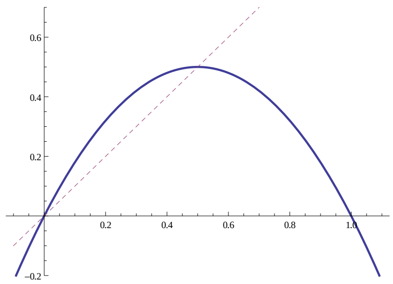
Name____	Math 5260: Dynamical Systems
ID # ____	Midterm 1, Fri. Oct. 18, 2013
Signature____	Prof. Bruce Peckham

Directions: Do all problems. Total is 50 points.

1. Consider the dynamical system defined by $x_{n+1} = f(x_n)$ where $f(x) = 2x(1-x)$. The graph of f is below.



   (a) (4pts) Determine all fixed points and their stabilities (attracting, repelling, or linearly neutral). Justify your answers with calculations.

   (b) (2pts) Determine the set of initial conditions for which the corresponding orbit stays bounded. Justify briefly either on the sketch above or with an analytic argument.

   (c) (2pts) Describe the fate of all bounded orbits.

2. (4pts) Sketch the graph of a continuous function which has an attracting fixed point and a repelling period-2 orbit.

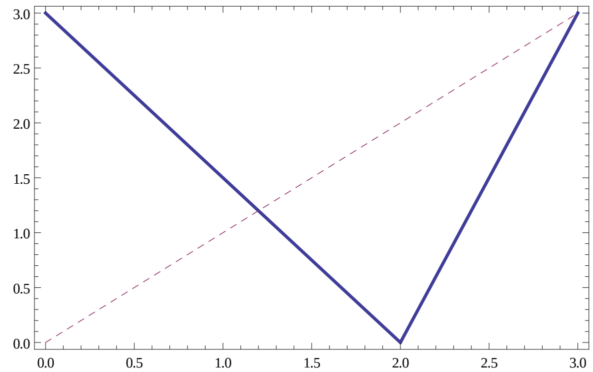
3. (3pts) Let $D(x) = 2x \pmod 1$. Define $S : [0,1) \to \Sigma$ (Σ is the space of sequences of 0's and 1's) by $S(x) = (s_0 s_1 s_2 ...)$ where

$$s_j = \begin{cases} 0 & \text{if } D^j(x) \in [0, .5) \\ 1 & \text{if } D^j(x) \in [.5, 1) \end{cases}$$

What is $S(\frac{2}{3})$? Explain briefly.

4. (3 pts) If a continuous map of the unit interval has a (prime) period 30 orbit, must it have an orbit of (prime) period 28? Explain briefly?

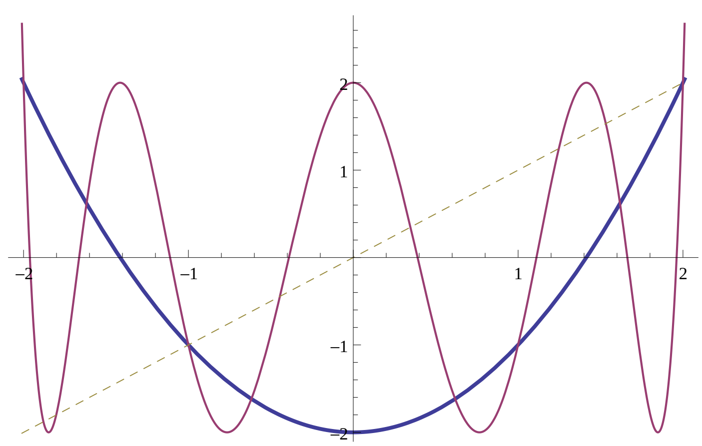
5. (3pts) How many prime period-11 *orbits* are there for the map $x^2 - 2$? Explain briefly how you determined your answer.

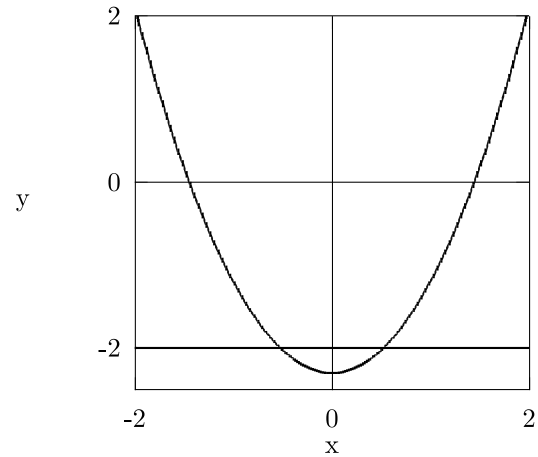


6. (4pts) Given the above graph of f, sketch the graph of f^2 (that is, $f \circ f$) on the same diagram.

7. (3pts) List the three properties necessary for a function $f : X \to X$ to be chaotic. (No formal definitions necessary; just list the names of the three properties.)

8. (3pts) The graphs of Q_{-2} and Q^3_{-2} are given below for $Q_{-2}(x) = x^2 - 2$. Label on the diagonal any three points A, B, C which are on the same prime period-3 orbit and for which $A \to B \to C \to A$ under iteration of the map Q_{-2}.





9. (3pts) Let $f(x)$ be defined by the graph above. Let $I_0 = [-2, -a], I_1 = [a, 2]$, where a is the x value of the right-hand intersection of the graph with the line $y = -2$. Define $I_{ij} = \{x \in [-2, 2] | x \in I_i, f(x) \in I_j\}$. Carefully sketch and label the set I_{01}.

10. (4pts) Let $t = (000\overline{0})$ and $s = (111\overline{1})$ be points in the sequence space Σ. Find a new point $z \in \Sigma$ and an integer N which satisfy $d(z, s) < 0.1$ and $d(\sigma^N(z), t) < 0.1$.

11. (3pts) List all prime period-3 *orbits* for $\sigma : \Sigma \to \Sigma$. (As in the text, Σ is the space of sequences of 0's and 1's, and σ is the shift map which drops the first term in the sequence. Indicate which the order in which points map to each other on these orbits.)

12. Consider the family of maps given by: $f_c(x) = x^2 + cx$.

   (a) (3pts) Find all fixed points for all maps in this family.

   (b) (3pts) Determine all intervals in the parameter c for which f_c has an attracting fixed point.

   (c) (3pts) Sketch a bifurcation diagram (in the phase $\times$ parameter space) including all fixed points with solid lines representing those that are attracting and dashed lines representing those that are repelling.

13. Extra Credit: (3pts). Consider the space Σ consisting of sequences 0's and 1's with the "usual" metric defined by $d(s,t) = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{2^i}$. Prove that if $s_i = t_i$ for $i = 0, 1, ..., n$, then $d(s,t) \leq \frac{1}{2^n}$.