Forces in Equilibrium

Goal: To study the conditions of static equilibrium, to graphically analyze forces, and to resolve forces into components, both experimentally and algebraically.

Lab Preparation

According to Newton's second law, $\Sigma\vec{F} = m\vec{a}$, if an object stays at rest, the sum of the forces acting on it must add up to zero. Such an object is said to be in equilibrium. The requirement that $\Sigma\vec{F} = 0$ will be evaluated three ways in this lab:

1. Experimentally you will find a third force $\vec{F}_3$ that balances two other specified forces by use of a force table.

2. Graphically you will find a third force by adding two specified forces using the head to tail method (so you might want to review what the head to tail method is).

3. Resolving forces into components algebraically and experimentally will be the third way that $\Sigma\vec{F} = 0$ will be evaluated. Thus you might want to review Newton's 2nd law equilibrium problems that use free body diagrams, components, and summing of forces. Recall that if $\Sigma\vec{F} = 0$, then $\Sigma\vec{F}_x = 0$ and $\Sigma\vec{F}_y = 0$.

Equipment

A circular force table will be used in this experiment (Figure 1). The forces act on a small metal ring in the middle of the table. Strings tied to the ring provide the forces. The tensions in the strings are provided by masses hanging over the edge of the force table.

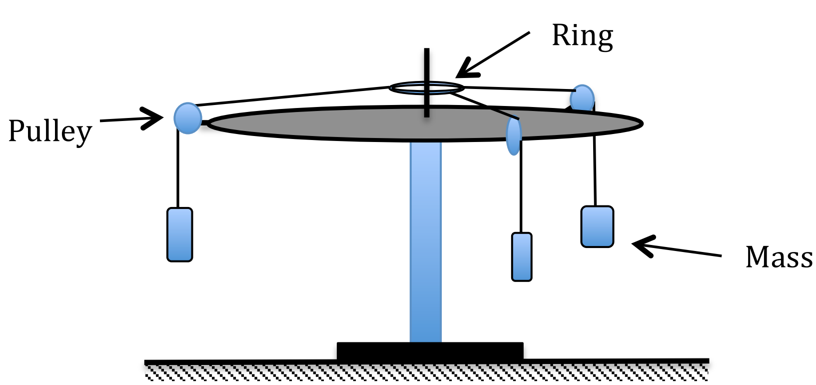

Figure 1

Procedure

Units. Since each force involved is directly proportional to the mass hanging on the string, $w = mg$, you can for convenience in this lab record forces in grams.

A card with your apparatus gives the sizes and directions of two forces $\vec{F}_1$ and $\vec{F}_2$. Record these values on your worksheet. For the first 3 parts of the lab you are to find the size and direction of a third force $\vec{F}_3$ so that all three forces produce equilibrium.

I. Experimental determination of $\vec{F}_3$.

Place the pin through the ring into the hole at the center of the force table. This will keep the ring at the center of the table as you are setting up the forces on the ring. Set one pulley at 180º and another at the angle given on your card for $\vec{F}_2$. Put the stings over the pulleys and hang the masses required to produce the forces $\vec{F}_1$ and $\vec{F}_2$ on the ends of the strings.

To find the direction of the third force $\vec{F}_3$ required to keep the ring in equilibrium at the center of the table, grip the third string in your hand and by trial and error find the direction of pull (parallel to the table) that will keep the ring at the center of the table. Place a third pulley at the required angle of pull.

To determine the magnitude of $\vec{F}_3$ place the third string over the pulley and by trial and error hang masses on the string to bring the ring to equilibrium at the center of the table. When you are near equilibrium remove the pin from the ring. Friction in the pulleys will allow the ring to be in equilibrium over a wide range of positions. To minimize the effects of friction, test for the equilibrium position by lifting the ring about 2 cm and releasing. The ring should return to the center of the table. Test several times and find an average equilibrium position.

The three forces must intersect at the center of the table. Use a ruler to check carefully that each string's direction, when extended, will pass through the hole at the center of the table. You can slide the string on the ring to make this adjustment. Make final adjustments to the pulley and masses if required.

Record the magnitude (F_3) and angle (θ_3) for $\vec{F}_3$.

II. <u>Graphical method for determining $\vec{F}_3$.</u>

Since $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ produce equilibrium, if you add them all up using the head to tail method they would form a closed triangle (Figure 2).

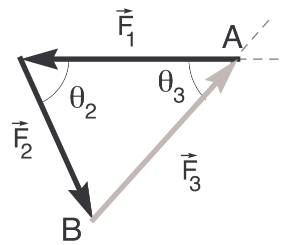

Figure 2

Use a sheet of unlined paper and start with point A (see Figure 2) in the upper right hand corner of the page and draw $\vec{F}_1$ and then $\vec{F}_2$ head to tail. Choose your scale (1 cm = *nn* grams) to make your drawing as large as possible but still fit on the paper.

$\vec{F}_3$ is found by closing the triangle by drawing a vector from B to A (see Figure 2). Measure the length of this line and record it on the diagram. Using your scale factor, determine the magnitude (F_3) of $\vec{F}_3$. Measure the angle θ_3 with a protractor. Record the graphical values of F_3 and θ_3.

III. <u>Algebraically finding $\vec{F}_3$ from components.</u>

Use Newton's 2nd law and the method of components to algebraically find $\vec{F}_3$. Use the following steps as a guideline.

A. Draw a free body diagram showing $\vec{F}_1$, $\vec{F}_2$, & $\vec{F}_3$.
B. Sum forces in the x and y direction.
C. Find the x and y components of $\vec{F}_1$ and $\vec{F}_2$.
D. Use the information above to calculate F_{3x} and F_{3y}.
E. Find the magnitude (F_3) of $\vec{F}_3$.
F. Find the angle θ_3.

IV. <u>Summarizing your results.</u>

Summarize your results by listing the experimental, graphical, and calculated values of F_3 and θ_3 in a table. In the last column list the percentage difference between the experimental and calculated values of F_3, and between the graphical and calculated values of F_3. Omit percentage differences between the values of θ_3.

V. Experimentally resolving $\vec{F}_3$ into components.

Set up on the force table the force $\vec{F}_3$ you calculated. Set a pulley for a new force $\vec{F}_4$ at 180°, and a pulley for a new force $\vec{F}_5$ at -90° (= +270°) as shown in Figure 4.

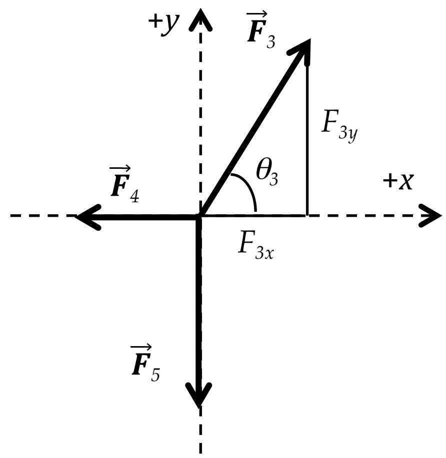

Figure 4

Adjust the masses producing $\vec{F}_4$ and $\vec{F}_5$ so that all three forces put the ring in equilibrium. From considering the sum of the components, $\vec{F}_4$ should be equal and opposite to F_{3x} and $\vec{F}_5$ should be equal and opposite to F_{3y}. Compare these values in a table and find the percent difference between the measured and the expected values.

*When finished with your lab return all masses to their proper containers and clean up your lab station.

Homework

The algebraic method used for the lab can be used to solve many different types of problems. Solve the following problem using the method used in the lab.

A person sleeps in a hammock tied between a couple of trees. The tension in the two ropes supporting the hammock is 684 N at an angle of 18° above the horizontal and 794 N at an angle of 35° above the horizontal. What is the mass of the person (ignore the mass of the hammock)?