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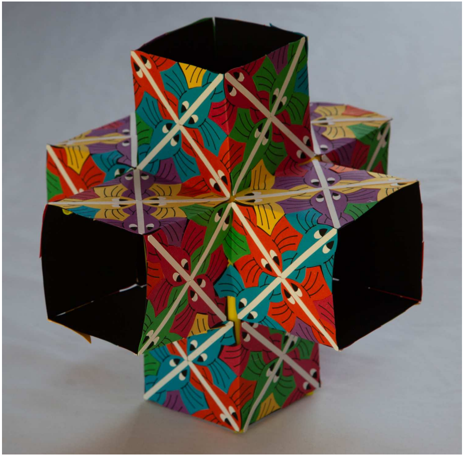
Papercrafted Mathematical Art

Douglas Dunham
Dept. of Computer Science
Univ. of Minnesota, Duluth
Duluth, MN 55812, USA

Lisa Shier
University of Maryland
Global Campus
New Market, AL 35761, USA



Outline

- Papercraft
- A papercrafted hyperbolic pattern of shells
- A papercrafted part of an infinite polyhedron
- Conclusions and future work
- Contact information

Papercraft

- Paper craft is the use of paper or card stock to create objects.
- We used a computer-controlled cutter/scorer/plotter to create mathematically inspired art works.
- Specifically, we used a Brother ScanNCut SDX225 papercrafting machine and Floriani Craft 'N Cut software to create input files for the pieces.

The Brother ScanNCut SDX225 cutter/scorer/plotter

A Papercrafted Hyperbolic Pattern of Shells

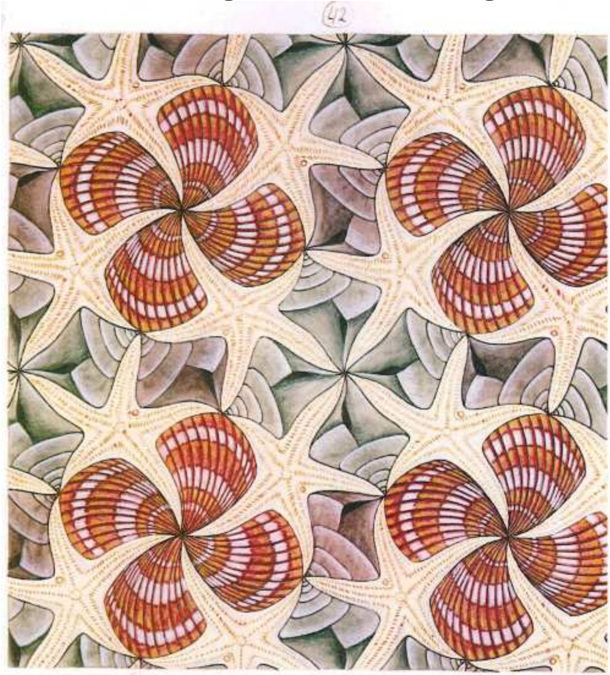
In 1941 Escher drew a Euclidean pattern of shells that had symmetry group $p4$ or 442 in orbifold notation.

In about 1990, Dunham designed a related hyperbolic pattern with symmetry group 552 (in orbifold notation) based on the regular $\{5, 5\}$ tessellation of the hyperbolic plane.

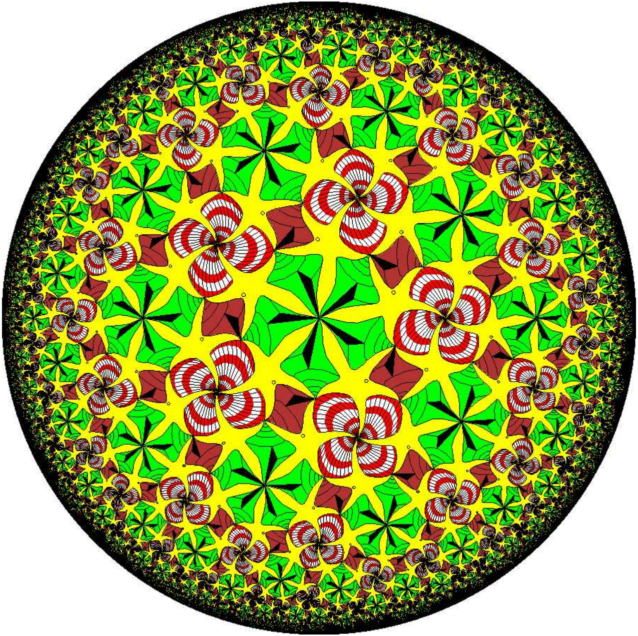
In 2019 Shier decided to implement that pattern using papercrafting, using the ScanNCut SDX225 cutter/scorer/plotter.

She made a couple of changes to Dunham's pattern. First she changed the colors of the "background" starfish to sea-star blue, the snails to pink, and the scallops to sunset red (a slight change). Second, by cutting out foam backing shapes, she raised scallops, conchs, and snails above the background starfish surface, giving the pattern a 3-dimensional effect.

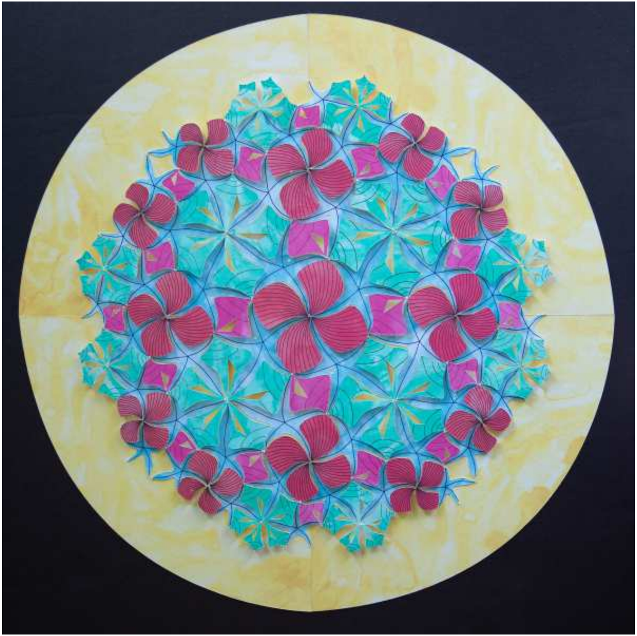
Escher's Regular Division Drawing 42



Dunham's hyperbolic 552 pattern of shells



Shier's papercrafted hyperbolic shell pattern



An oblique view showing the 3D effect

A Papercrafted Part of an Infinite Regular Polyhedron

In 1926 H.S.M. Coxeter defined *regular skew polyhedra* to be infinite polyhedra repeating in three independent directions in Euclidean 3-space.

Coxeter denoted them by the extended Schläfli symbol $\{p, q \mid r\}$ which denotes the polyhedron composed of p-gons meeting q at each vertex, with regular r-sided polygonal holes.

Coxeter and John Flinders Petrie proved that there are exactly three of them: $\{4, 6 \mid 4\}$, $\{6, 4 \mid 4\}$, and $\{6, 6 \mid 3\}$.

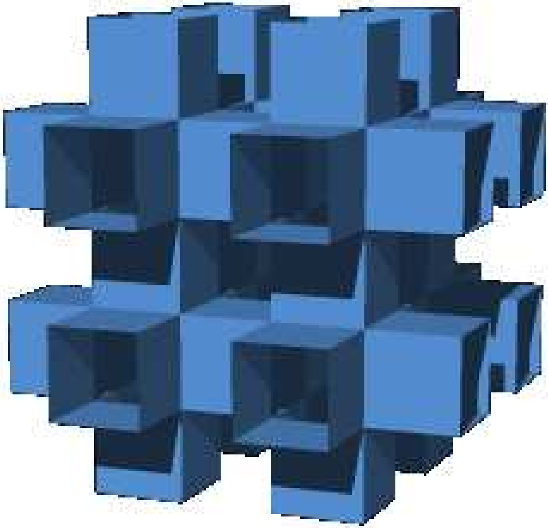
Since the sum of the vertex angles is greater than 2π, they are considered to be the hyperbolic analogs of the Platonic solids and the regular Euclidean tessellations $\{3, 6\}$, $\{4, 4\}$, and $\{6, 3\}$

In 2012 Dunham was the first person to decorate those solids with Escher-inspired patterns.

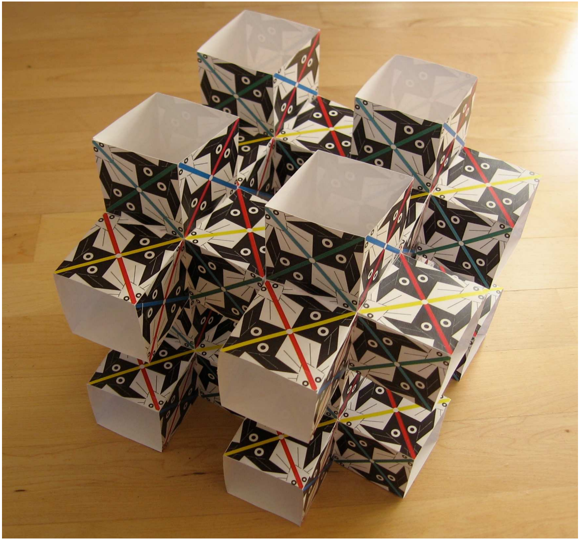
One of these, a patterned $\{4, 6 \mid 4\}$ polyhedron, had some flaws that Dunham thought could be fixed by re-implementing it using papercrafting, which Shier did using the ScanNCut SDX225.

The simplest regular skew polyhedron: $\{4, 6 \,|\, 4\}$

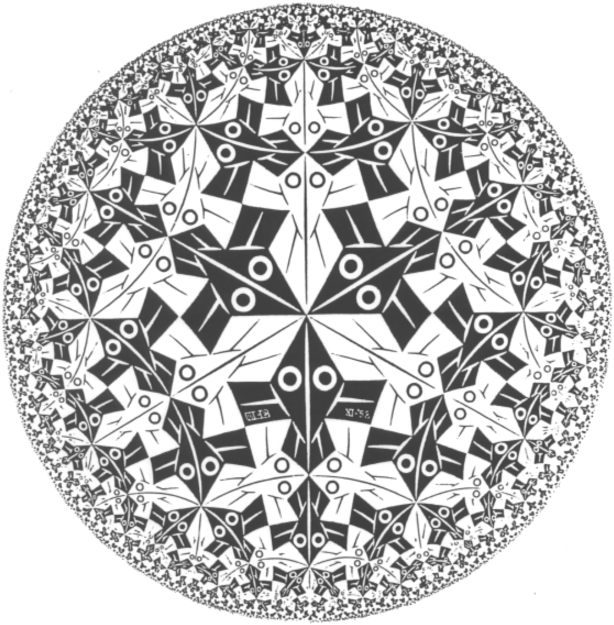
Also called the *Mucube* (for Multi cube). It consists of invisible "hub" cubes connected by "strut" cubes, hollow cubical cylinders with their open ends connecting neighboring hubs.



Dunham's patterned $\{4, 6 \,|\, 4\}$ with fish



Inspiration: Escher's Woodcut Circle Limit I



Problems Circle Limit I according to Escher

1. The fish were not consistently colored along backbone lines — they alternated from black to white and back every two fish lengths.
2. The fish also changed direction every two fish lengths — thus there was no "traffic flow" (Escher's words) in a single direction along the backbone lines.
3. The fish are very angular and not "fish-like"

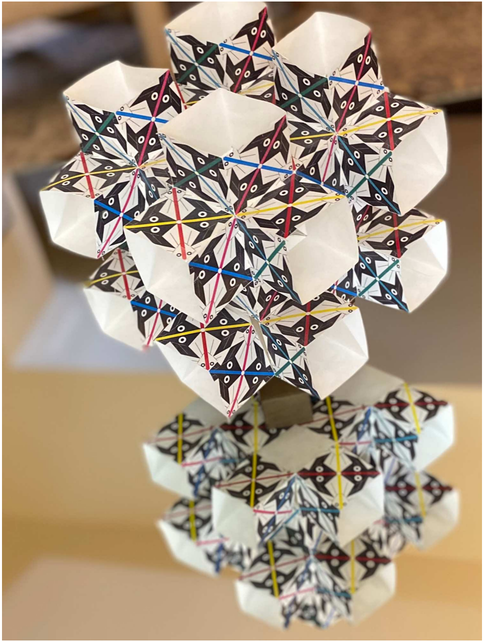
Cure: Escher's Woodcut Circle Limit III

— solved the problems

Problems with Dunham's fish polyhedron

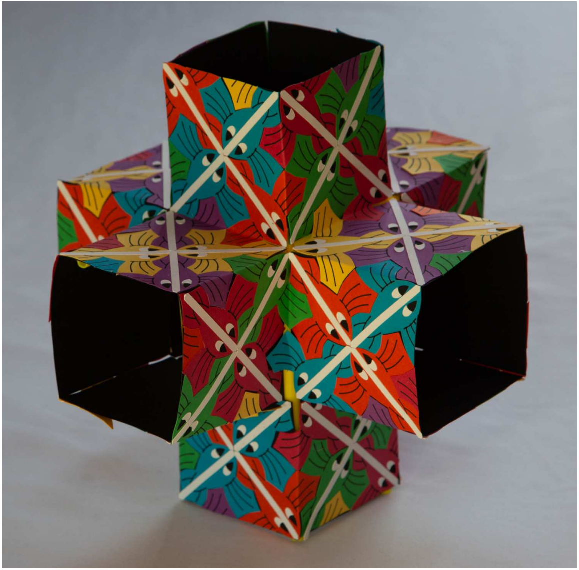
1. Same problems as Escher saw in *Circle Limit I*.
2. The backbone lines of a particular color are not parallel — as can be seen in a mirror.

Dunham's fish polyhedron on a mirror



Shier's new implementation

Fixes the first and third problems.



Shier's new polyhedron on a mirror

Also fixes the fourth problem.

Future Work

- We would also like to explore papercrafting 2D hyperbolic circle patterns and patterns on triply repeating polyhedra such as the regular skew polyhedra.
- Specifically, we would like to try creating a fish pattern on the $\{6, 6 \,|\, 3\}$ polyhedron which could also fix the second problem with Dunham's $\{4, 6 \,|\, 4\}$ polyhedron — so the fish all go the same direction along a backbone line.

Acknowledgements and Contact

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Contact Information:

Doug Dunham
Email: ddunham@d.umn.edu
Web: http://www.d.umn.edu/~ddunham

Lisa Shier
Email: kwajshier@yahoo.com
Blog: "Fun with a Sewing Machine"
http://funwithasewingmachine.blogspot.com/