Math 3280: Differential Equations with Linear Algebra, Sample Test 2, B. Peckham

1. Obtain the general solutions to the following differential equations.

   (a) (7 pts) $y'' - 2y' - 3y = 0$.

   (b) (9 pts) $y'' - 2y' + 2y = 2x$. Hint: Use the fact that *one* solution is $y_p(x) = x + 1$.

2. (6 pts) Given that $y(x) = c_1 \cos(2x) + c_2 \sin(2x)$ is the general solution to $y'' + 4y = 0$. Find the specific solution to the differential equation which satisfies the initial conditions $y(0) = 1$ and $y'(0) = 1$.

3. Evaluate the following determinants. Show your work.

   (a) (3 pts) $\begin{vmatrix} 3 & -2 \\ -3 & -2 \end{vmatrix}$ (7 pts) $\begin{vmatrix} 0 & 3 & -1 & 2 \\ 1 & 4 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -2 & 2 \end{vmatrix}$

4. (8 pts) Solve the following linear system USING GAUSSIAN ELIMINATION. Leave your answers as exact fractions - not calculator approximations.

$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & 3 & 2 \\ 4 & 1 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$$

5. (10 pts) Find a basis for the space of solutions to $\begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 3 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

6. (8 pts) Let $A = \begin{pmatrix} 3 & 9 \\ -1 & 1 \end{pmatrix}$. Find A^{-1} using the Gauss-Jordan (row reduction) technique. Check your answer.

7. TRUE-FALSE. Justify your answer briefly. A formal proof is not required.

   (a) (5 pts) $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right\}$ is a basis for $\Re^2$.

   (b) (5 pts) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \right\}$ is a basis for $\Re^3$.

   (c) (7 pts) $\{1 + x, x, x^2\}$is a basis for $\mathcal{P}_2 \equiv \{a_0 + a_1 x + a_2 x^2 : a_i \in \Re\}$?

8. (7 pts) Let $S = \{\mathbf{x} = (x_1, x_2) \in \Re^2 : x_2 = x_1^2\}$. Is S a vector subspace of $\Re^2$? Justify briefly.

9. (6pts) Write down the system of equations needs to be solved in order to show DIRECTLY FROM THE DEFINITION of linear independence that the set of vectors $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\}$ is linearly independent in $\Re^3$. DO NOT SOLVE the system, but state what properties of the solution would determine whether the three vectors are lineaerly independent in $\Re^3$.

10. (12 pts) Consider the following subset T of $\Re^3$. PROVE that T is a vector subspace of $\Re^3$.

$$T = \{(x_1, x_2, x_3) : x_2 = x_3\}$$