A Papercrafted Pattern on a Triply Periodic Polyhedron

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Outline

- Background and motivation
  - M.C. Escher’s *Circle Limit I* and *Circle Limit III*
  - Triply periodic polyhedra
  - The previous polyhedron and its problems
  - The computer-controlled cutter/plotter
- The new papercrafted part of the triply periodic polyhedron
- Future work
- Contact information
Escher’s Woodcut Circle Limit I
Problems Circle Limit I per Escher

1. The fish were not consistently colored along backbone lines — they alternated from black to white and back every two fish lengths.

2. The fish also changed direction every two fish lengths — thus there was no “traffic flow” (Escher’s words) in a single direction along the backbone lines.

3. The fish are very angular and not “fish-like”
Escher’s Woodcut Circle Limit III

— solved the problems
Regular Triply Repeating Polyhedra

In 1926 H.S.M. Coxeter defined *regular skew polyhedra* (apeirohedra) to be infinite polyhedra repeating in three independent directions in Euclidean 3-space.

Coxeter denoted them by the extended Schl"afli symbol \{p, q | r\} which denotes the polyhedron composed of $p$-gons meeting $q$ at each vertex, with regular $r$-sided polygonal holes.

Coxeter and John Flinders Petrie proved that there are exactly three of them: \{4, 6 | 4\}, \{6, 4 | 4\}, and \{6, 6 | 3\}.

Since the sum of the vertex angles is greater than $2\pi$, they are considered to be the hyperbolic analogs of the Platonic solids and the regular Euclidean tessellations \{3, 6\}, \{4, 4\}, and \{6, 3\}.

In 2012 Dunham was the first person to decorate those solids with Escher-inspired patterns.
The simplest regular skew polyhedron: \( \{4, 6 \mid 4\} \)

Also called the *Mucube* (for Multi-cube). It consists of invisible “hub” cubes connected by “strut” cubes, hollow cubical cylinders with their open ends connecting neighboring hubs.
Dunham's patterned \{4, 6 | 4\} with fish
Problems with Dunham’s fish polyhedron

1. The same three problems Escher saw in *Circle Limit I*.

2. A fourth problem: the backbone lines of a particular color are not parallel — which can be seen in a mirror.
Dunham’s fish polyhedron on a mirror
The Brother ScanNCut SDX225 cutter/scorer/plotter
Uses Floriani Craft ’N Cut software to create the input file.
Shier’s new implementation

Fixes the first and third problems.
Shier’s polyhedron on a mirror
Also fixes the fourth problem.
Future Work

- We would like to explore papercrafting patterns on other triply repeating polyhedra.

- Specifically, we would like to try creating a fish pattern on the \( \{6, 6 | 3\} \) polyhedron which could also fix the second problem with Dunham’s \( \{4, 6 | 4\} \) polyhedron — so the fish all go the same direction along a backbone line.
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