MIDTERM TOPIC LIST
Dynamical Systems
Math 5260
Bruce Peckham
September 30, 2019
For midterm on Monday, Oct. 7, 2019.

In general, the midterm will cover any topics we covered in Chapters 1-12. The focus will be on "basic" material. Homework type questions from previous assignments will be emphasized. I will attempt to make the problems noncomputationally intensive. The following list of topics should give you a more specific idea of what kinds of questions will be asked.

1. Definitions to know:
    - Fixed point, periodic point, orbit, cycle, period, prime or least period
    - Attracting, superattracting, repelling, (linearly) neutral periodic point (orbit, cycle), weakly attracting, weakly repelling; use of chain rule in determining these adjectives
    - Phase portrait
    - Graphical analysis
    - Discrete dynamical system vs. continuous dynamical system
    - Bifurcation, incl. esp. saddle-node and period doubling (nondegeneracy conditions not necessary to memorize)
    - The sequence space Σ, the "usual" metric on Σ, the shift map σ.
    - Three properties of a chaotic system and all terminology used in the def. of the properties.
    - A *dense* in B for $A \subset B$.
    - Homeomorphism, topological conjugacy, topological semiconjugacy
2. Results to know:
    - Relationship between the shift map on the symbol sequence space and the quadratic map on the invariant Cantor set (for $c < -2$).
    - Sarkovkii's theorem, including Sarkovskii's ordering
    - "Negative Schwarzian Derivative Property" (which is true for any quadratic): Any attracting periodic orbit must attract a critical orbit.

3. Techniques to know:

   - Locating fixed and periodic pts/orbits analytically (for individual maps and for families of maps)
   - Locating period-n pts/orbits of f from graphs of f and f^n.
   - Interpreting orbit diagrams (identifying, for example, parameter values corresponding to maps with attracting orbits of a certain period, or saddle-node bifurcations or period-doubling bifurcations)
   - Determining stability of periodic orbits either analytically or from graphs.
   - Constructing a graph of f so that f has a point p that is, for example, a period-n point, and the derivative $(f^n)'(p)$ has a specificed value. (Hint: use the chain rule!)
   - Constructing the graph of iterates of f given the graph of f.
   - The construction of the invariant Cantor set Λ for $x^2 + c$ with $c < -2$.
   - The construction of an itinerary map.
   - Given a map and a change of variables, find the equation for the map in the new variables.
   - Recognizing a saddle-node and/or period doubling bifurcation from a sequence of graphs of a family of maps as a parameter changes
   - Determining the number of prime periodic orbits of each period for $x^2 - 2$ (equivalently $2x \pmod 1$, $4x(1-x)$, σ)
   - Determining the number of period-n windows for each n in the orbit diagram for the family $x^2 + c$.
   - Locating each period-n window in the orbit diagram for the family $x^2 + c$.

4. Proofs to know:

   - If p is a period-n point for f, then $f^{n\prime}(p) = f^{n\prime}(q)$ where q is any iterate of p (that is, $q = f^k(p)$ for some natural number k).
   - Prove that if f is continuous, f has a periodic point implies f has a fixed point.
   - Prove either direction of the Proximity Theorem.
   - Prove $\sigma : \Sigma \to \Sigma$ is chaotic. (Prove any or all 3 properties.)

5. Anything else we've covered that I think is easy.