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A Papercrafted Pattern on a Triply Periodic Polyhedron

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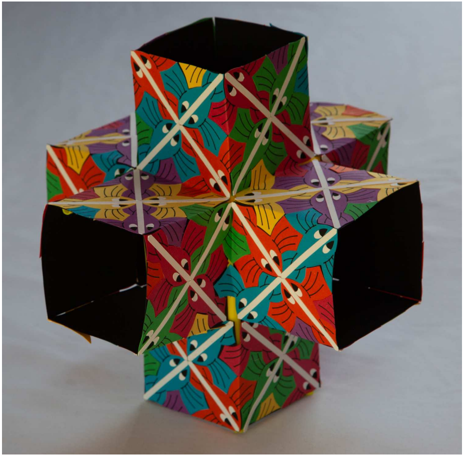
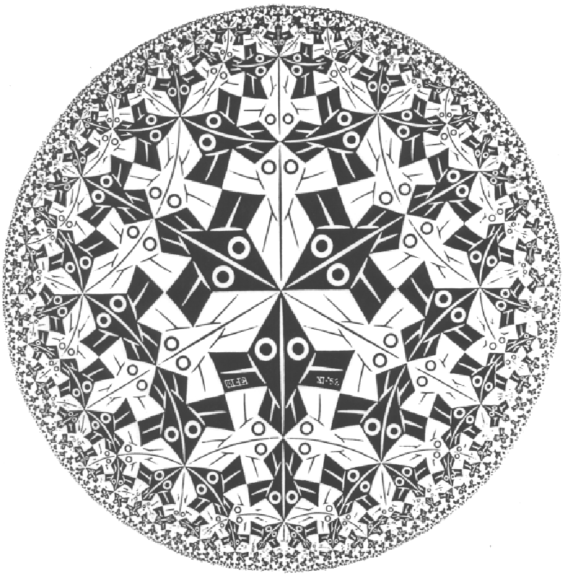


Outline

- Background and motivation
  - M.C. Escher's *Circle Limit I* and *Circle Limit III*
  - Triply periodic polyhedra
  - The previous polyhedron and its problems
  - The computer-controlled cutter/plotter
- The new papercrafted part of the triply periodic polyhedron
- Future work
- Contact information

Escher's Woodcut Circle Limit I



Problems Circle Limit I per Escher

1. The fish were not consistently colored along backbone lines — they alternated from black to white and back every two fish lengths.
2. The fish also changed direction every two fish lengths — thus there was no “traffic flow” (Escher’s words) in a single direction along the backbone lines.
3. The fish are very angular and not “fish-like”

Escher's Woodcut Circle Limit III

— solved the problems

Regular Triply Repeating Polyhedra

In 1926 H.S.M. Coxeter defined *regular skew polyhedra* (apeirohedra) to be infinite polyhedra repeating in three independent directions in Euclidean 3-space.

Coxeter denoted them by the extended Schläfli symbol $\{p, q \mid r\}$ which denotes the polyhedron composed of p-gons meeting q at each vertex, with regular r-sided polygonal holes.

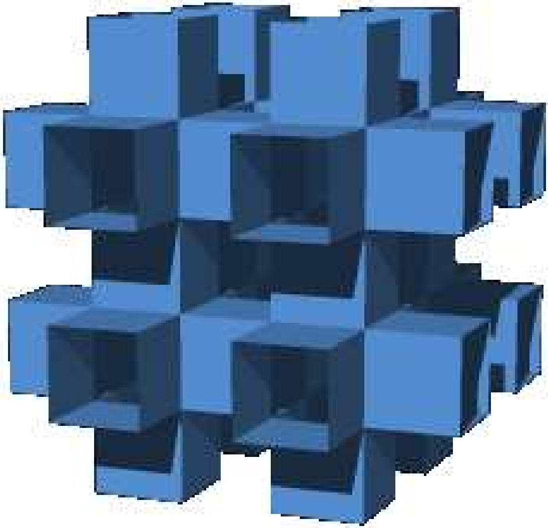
Coxeter and John Flinders Petrie proved that there are exactly three of them: $\{4, 6 \mid 4\}$, $\{6, 4 \mid 4\}$, and $\{6, 6 \mid 3\}$.

Since the sum of the vertex angles is greater than 2π, they are considered to be the hyperbolic analogs of the Platonic solids and the regular Euclidean tessellations $\{3, 6\}$, $\{4, 4\}$, and $\{6, 3\}$

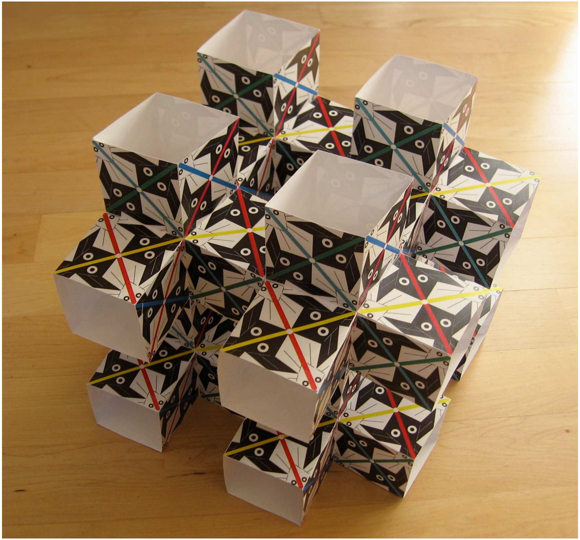
In 2012 Dunham was the first person to decorate those solids with Escher-inspired patterns.

The simplest regular skew polyhedron: $\{4, 6 \,|\, 4\}$

Also called the *Mucube* (for Multi-cube). It consists of invisible "hub" cubes connected by "strut" cubes, hollow cubical cylinders with their open ends connecting neighboring hubs.



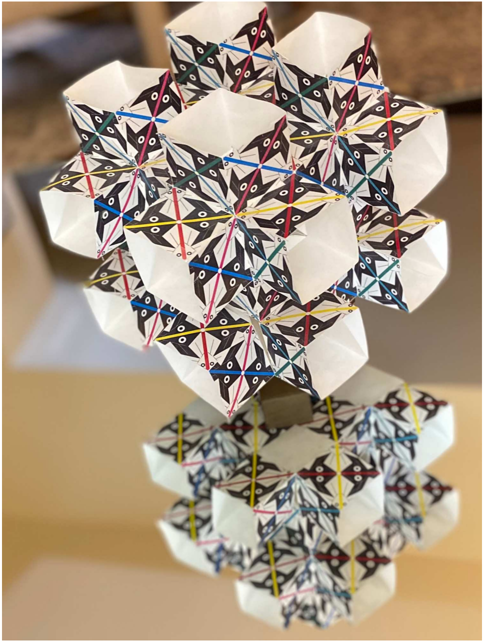
Dunham's patterned $\{4, 6 \,|\, 4\}$ with fish



Problems with Dunham's fish polyhedron

1. The same three problems Escher saw in *Circle Limit I*.
2. A fourth problem: the backbone lines of a particular color are not parallel — which can be seen in a mirror.

Dunham's fish polyhedron on a mirror

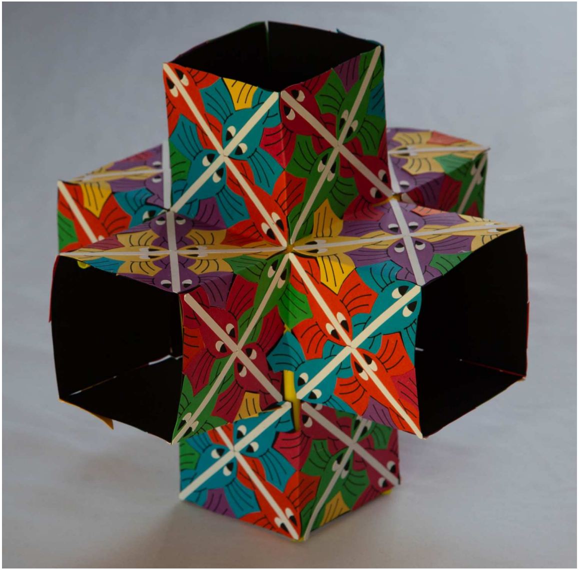


The Brother ScanNCut SDX225 cutter/scorer/plotter

Uses Floriani Craft 'N Cut software to create the input file.

Shier's new implementation

Fixes the first and third problems.



Shier's polyhedron on a mirror

Also fixes the fourth problem.

Future Work

- We would like to explore papercrafting patterns on other triply repeating polyhedra.
- Specifically, we would like to try creating a fish pattern on the $\{6, 6 \mid 3\}$ polyhedron which could also fix the second problem with Dunham's $\{4, 6 \mid 4\}$ polyhedron — so the fish all go the same direction along a backbone line.

Acknowledgements and Contact

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