval $0 \leq x \leq 2\pi$. Example: Plot[$x$ˆ2, {x,-1,1}]. More plotting features can be found by looking up the Plot command with the online Help.

(b) Repeat part (a) for $f'(x)$ and $f''(x)$.

(c) Obtain a simultaneous plot of $f, f'$, and $f''$ over the interval $0 \leq x \leq 2\pi$. Example of Plot command: Plot[{$x$ˆ2, .3 x, 1-x}, {x,-1.5,1}].

(d) Use Mathematica to show that $f(x)$ satisfies the initial value problem:

$$y''(x) + y'(x) + \frac{37}{4}y(x) = \frac{37}{2}, \qquad y(0) = 5, y'(0) = -\frac{3}{2}$$

Hint: Compute the left hand side of the differential equation with the $f(x)$ given above replacing the unknown function in the differential equation, subtract the right hand side, and see whether it simplifies to zero. (Use the Simplify[ ] command.) If it does, the differential equation is "satisfied by $f(x)$." Do the same for the two initial conditions: see if the $f(x)$ given above satisfies the two initial conditions.