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SINGULAR PERTURBATIONS AND TIME SCALES IN CONTROL THEORIES AND APPLICATIONS: AN OVERVIEW 2002-2012

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Abstract. This paper presents an overview of singular perturbations and time scales (SPaTS) in control theory and applications during the period 2002-2012. The previous overviews/surveys were provided for the period up to 1976 [241], 1976-1983 [377], and 1984-2001 [312]. Due to the limitations on the scope and space, this is in no way intended to be an exhaustive survey on the topic.

Key Words. Singular perturbation, time scale, control system, order reduction, control theories and applications.

1. Introduction

From the perspective of systems and control, Kokotovic and Sannuti [243, 384, 385] were the first to explore the application of the theory of singular perturbations to continuous-time optimal control, both open-loop formulation leading to two-point boundary value problem [243] and closed-loop formulation leading to the matrix Riccati equation [385]. The methodology of singular perturbations and time-scales (SPaTS), "gifted" with the remedial features of both dimensional reduction and stiffness relief, is considered as a "boon" to systems and control engineers. Thus the goal of SPaTS technqiues is to reduce and simplify the software and hardware implementation.

The technique has now attained a high level of maturity in the theory of continuous-time and discrete-time control systems described by ordinary differential and difference equations, respectively. The growth of research activity in the field of SPaTS resulted in the publication of excellent survey papers [438, 338, 55, 254, 156, 241, 439, 236, 377, 237, 316, 313, 209, 251, 310, 60, 440, 314, 311, 315, 312, 204, 206], reports and proceedings of special conferences [242, 122, 13]. Also, see research monographs and books (including the general area of singular perturbation theory) [126, 119, 463, 217, 95, 120, 320, 441, 339, 121, 288, 292, 433, 226, 321, 94, 191, 73, 317, 432, 322, 411, 239, 240, 169, 238, 32, 309, 131, 146, 253, 180, 306, 340, 54, 447, 147, 2, 442, 227, 230, 31, 387, 7, 144, 410, 481, 182, 402, 491, 408], encyclopedia [409] and control handbook [229].

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overviews and/or surveys were provided for the periods up to 1976 [241], 1976-1983 [377], and 1984-2001 [312]. Due to the limitations on the scope and space, this is in no way intended to be an exhaustive survey on the topic.

Those readers who do not have some basic background in SPaTS, need to refer to any of the above references, in particular, [240, 309, 147, 312].

2. Modeling

About the basic singular perturbation modeling background, please refer to the reference [312]. Besides, a new unified modeling method is developed using δ-operators in [258] and a bond graph model is presented in an integral causality assignment in [163]. Liyu Cao introduced a new reduced-order model which is based on the actual value of ϵ [62].

In SPSs, fast subsystems produce a limiting system for the slow subsystem. It is shown in [168] that if the flows produced by the unperturbed fast subsystems are "chain transitive", the limiting system can be approximated. And "the reachable sets are contained in the reachable sets of the slightly inflated singularly perturbed system".

2.1. Linear Time Varying (LTV) Systems.

In [484] a larger bound on ϵ is provided which is a gauge on the validity of the Chang transformation. Gauss-Seidel iteration method is used to investigate the exponential stability of singularly perturbed LTV systems in [75], and a method is proposed to compute the upper bound of ϵ under which the system is exponential stable.

2.2. Nonlinear Control Systems.

A bond graph model for a nonlinear singularly perturbed system is presented [22]. A new discretization scheme for two-time-scale nonlinear continuous-time systems is proposed based on Euler's methodology in [35].

For singularly perturbed Hodgkin-Huxley system, Neumann boundary conditions are given from [40]. In [92] gain scheduling control is designed for a nonlinear singularly perturbed time-varying system. In [271], a class of nonlinear memoryless controllers is synthesized for a class of imperfectly known nonlinear SPSs with discrete and distributed delays.

In [319] a holographic explanation is given to show how the renormalization group approach to singular perturbations in non-linear differential equations proposed by Chen, Gold-enfeld and Oono is indeed equivalent to a renormalization group method in quantum field theories proposed by Gell-Mann and Low via AdS/CFT correspondence.

2.3. Hybrid Control Systems.

If a singularly perturbed system includes both the continuous and discrete states, or both the continuous-time and discrete-time, or both the time driven and event dirven properties, then it is called a singularly perturbed hybrid systems.

The stability of singularly perturbed hybrid systems is analyzed in [383, 76, 457]. The oscillation conditions of a second-order singularly perturbed hybrid linear delay dynamic equation are discussed on different time scales in [125].

In [456, 457] the solutions of a class of singularly perturbed hybrid linear delay dynamic equations are discussed.

In [171] singular perturbation theory is used to decompose a hybrid system and the global bifurcations of the forced van der Pol equation are studied based on the reduced systems. Similarly, in [390, 490, 97] singular perturbation is used to deal with hybrid systems.

Also, see [489, 259] for further results on this topic.

3. Boundary Value Problem (BVP)

Boundary value problem of SPSs has attained much attention because one of the boundary conditions has to be sacrificed in the process of degeneration.

A survey [247] presents computational techniques for solving singularly perturbed BVP developed by numerous researchers between 2000 and 2005. Also see [437, 208, 362, 177] for some more results regarding numerical algorithms of solving BVPs.

New approaches are developed to find solutions of singularly perturbed BVP [50, 296, 295, 371, 363, 11, 197]. A Dirichlet problem is studied for a singularly perturbed parabolic reaction-diffusion equation in [403, 404].

In [203, 205, 200] difference schemes for singularly perturbed two point BVP are derived using spline in compression on non-uniform mesh.

Methods for solving reaction-diffusion singularly perturbed BVPs are studied in [250, 449]. In [49] for a singularly perturbed reaction-diffusion problem, the well known apriori Bakhvalov and Shishkin meshes are compared with the adaptive mesh based on the aposteriori dual error estimators.

The proof of the existence of a positive solution of singular discrete third-order BVP with mixed boundary conditions is given in [108]. For some singular perturbation problems which consist of the fourth order differential equations in [386], sufficient conditions are obtained such that the boundary layer can be ignored.

Also see [207, 24, 293, 360, 508] for more works on linear singularly perturbed BVP.

In [51, 53, 59, 246, 106, 66], methods to solve non-linear BVPs are studied and the solutions are analyzed.

A rationalization of the singularly perturbed continuous system of single and multiple time delays is presented by setting a delay coefficient in [397].

The author of [352] obtained an upper bound, in the spirit of Γ lim sup, by multidimensional profiles, for some classes of singular perturbation problems.

4. Time Scale Analysis

In [113], necessary and sufficient conditions for the existence of uniform exponential stability are derived of LTI systems on arbitrary time scales. In [382] a novel time-frequency method to analyze the phase-locked loops is presented. Singular perturbation method is used for diagnosability of linear two-time scale systems in [162] and reduction of the order of unstable linear time invariant systems is done in [467].

In [188], considering a class of second order nonlinear dynamic equations on time scales, a condition is developed that ensures the existence and uniqueness of solutions.

Also see [353, 337, 346] for works on two-time-scale discrete-time systems.

Further, for multiparameter (multi-time-scale) deterministic and stochastic systems, we decompose a full-order system with several small parameters into one low-order slow subsystem and several low-order fast subsystems. See [33, 290, 354, 44] for recent results on multi-time-scale method.

Multi-time-scale method is applied to the network of livestock movements and the dynamics of diseases [214], fractal dynamics in physiology [161], a small set of plant, animal, and abiotic processes structure ecosystems [181].

5. Stability Analysis

5.1. Stability Analysis of Linear Singularly Perturbed Systems. The stability and stabilization problems of continuous-time linear SPSs are studied in [133, 47, 172, 278]. For singularly perturbed Stokes problem, a new stabilized finite element method is developed in [81].

In [116] considering two-time scale systems, an explicit state-space solution of the robust stabilization problem is presented based on Riccati equations. Also the robust stability problem of uncertain two-time-scale systems is studied using a state transformation and Lyapunov theory in [393, 394].

And the authors of [510] studied global exponential stability of singularly perturbed descriptor systems with nonlinear perturbation using fixed-point principle and LMI. In [213] the exponential stability of SPS with both time delay and uncertainties is investigated. In [117] the asymptotic stability of the stability radius is described as ϵ tends to zero. Considering impulsive SPSs under nonlinear perturbation, a sufficient condition that ensures robust exponential stability for sufficiently small ϵ is derived in [77].

Stability bound problems are studied for multi-parameter singularly perturbed time-delay systems [91], linear SPSs [63], discrete multiple time-delay SPSs [89], SPSs with nonlinear uncertainties [511], and nonstandard SPS with time-varying delay [425].

D-stability problem is studied for discrete-time singularly perturbed systems (DSPS) in [186] where a system is called D-stability if the poles of the system are within the specific disk D(a,r) centered at (a,0) with radius r, in which $|a| + r < 1$.

[279] and [173] are both about switched singularly perturbed systems. In [279] the relationship between the stability of subsystems and that of original system is studied for switched linear SPS. In addition, a state-feedback controller is designed. In [173], stability of the planar linear switched SPS is analyzed. Futher, stability of the switched linear DSPS is studied by Ivan Malloci in 2010 using LMIs and switched quadratic Lyapunov functions[280].

In [430], singularly perturbation theory is used to study the input-to-state stability (ISS) of general systems. Assuming the system can be separated to slow and fast subsystems, "the main results establish that if the boundary layer and averaged systems are ISS then the ISS bounds also hold for the actual system with an offset that converges to zero with the parameter that characterizes the separation of time-scales".

Singular perturbation approach is applied for time-domain assessment of the Phase Margin (PM) of an SISO LTI system, whose fast loop system is considered as a singular perturbation with a singular perturbation (time-scale separation) parameter ϵ in [513]. A bijective relationship between the Singular Perturbation Margin (SPM) ϵ_{max} and the PM of the nominal (slow) system is revealed as well as the phase of the fast system.

5.2. Stability Analysis of Nonlinear Singularly Perturbed Systems. Here, we review the SPaTS methodology as applicable to nonlinear systems and the related stability problems.

Robust regulation of a class of nonlinear SPS, is considered via nonlinear H_∞ approach in [10]. See [9, 347] for more results regarding robust stability of nonlinear SPSs.

Exponential stability condition of nonlinear SPS with uncertainties which has upper norm bounds for enough small ϵ is developed and a stabilizing controller is proposed in [414]. Also exponentially stability of non-standard nonlinear SPSs is studied in [93, 345].

Using singular perturbation theory, sufficient conditions of the global asymptotic stability of a class of scalar nonlinear difference equations is given [192]. The absolute stability problem for Lur'e SPS with multiple nonlinearities is studied in [478, 479].

A new concept of mesh stability is proposed for a class of interconnected nonlinear systems in [344] and a set of sufficient conditions of mesh stability are derived. Two different definitions of semiglobal practical external stability are discussed in [12].

Also see [389, 427, 375, 71] for more related works on stability of nonlinear SPSs.

Besides stability problem, more properties of nonlinear SPSs are analyzed during 2002-2012. In [42] an averaged system is constructed to approximate the slow dynamics of a two time scale nonlinear stochastic control system and the approximation is shown valid. Similarly, in [143] an averaging technique is developed.

The limit occupational measures set is presented in terms of the vector function defining the system's dynamics in [142]. In [202] an exponentially fitted difference scheme using cubic splines for a singularly perturbed ordinary differential equation is derived.

The proof of the existence of critical points with semi-stiff boundary conditions for singular perturbation problems in simply connected planar domains is given in [256]. 'Two-stages' strategy used with singular perturbations is extended to compute a balancing form of nonlinear singularly perturbed system in [110, 112].

In [15, 159, 166, 157], order reduction approaches to singularly perturbed nonlinear systems are presented.

And about the solutions of the nonlinear SPS, the conditions of existence are presented in [464, 52, 218], estimate solutions are developed in [448].

Singular Perturbation Margin (SPM) is proposed in term of ϵ in [485]. The SPM relationship between LTI and NLTI systems at the equilibrium is presented. In [286] a new concept of the point-wise eigenvalues and eigenvectors are defined and exploited "as an indicator of the local rate of change of the state of a nonlinear system".

The problem of passivity and passification for a class of nonlinear SPSs is studied via neural network in [465]. The upper bound of perturbation parameter can be obtained by solving algebra inequalities, and the proposed controller can make the singularly perturbed nonlinear system passive. Compared with [465], time-varying delays and polytopic uncertainties are added to the singularly perturbed nonlinear systems in [466]. And the results are generalized.

In [303], singular perturbation technique is used to deal with nonminimum-phase multiple-input-multiple-output (MIMO) nonlinear systems. In order to design oscillation controller in non-linear systems, singular perturbation method is used to analyze the stability of high order systems [61].

6. Observers

In [160] observability of singularly perturbed linear time-dependent differential systems with distributed time-delays in state variables is studied. A new recursive algorithm for solving the multiparameter algebraic Riccati equations to obtain the optimal Kalman filter for multiparameter SPS is developed in [300].

Singular perturbation theory is used to study the well posedness of observer-based fault detection filters in [334], observer design for second order mechanical systems [109], and sliding mode observer design [291].

See [470, 165] for more works related to observers of SPSs.

7. SPaTS in Optimal Control

7.1. Open-Loop Optimal Control. Hiroaki Mukaidani revised Kleinman algorithm based on Newton-Kantorovich theorem to solve algebraic Riccati equation of SPSs, and its quadratic convergence property is proved [257]. In [486], singular perturbation method is used to get a reduced system based on which optimal poles are found.

Optimal control problems are studied for discrete-time and continuous-time SPSs in [123, 58, 376, 101]. [145] is a view on optimal control of linear SPSs and applications.

7.2. Closed-Loop Optimal Control. The closed-loop optimal control has some very elegant results for singularly perturbed systems.

See [153] for study on LQ (linear quadratic) decentralized pole location for SPS, and composite LQ control for SPSs [263].

In SPS, “the variational limit, as the ratio of time scales grows, is best depicted as a trajectory in a probability measures space”. In [16], the variational limit in the form of the Pontryag in Maximum Principle is presented. Also its relationship with the Maximum Principle of the system is discussed.

See [400, 14, 304, 219, 379, 487, 355, 327] for more works on closed-loop optimal control problems of different kinds of linear continuous-time SPSs.

A generalized approach from continuous time system to discrete systems of designing a state feedback controller to get a specified insensitivity of the closed-loop trajectory by the singularly perturbed unified system is developed in [401] and necessary conditions for optimality are derived.

Also, in [36, 231, 25] optimal control for Discrete Singularly Perturbed Systems (DSPSs) is studied.

In [299, 301], “the linear quadratic Nash games for infinite horizon multiparameter SPS with uncertain singular perturbation parameters are discussed”. Based on successive approximation, a construction of high-order approximations to a strategy that guarantees a desired performance level is presented, which improves the cost performance.

8. Other Control Problems

8.1. Robust Control problems. The robust control is studied for SPSs with nonlinear uncertainties in [493], with delay in [469]. The time-delay effect on the robust stabilization of an uncertain SPS via a networked feedback is presented when the time delay is smaller than the sampling time in [459].

Grammel considered the nonlinear SPS with small time delays in the slow variables in [167], and sufficient conditions of exponential stability of the slow subsystem being robust is established using averages of the fast variables.

The author of [70] presented a three-time-scale redesign which stabilizes nonlinear systems with input uncertainties and recovers the nominal closed loop trajectories. This is achieved by designing a high filter which estimates the uncertainty over a fast time scale, and then forcing them to converge to the nominal input manifold by another set of fast filters.

8.2. Fuzzy Control Problems. In [499], delta operator is used to construct a fuzzy singular perturbed unified model which applies to both continuous-time domain and discrete-time domain and the robust control is presented for the proposed model framework. Singular perturbation technique is used for order reduction of linear complex systems described by TSK fuzzy models in [46].

In [185], a robust fuzzy controller is designed for nonlinear multiple time-delay SPSs and in [18] an H_∞ fuzzy controller is designed for a class of nonlinear SPSs, which can be used for both standard and unstandard nonlinear SPSs.

Similar results exist for nonlinear singularly perturbed Takagi-Sugeno (TS) fuzzy models, designing fuzzy H_∞ filter, H_∞ output feedback controller and composite fuzzy controllers in [17, 21, 20, 19, 261, 323].

Also see [124] for a fuzzy logic algorithm to optimize sliding surface parameter. In [273] based on the stability analysis of both continuous-time and discrete-time fuzzy SPS, stabilizing feedback controllers are designed separately.

A novel high gain observer-based decentralised indirect adaptive fuzzy controller is developed for a class of uncertain affine large-scale nonlinear systems in [187]. Multi-objective control which consists of H_∞ control, pole placement and singular perturbation bound design for T-S fuzzy SPS is presented in [480]. Also in [498] the decentralized multi-objective robust control problem is discussed for interconnected fuzzy singular perturbed model (FSPM) with multiple perturbation parameters.

8.3. Network Control Problems.

For some results on the topic of network control problems, see [27, 305, 372, 276] for singular perturbation theory applications in network systems.

A two-time-scale plant is analyzed in [494] whose sensor is connected to a linear controller/actuator via a network.

In addition, the model-based networked control is studied in [496, 458] for SPS and SPS with uncertainties. The neural network-based control and observer design for a class of nonlinear SPSs are studied with guaranteed H_∞ control performance in [269].

8.4. H_∞/H_2 control problems. New methods are presented to solve the H_2/H_∞ control problems for continuous-time SPS in [302, 128].

Also, H_∞ controllers are designed using different approaches for multiparameter SPSs, SPS with norm-bounded uncertainties, nonlinear SPS of TS fuzzy model, and uncertain SPS in [477, 431, 134, 135, 482, 270].

Robust multi-objective H_∞ control is studied for linear two-time scale systems in which fast dynamics are assumed of norm-boundedness in [220]. H_∞ control is studied using singular perturbation theory for inclusion nonlinear systems in [294].

For standard DSPS with polytopic uncertainties, state feedback H_∞ controller is designed by using LMI in two ways in [114]. In [105] a mixed H_2/H_∞ linear state variable feedback suboptimal controller is designed for a DSPS based on reduced order slow and fast subsystems. See [115, 272] for more results regarding to H_∞ control problem of DSPS.

See [170, 483] for more works on H_∞/H_2 control and SPaTS.

8.5. Other Control Problems. For two-time-scale systems, an approximate controller is designed in [398], ergodic control is considered in [43].

Considering linear SPS with time-delay in [88], reduced subsystems are obtained via singular perturbation techniques and the relationship of controller and observer design between the original and the subsystems obtained is presented.

In [154] composite and reduced input bounded controller are designed for linear SPS and constructive geometric conditions are proposed to drive a stabilizing controller in both composite and reduced contexts. For a class of interconnected SPSs in [335], a controller is designed that reduces the trajectory sensitivity to small feedback delays using the singular perturbation and sensitivity theories.

Also, sliding-mode control is studied of SPSs in [152, 139, 3]. Observer-based feedback controller is designed for continuous-time SPS in [267].

Besides, guaranteed cost control, composite control, passivity-based control, adaptive control, switched output feedback control, admissible control, and variable structure control are studied for different kinds of SPSs in [264, 504, 90, 151, 497, 268, 5, 495, 41, 329].

Singular perturbation theory is applied to other control problems. See [211, 381, 190, 399, 415, 333, 328, 136, 176] for applications of singular perturbation approach to study dynamical feedback control, observer-based control, high-gain feedback control, variable structure control, indirect adaptive control, sliding mode control, cost control, and tracking control problems for various systems.

Singular perturbation theory is used to derive the structure of controllability and observability energy functions of bilinear SPS in [111].

About the control problem of nonlinear SPSs, fault tolerant control, optimal control, closed-loop composite control, PI control, feedback control, PID control, adaptive control, and tracking control are studied in [512, 232, 233, 452, 500, 501, 502, 406, 407] respectively. Model predictive control problem is studied in [79, 78, 468, 331].

9. Numerical Algorithm for SPSs

Because of the stiffness and high order property of the SPSs, it is difficult to get the analytical solutions for singularly perturbed equations, therefore numerical algorithms are developed and applied to SPSs.

Computational algorithms are developed to solve self-adjoint singularly perturbed BVPs in [29, 221]. Please refer to [436, 82, 392, 4, 34, 342, 503] for more results about numerical algorithms to solve singularly perturbed BVPs.

The author of [378] described numerical methods for solving stiff initial-value problems (IVP) using one-step schemes of exponential type.

A computational algorithm is developed to get the solution of a singularly perturbed differential-difference equation with turning points which can result in boundary or interior layer in [359]. In [104, 102, 103], an efficient numerical scheme is proposed to solve SPSs of Robin type reaction-diffusion problems.

More algebraic and numerical approaches are presented to get the solutions of singular perturbation problems in [201, 429, 175, 199, 210, 298, 507].

Nonstandard finite difference method is introduced to solve the singularly perturbed differential equations by [349]. Works [248, 249] present a survey of the most effective computational techniques for solving singularly perturbed partial differential equations. A computational method is presented for solving singularly perturbed delay differential equations with negative shift whose solution has boundary layer [129].

The Benoit's theorem is extended in [158] for the generic existence of solutions of SPSs of dimension three with one fast variable to those of dimension four. For a singularly perturbed nonlinear Robin problem in a periodically perforated domain,

[107] proved the existence of a family of solutions for ϵ sufficiently small. Considering the Helmholtz equation, $\epsilon^2 u_x x - u = f(x)$ where ϵ is small, two methods are developed to get the solution in [48].

9.1. Asymptotic Approximation for Solutions of SPSs. Via numerical methods, we obtain the asymptotic approximations to give the qualitative behavior of the solutions. The details of obtaining the approximated solutions are given in [463, 441, 309].

The asymptotic expression of a kind of vector singularly perturbed delay-differential equations in [451] is constructed and the uniform validity of asymptotic solution is also proved. Asymptotic approximations of the solutions for different classes of SPSs are also obtained in [471, 287, 184].

In [252], asymptotic expansion of solutions of optimal control problems for singularly perturbed systems (SPS) are constructed. For Markov random process, asymptotic expansions and probability distributions are studied using SPaTS theory in [6, 326, 491].

Also explicit bounds on the convergence rate of the trajectories to the slow manifold and on the asymptotic error between the trajectories of the SPS and those of the reduced system are obtained in [444, 445].

10. Applications

10.1. Aerospace. The theory of SPaTS has its roots in fluid dynamics and naturally found its wide applications in the area of aerospace systems. See a survey [361] on applications of SPaTS on aerospace.

See [422, 357, 285] for applications of singular perturbation to digital flight control systems.

For two-time-scale aircraft dynamics systems, see interesting applications of SPaTS in [164, 398]. Singular perturbation approach is utilized to aircraft systems in [473, 127] for multi-time-scale decomposition.

In [45], considering a class of nonlinear systems actuated by actuators whose actuator dynamics are assumed fast, baseline controller is designed. Singular perturbation is applied to show that the closed loop system achieves the control objective.

A postbuckling analysis is presented for nano-composite cylindrical shells reinforced by single-walled carbon nano-tubes subjected to combined axial and radial mechanical loads in thermal environment in [396], and a boundary layer theory and associated singular perturbation technique are employed to determine the buckling loads and postbuckling equilibrium paths.

Also, see [183, 118, 358] for applications of singular perturbation and time-scale methodologies to missile systems.

10.2. Mechanical Systems.

Another interesting area of the application of SPaTS is mechanical dynamics and control

For a multi-link flexible robot with uncertainties, an improved composite controller is designed based on singular perturbation theory in [86]. In [380] a new control strategy for flexible-joint manipulators with joint friction is proposed. The proposed controller includes two main components: a friction compensating torque, and a composite controller torque which is designed using singular perturbation theory.

Also see [84, 99, 341, 266, 348, 100, 274, 428, 424, 64, 423, 265, 85, 405, 395, 435] for more results about applications of SPaTS to flexible robots.

See [69, 283, 453, 150] for more results of SPaTS on other kinds of robots.

In [98, 462, 461], closed kinematic chains (CKC) are modeled as singular perturbation systems and the properties of validity domain, error characterization and stability are analyzed.

In [130] results on partial stability of the differential form of speed-gradient control for SPS are generalized to the case of speed-gradient control in finite form.

Based on singular perturbation techniques, direct torque control (DTC) is derived in [417] and a link between DTC and feedback linearization is presented. “An explicit relationship between DTC performance and machine characteristics has been revealed”, which can be used to improve DTC performance by designing an induction motor.

In [460] the singular perturbation formulation is compared to control based on input-output linearization (IOL) and advantages and disadvantages of each method are described.

Traveling wave solutions of viscous conservation laws are studied in [1]. The eigenvalue problem corresponding to the linearization around a viscous shock wave is viewed as a singularly perturbed problem, and geometric singular perturbation theory is used for the analysis of the Evans function. And the Gardnerand Zumbrun results about the first derivative of the Evans function are proved at the origin.

A new coning correction algorithm, based on the singular perturbation technique, is proposed for the attitude update computation with non-ideal angular rate information [141]. Singular perturbation theory is applied to show that the Boltzmann C Enskog equation results in the Navier C Stokes equation for incompressible fluids together with two different Boussinesq relations and temperature fluctuation equations in [193] and the proof of a rigorous result is given in [194].

In addition to the above applications, singular perturbation theory is also applied to air-conditioning systems [364], an axially moving cable with large sag [368], a four-wheeled steering and four-wheeled drive vehicle [351], early detection systems with multiple-bottleneck links [462], harmonic drive systems [149], pneumatic vibration isolators [174], the voice coil motor [343], hypersonic vehicles [307], hydraulic systems citeLongke-Wang-et-al2012, a flexible beam used in underwater exploration [308], infinite-dimensional mechanics of fluids and plasmas [492], dual-loop exhaust gas recirculation air-path systems [476], underactuated biped robots [87], hydrostatic drive or cylinder [281], single-axis rate gyro [74], bimolecular association mechanism [132], 2D thermal convection loop [443], and so on.

10.3. Electrical and Electronic Circuits and Systems. It is common to neglect dynamic saliency in synchronous machines of power systems [275]. This paper [80] is a summary which tells how to model power systems using singular perturbation approach, and neglect the fast dynamics to get a simplified power system model. The error associated with neglecting dynamic saliency is eliminated by inserting a singular perturbation into the machine model [350].

In [434, 420, 418, 419] singularly perturbation method is used for synchronous generator systems to sliding mode control. In [262], a doubly-fed induction generator is considered to design a controller based on multi-time scale theory.

For singularly perturbed relay systems [138, 137], a theorem about existence and stability of the periodic solutions is proved and an algorithm of asymptotic representation for the periodic solutions is presented using boundary layer method.

Forced singular perturbations are proposed to reduce computations of multirate strapdown terrestrial navigation algorithm in [450]. The transmission problem is studied for the system of piezoelectricity having piecewise constant coefficients in [216].

For a class of DC-DC power converters, current-mode control problem is studied in [8]. Singular perturbation is used to separate the fast and slow states of dc-dc converters systems in [234], and a relationship between inductance, capacitance, load resistance, and loss resistance is obtained from an analysis of an approximate model. Compared with [234], discrete-time analysis is added in [235].

The papers [324, 325] studied the singular perturbation analysis and synthesis of wind energy conversion systems. SPaTS theory is used to analyze wind energy conversion systems in [195] and time-scale method and MPC are combined to control wind energy conversion systems in [509].

Also, see [374, 366, 140] for more applications of SPaTS to wind power systems.

For servomotor systems [506] a general PIV controller is taken into the original system, and the closed loop is decomposed to two subsystems using singular perturbation theory which stand for the position control loop and the high frequency RV dynamics, respectively.

Singular perturbation theory is used to design an observer of sliding mode type for the flux estimation of an induction machine in [332]. In order to provide insight into the connections between the different nodes of a power network, a method based on differential geometric control theory is obtained in [28]. In [179] circuit-averaging techniques are applied to simplify a lumped-parameter model of the cardiovascular system.

Considering the noncoherent digital delay lock loops on chip timing synchronisation in [178], the mean time to lose lock is calculated using diffusion approximation and the singular perturbation method. Loop bandwidth is optimized for the first order loop.

The stability of a large-scale power system is analyzed by Jacobian analysis based on singular perturbation in [472]. Considering the converter-interfaced wind turbines in [367], singular perturbation theory is used to decompose the system dynamics based on which a controller is designed to isolate wind-power fluctuations from the power grid.

In [454] a singularly perturbed model is developed for AIMD/RED systems with multiple bottlenecks and feedback delays, of which stability is analyzed. The delay-dependent LMI conditions for the stability are established and it is proved that sufficiently small parameters exist to guarantee the asymptotic stability of the system considered above.

In [282] singular perturbation technique is applied to the permanent magnet synchronous machine (PMSM) system. Based on the decoupled subsystems, "the control speed and the I_d current are carried out by neuro-fuzzy regulators (ANFIS)".

Based on the combination of PMSG and super sparse matrix converter, a novel Variable-speed wind energy generation scheme is developed for the wind energy conversion system in [488].

Emitter-coupled multivibrators are modeled and analyzed using the singular perturbation theory in [356]. A sampled-data strategy for a boundary control problem of a heat conduction system modeled as PDE is developed in [83]. Using singular perturbation theory, the reduced subsystems are presented.

Considering the power system model in [260], singular perturbation method is used to decompose the system into slow and fast subsystems and the relationship between the stability of reduced-order system and original system is analyzed.

In [474] a class of power systems with detailed excitation and power system stabiliser (PSS) controller is modeled as singular perturbation system. An LMI-based approach is developed to estimate the stability region. Considering multi-machine power systems with matched additive uncertainty and input multiplicative uncertainty in [72], several time-scale separation designs are used for robust stabilization and performance recovery.

A complementary controller is designed to improve power systems stability in [426] based on singular perturbation theory. Considering an oscillator model which is composed of a fast membrane potential dynamics and a slow recovery dynamics in [318], phase response curves (PRCs) for both the dynamics and plausibility of feedback inputs to the slow dynamics rather than the fast dynamics are shown using singular perturbation theory.

10.4. Chemical Reactions and Reactors.

Two different reducing order methods are compared to singular perturbation method for chemical kinetics equations in [215]. And singular perturbation method is applied for model reduction of stiff chemical Langevin equations and chemical kinetics problems in [96] and [255]. Global Quasi-Linearization (GQL) method which is based on two-time-scale theory is presented for an automatic reduction of chemical kinetics models in [56].

Equations for the description of chemical reactions of dissociation and recombination are transformed into singularly perturbed equations in [148]. A new concept of critical simplification for chemical kinetics is proposed, which is valid in the presence of a dominant competitive reaction and critical phenomena in [475].

Singular perturbation method is used to analyze and synthetize the model of thermal explosion in a gas-droplets mixture [57], a chemical reactor and a feed effluent heat exchanger in [198].

Viewing the prompt jump approximation (PJA) of nuclear reactor dynamics as the zeroth-order approximation of an asymptotic expansion to SPS of ordinary differential equations, the equations describing its first-order approximation is derived [30].

Two-point linear controllers for binary distillation columns are designed based on singular perturbation theory in [65]. Considering a singularly perturbed convection-diffusion equation with constant coefficients in a half plane, with Dirichlet boundary conditions [225], precise pointwise bounds for the derivatives of the solution are obtained.

10.5. Biology.

Models describing the biotechnical process behaviour are usually high order and nonlinear with time-varying parameters. In [223], decomposition techniques based on singular perturbation analysis or batch phase analysis are used to simplify the model. Also, it is shown that the essential initial values can be effectively obtained from the data.

Considering a host-vector model for a disease without immunity in [373], the stability of the steady states using the contracting-convex-sets technique is studied and using the geometric singular perturbation method, existence of travelling wavesolutions is established.

The work of [505] shows that travelling wave solutions exist for a modified vector-disease model using the geometric singular perturbation theory. Models that incorporate local and individual interactions are introduced in [416]. In addition, epidemiological time scales are used to reduce the dimension of the model and singular perturbation methods are used to corroborate the results of time-scale approximations.

In [365] a mathematical model is proposed for the differentiation of osteoblastic and osteoclastic populations in bone, based on the differential effects of PTH, and singular perturbation theory is used to analyze the highly diversified dynamics.

Considering the model describing the growth of microalgae, the authors of [67, 68] maximized the specific growth rate of microalgae by manipulating the irradiance using singular perturbation theory.

The topic of [391] focuses on the analysis of a nonlinear dynamical model of a class of bioprocesses, and in order to obtain reduced order models, the singular perturbations method and quasi-steady state assumption are used.

A novel ion channel biosensor is modeled in [297], and singular perturbation theory is used to designed an optimal input voltage to the biosensor to minimize the covariance of the estimation error.

A mechanism is proposed in bio-molecular systems to attenuate retroactivity in [196]. Using coodinating transforms and singular perturbation theory retroactivity can be arbitrarily attenuated by internal system gains. Singular perturbation theory is applied to analyze the bio-molecular feedback systems in [446].

In [244], the model is analyzed to predict the performance of the biosensor in transient and steady-state regimes. Singular perturbation is used to determine the conditions for globally uniformly stability of a class of biological networks with different time-scales under parameter perturbations in [289].

Based on singular perturbation theory, dynamical system theory, and differential-algebraic equations, a mathematical framework is developed to analyze and design on-line schemes for fixed point recur-rent neural learning in [369].

10.6. Other Areas.

In [413] the moisture-induced deformation in an elastic panel is modeled as singularly perturbed system, which is approximated by singular perturbation method. Compared with experimental results, the solution is analyzed based on the material of the elastic panel. Optimal climate control problem is studied for a potato storage facility to exploit the favorable weather conditions in [224]. Using singular perturbations and based on Fenichel's theorems, extensions of Hirsch's generic convergence theorem for monotone systems are studied in [455].

Considering a large-scale nonlinear network system, singular perturbation is used to decompose the states into fast and slow subsystems in [37, 38], and the validity of the reduced-model approximation is proved on the infinite time interval.

The notion of two-time-scale (TTS) distributions is introduced [336] and TTS distribution is analyzed in two different time scales. Singular perturbation is applied to choose the Page Rank factor in a bow-tie web graph in [23].

Boundary value theory is applied to analyze the gravitational-tidal evolution of planetary subsystems of the Sun in [39]. The topic of [26] is about the spectral properties of the Neumann-Laplacian on the singularly perturbed periodic quasi-cylinder. Singular perturbation theory is used to analyze the global asymptotic stability of positive equilibria of ratio-dependent, predator-prey models with stage structure for the prey in [330].

In [155], singular perturbation theory is applied to analyzed the quadratic family with multiple poles. The author of [370] analyzed the stokes flow in a singularly perturbed exterior domain. Considering static and dynamic behaviour of two-dimensional droplets in [388], an evolution equation for the droplet thickness is obtained using singular perturbation theory.

In [421], "vorticity distributions over the diffracted shock both from Lighthills theory applicable for small bends and Sakurai and Takayamas theory applicable for larger bends have been investigated for Mach numbers 1.80 and 1.95" using singular perturbation theory.

11. Conclusions and Future Directions

11.1. Conclusions. This paper presents an overview of singular perturbations and time scales (SPaTS) in control theory and applications during the period 2002-2012. The works on optimal control, robust control, fuzzy control, network control, H_2/H_∞ control, stability analysis, numerical algorithms and other control problems of SPSs during 2002-2012 are reviewed. In the end, applications of SPaTS theory to aerospace, mechanical, electrical and electronic systems, chemical reactions, biology and other areas are presented.

11.2. Future Directions. Some of the areas for future investigations are suggested below.

(1) One area is to investigate singular perturbation techniques for *nonlinear* systems in general and explore the possible applications to a variety of systems such as wind energy systems and life science problems. In particular, this may lead to exploring the singularly perturbed Sate-Dependent Riccati Equation (SDRE) - both differential and algebraic for continuous-time and discrete-time systems with time-scale character.
(2) Another interesting and recent area is *Model Predictive Control (MPC)* of linear and nonlinear and deterministic and stochastic Time Scale Systems [78, 79].
(3) Also, another area worth looking is the absolute stability for uncertain Lur'e SPS based on the Lyapunov theory, and Linear Matrix Inequalities (LMIs).
(4) Control for singularly perturbed *descriptor system* with nonlinear perturbation [510] seems an interesting area.
(5) Control for singularly perturbed *switched linear control systems* [173].
(6) Since time-scale character occurs so naturally in many science and engineering systems, it is worth exploring the application spectrum to areas in science (biology, economics, management, etc.) and engineering (Aerospace, Biomedical, Chemical, Electrical, Mechanical (including robotics), Nuclear Engineering, etc.).

References

[1] F. Achleitner and P. Szmolyan. Saddle-node bifurcation of viscous profiles. *Physica D: Nonlinear Phenomena*, 241(20):1703 – 1717, 2012.

[2] Z. Aganović and Z. Gajić. *Linear Optimal Control of Bilinear Systems with Applications to Singular Perturbations and Weak Coupling*, volume 206 of *Lecture Notes in Control and Information Sciences*. Springer-Verlag, London, UK, 1995.

[3] A.E. Ahmed, H.M. Schwartz, and V.C. Aitken. Sliding mode control for singularly perturbed system. In *2004. 5th Asian Control Conference*, volume 3, pages 1946 –1950, 2004.

[4] G. Akram and A. Naheed. Solution of fourth order singularly perturbed boundary value problem using septic spline. *Middle-East Journal of Scientific Research*, 15(2):302–311, 2013.

[5] A. N. Al-Rabadi. Soft computation using artificial neural estimation and linear matrix inequality transmutation for controlling singularly-perturbed closed time-independent quantum computation systems, part a: Basics and approach. *Intelligent Automation and Soft Computing*, 18(1):75–95, 2012.

[6] S. Albeverio, V.S. Koroliuk, and I.V. Samoilenko. Asymptoti c expansion of semi-markov random evolutions. *Stochastics: An International Journal of Probability and Stochastics Processes*, 81(5):477–502, Oct. 2009.

[7] S. Albeverio and P. Kurasov. *Singular Perturbations of Differential Operators: Solvable Schrödinger Type Operators*, volume 271 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, Cambridge, UK, 2000.

[8] J. Alvarez-Ramirez, G. Espinosa-Perez, and D. Noriega-Pineda. Current-mode control of dc-dc power converters: a backstepping approach. In *International Journal of Robust and Nonlinear Control*, pages 421–442, 2003.

[9] R. Amjadifard, M. T. H. Beheshti, and M. J. Yazdanpanah. Robust stabilization for a class of nonlinear singularly perturbed systems. *Journal of Dynamics Systems, Measurements, and Control*, 2011.

[10] R. Amjadifard, M. J. Yazdanpanah, and M. T. H. Beheshti. Robust regulation of a class of nonlinear singularly perturbed systems. *Proceeding 16th IFAC World Congress*, 2005.

[11] N. Ratib Anakira, A. K. Alomari, and I. Hashim. Numerical scheme for solving singular two-point boundary value problems. *Journal of Applied Mathematics*, 2013.

[12] D. Angeli and D. Nesic. A trajectory-based approach for the stability robustness of nonlinear systems with inputs. *Mathematics of Control, Signals, and Systems (MCSS)*, 15:336–355, 2002.

[13] M.D. Ardema, editor. *Singular Perturbations in Systems and Control*. Springer-Verlag, Wien, Austria, 1983.

[14] Z. Artstein. An occupational measure solution to a singularly perturbed optimal control problem. *Control and Cybernetics*, 31:623–642, 2002.

[15] Z. Artstein. Singularly perturbed control systems with one-dimensional fast dynamics. *SIAM journal on control and optimization*, 41, 2002.

[16] Z. Artstein. Pontryagin maximum principle for coupled slow and fast systems. *Control and cybernetics*, 38, 2009.

[17] W. Assawinchaichote and S. K. Nguang. H_∞ filtering for fuzzy singularly perturbed systems with pole placement constraints: an lmi approach. *IEEE Transactions on Signal Processing*, 52:1659 – 1667, 2004.

[18] W. Assawinchaichote and S. K. Nguang. H_∞ fuzzy control design for nonlinear singularly perturbed systems with pole placement constraints: An lmi approach. *IEEE Transactions on Signal Processing*, 34:579 – 588, 2004.

[19] W. Assawinchaichote and S. K. Nguang. Fuzzy H_∞ output feedback control design for singularly perturbed systems with pole placement constraints: an lmi approach. *IEEE Transactions on Fuzzy Systems*, 14:361 – 371, 2006.

[20] W. Assawinchaichote, S. K. Nguang, and P. Shi. output feedback control design for uncertain fuzzy singularly perturbed systems: an lmi approach. *Automatica*, 40:2147 – 2152, 2004.

[21] W. Assawinchaichote, S. K. Nguang, P. Shi, and R.K. Agarwal. Robust H_∞ fuzzy filter design for uncertain nonlinear singularly perturbed systems with markovian jumps: an lmi approach. In *43rd IEEE Conference on Decision and Control*, volume 2, pages 1773 – 1777 Vol.2, dec. 2004.

[22] G. G. Avalos and N. B. Gallegos. Quasi-steady state model determination for systems with singular perturbations modelled by bond graphs. *Mathematical and Computer Modelling of Dynamical Systems*, 19(5):483–503, 2013.

[23] K. Avrachenkov, N. Litvak, and K. S. Pham. A singular perturbation approach for choosing the pagerank damping factor. *Internet Mathematics*, 5(1):47–69, 2008.

[24] A. Ramesh Babu and N. Ramanujam. The sdfem for singularly perturbed convectioncdiffusion problems with discontinuous source term arising in the chemical reactor theory. *International Journal of Computer Mathematics*, 88(8):1664–1680, May 2011.

[25] G. Badowski, G. Yin, and Q. Zhang. Near-optimal controls of discrete-time dynamic systems driven by singularly-perturbed markov chains. *Journal of Optimization Theory and Applications*, 116:131–166, 2003.

[26] F. L. Bakharev, S.A. Nazarov, and K.M. Ruotsalainen. A gap in the spectrum of the neumannclaplacian on a periodic waveguide. *Applicable Analysis*, pages 1–27, Aug. 2012.

[27] M. Baldea and P. Daoutidis. Model reduction and control for networked systems with high energy throughput. *Proceedings of the 2006 American Control Conference*, pages 6008–6013, June 2006.

[28] E. Barany, S. Schaffer, K. Wedeward, and S. Ball. Nonlinear controllability of singularly perturbed models of power flow networks. *43rd IEEE Conference on Decision and Control*, 2004.

[29] R.K. Bawa and A. Walia. Parallel implementation of numerov's method based algorithm for singularly perturbed boundary value problems. In *International Conference on Computing Informatics*, pages 1 –4, 2006.

[30] R. Beauwens and J. Mika. The improved prompt jump approximation. *Transport Theory and Statistical Physics*, 36(1-3):211–225, 2007.

[31] C.M. Bender and S.A. Orszag. *Advanced Mathematical Methods for Scientists and Engineers I: Asymptotic Methods and Perturbation Theory.* Springer-Verlag, New York, NY, 1999.

[32] A. Bensoussan, editor. *Perturbation Methods in Optimal Control.* John Wiley & Sons, Chichester, UK, 1988.

[33] N. Berglund and B. Gentz. Geometric singular perturbation theory for stochastic differential equations. *Journal of Differential Equations*, 191(1):1 – 54, 2003.

[34] C. Besse, F. Deluzet, C. Negulescu, and C. Yang. Efficient numerical methods for strongly anisotropic elliptic equations. *Journal of Scientific Computing*, 55(1):231–254, 2013.

[35] M. Bidani and M. Djemai. A multirate digital control via a discrete-time observer for nonlinear singularly perturbed continuous-time systems. *International journal of control*, 75:591–613, 2002.

[36] M. Bidani, N. E. Radhy, and B. Bensassi. Optimal control of discrete-time singularly perturbed systems. *International journal of control*, 75:955–966, 2002.

[37] E. Biyik and M. Arcak. Area aggregation and time scale modeling for sparse nonlinear networks. In *45th IEEE Conference on Decision and Control, 2006*, pages 4046 –4051, 2006.

[38] E. Biyik and M. Arcak. Area aggregation and time-scale modeling for sparse nonlinear networks. *Systems and Control Letters*, 57:142 – 149, 2008.

[39] V. V. Bondarenko, Y. G. Markov, A. M. Mikisha, L. V. Rykhlova, and I. V. Skorobogatykh. Gravitationalctidal evolution of planetary subsystems of the sun. *Astronomical and Astrophysical Transactions*, 25(4):275–290, Aug. 2006.

[40] A. Bonfoh. Dynamics of hodgkinchuxley systems revisited. *Applicable Analysis*, 89(8):1251–1269, August 2010.

[41] V. Borkar and V. Gaitsgory. Existence of limit occupational measures set used for averaging of singularly perturbed controlled stochastic differential equations. *45th IEEE Conference on Decision and Control*, pages 326 – 331, 2006.

[42] V. Borkar and V. Gaitsgory. On averaging of singularly perturbed controlled stochastic differential equations. *Applied mathematics and optimization*, 56, 2006.

[43] V. S. Borkar and V. Gaitsgory. Singular perturbations in ergodic control of diffusions. *SIAM journal on control and optimization*, 46, 2007.

[44] V. S. Borkar and K. S. Kumar. Singular perturbations in risk-sensitive stochastic control. *SIAM Journal on Control and Optimization*, pages 3675–3697, 2010.

[45] J. D. Boskovic and R.K. Mehra. A decentralized fault-tolerant control system for accommodation of failures in higher-order flight control actuators. *IEEE Transactions on Control Systems Technology*, 18(5):1103 –1115, Sep. 2010.

[46] A. Bouazza, A. Sakly, and M. Benrejeb. Order reduction of complex systems described by tsk fuzzy models based on singular perturbations method. *International Journal of Systems Science*, 44(3):442–449, 2013.

[47] E. K. Boukas and Z. K. Liu. Delay-dependent stabilization of singularly perturbed jump linear systems. *International Journal of Control*, 77(3):310–319, 2004.

[48] J. Boyd. Chebyshev solution of the nearly-singular one-dimensional helmholtz equation and related singular perturbation equations: multiple scale series and the boundary layer rule-of-thumb. *Numerical Algorithms*, 38:197–207, 2005.

[49] A. Bugajev and R. Ciegis. Comparison of adaptive meshes for a singularly perturbed reaction-diffusion problem. *Mathematical Modelling and Analysis*, 17(5):732–748, Nov. 2012.

[50] E. E. Bukzhalev. On the construction of upper and lower solutions by the nagumo method. *Differential Equations*, 40(6):723–730, 2002.

[51] E. E. Bukzhalev. On the construction of upper and lower solutions by the nagumo method. *Differential Equations*, 40:723–730, 2004.

[52] E. E. Bukzhalev. On an application of the method of differential inequalities to equations of parabolic type whose right-hand side grows faster than quadratically with respect to the space gradient. *Differential Equations*, 41:356–365, 2005.

[53] E. E. Bukzhalev and A. B. Vasil'eva. Solutions to a singularly perturbed parabolic equation with internal and boundary layers depending on stretched variables of different orders. *Computational Mathematics and Mathematical Physics*, 47:424–437, 2006.

[54] A.W. Bush. *Perturbation Methods for Engineers and Scientists.* CRC Press, Boca Raton, FL, 1992.

[55] V.F. Butuzov, A.B. Vasil'eva, and M.V. Fedoryuk. Asymptotic methods in the theory of ordinary differential equations. In R.V. Gamkrelidge, editor, *Progress in Mathematics*, volume 8, pages 1–82. Plenum Publishing, New York, New York, 1970. (Review article with 504 references).

[56] V. Bykov, V. Goldshtein, and U. Maas. Simple global reduction technique based on decomposition approach. *Combustion Theory and Modelling*, 12(2):389–405, April 2008.

[57] Viatcheslav Bykov, Igor Goldfarb, Vladimir Goldshtein, and J Barry Greenberg. Thermal explosion in a hot gas mixture with fuel droplets: a two reactant model. *Combustion Theory and Modelling*, 6:339–359, 2002.

[58] C. X. Cai and Y. Zou. Sub-optimal linear quadratic control for non-standard discrete-time singularly perturbed systems. In *2004 8th International Conference on Control, Automation, Robotics and Vision*, volume 3, pages 1649–1653, 2004.

[59] M. Cakir and G. M. Amiraliyev. Numerical solution of a singularly perturbed three-point boundary value problem. *International Journal of Computer Mathematics*, 84(10):1465–1481, 2007.

[60] A.J. Calise. Singular perturbations in flight mechanics. In A. Miele and A. Salvetti, editors, *Applied Mathematics in Aerospace Science and Engineering*, pages 115–132. Plenum Press, New York, New York, 1994.

[61] L. Cao and H. M. Schwartz. Oscillation control in non-linear systems using a first-order filter. *International Journal of Control*, 75(18):1504–1524, 2002.

[62] L. Cao and H. M. Schwartz. Output feedback stabilization of linear systems with a singular perturbation model. *American Control Conference, 2002.*, 2:1627–1632, 2002.

[63] L. Cao and H. M. Schwartz. Complementary results on the stability bounds of singularly perturbed systems. *IEEE Transactions on Automatic Control*, 49:2017 – 2021, 2004.

[64] X. T. Cao and Y. C. Li. Distributed parameter singular perturbation model and cooperative control of flexible manipulators. In *Proceedings of 2005 International Conference on Machine Learning and Cybernetics*, volume 2, pages 1009 –1014, 2005.

[65] E. Castellanos-Sahagun and J. Alvarez. Synthesis of two-point linear controllers for binary distillation columns. *Chemical Engineering Communications*, 193(2):206–232, 2006.

[66] A. Cebers and H. Kalis. Mathematical modelling of an elongated magnetic droplet in a rotating magnetic field. *Mathematical Modelling and Analysis*, 17(1):47–57, Feb. 2012.

[67] S. Celikovsky, S. Papacek, A. Cervantes Herrera, and J. Ruiz Leon. Singular perturbation based solution to optimal microalgal growth problem and its infinite time horizon analysis. In *47th IEEE Conference on Decision and Control*, pages 2662 –2667, Dec. 2008.

[68] S. Celikovsky, S. Papacek, A. Cervantes Herrera, and J. Ruiz Leon. Singular perturbation based solution to optimal microalgal growth problem and its infinite time horizon analysis. In *IEEE Transactions on Automatic Control*, pages 767–772, March 2010.

[69] I. Cervantes, R. Kelly, J. Alvarez-Ramirez, and J. Moreno. A robust velocity field control. *IEEE Transactions on Control Systems Technology*, 10(6):888 – 894, Nov. 2002.

[70] A. Chakrabortty and M. Arcak. A three-time-scale redesign for robust stabilization and performance recovery of nonlinear systems with input uncertainties. *Proceedings of the 46th IEEE Conference on Decision and Control*, pages 3484–3489, 2007.

[71] A. Chakrabortty and M. Arcak. Robust stabilization and performance recovery of nonlinear systems with unmodeled dynamics. *IEEE Transactions on Automatic Control*, 54(6):1351 –1356, June 2009.

[72] A. Chakrabortty and E. Scholtz. Time-scale separation designs for performance recovery of power systems with unknown parameters and faults. *IEEE Transactions on Control Systems Technology*, 19(2):382 –390, Mar. 2011.

[73] K.W. Chang and F.A. Howes. *Nonlinear Singular Perturbation Phenomena: Theory and Application.* Springer-Verlag, New York, New York, 1984.

[74] H. H. Chen. Stability and chaotic dynamics of a rate gyro with feedback control under uncertain vehicle spin and acceleration. *Journal of Sound and Vibration*, 273:949 – 968, 2004.

[75] W. H. Chen, W. Fu, R. Du, and X. M. Lu. Exponential stability of a class of linear time-varying singularly perturbed systems. *International Conference on Information Science and Technology*, pages 778–783, 2011.

[76] W. H. Chen, G. Yuan, and W. X. Zheng. Robust stability of singularly perturbed impulsive systems under nonlinear perturbation. *IEEE Transactions on Automatic Control*, pages 1–6, 2012.

[77] W. H. Chen, G. Yuan, and W. X. Zheng. Robust stability of singularly perturbed impulsive systems under nonlinear perturbation. *Automatic Control, IEEE Transactions on*, 58(1):168–174, 2013.

[78] X. Z. Chen, M. Heidarinejad, J. F. Liu, and P. D. Christofides. Composite fast-slow mpc design for nonlinear singularly perturbed systems: Stability analysis. In *American Control Conference (ACC)*, pages 4136 –4141, June 2012.

[79] X. Z. Chen, M. Heidarinejad, J. F. Liu, D. M. Pena, and P. D. Christofides. Model predictive control of nonlinear singularly perturbed systems: Application to a reactor-separator process network. *2011 50th IEEE Conference on Decision and Control and European Control Conference*, pages 8125–8132, Dec. 2011.

[80] Y. Chen and Y. Q. Liu. Summary of singular perturbation modeling of multi-time scale power systems. In *2005 IEEE/PES Transmission and Distribution Conference and Exhibition: Asia and Pacific*, pages 1–4, 2005.

[81] Y. M. Chen. A stabilized finite element method for darcy-stokes problems. In *Third International Joint Conference on Computational Science and Optimization (CSO),*, volume 1, pages 201 –204, May 2010.

[82] Z. Y. Chen, C. N. H, and B. Wu. High order finite volume methods for singular perturbation problems. *Science in China Series A: Mathematics*, 51:1391–1400, 2008.

[83] M. B. Cheng, V. Radisavljevic, C. C. Chang, C. F. Lin, and W. C. Su. A sampled-data singularly perturbed boundary control for a heat conduction system with noncollocated observation. *IEEE Transactions on Automatic Control*, 54(6):1305 –1310, June 2009.

[84] J. Cheong, W. K. Chung, and Y. Youm. Pid composite controller and its tuning for flexible link robots. *Proceeding of the 2002 IEEE Intl. Conference of Intelligent Robots and Systems*, 19(8):2122–2127, 2002.

[85] J. Cheong, W. K. Chung, and Y. Youm. Inverse kinematics of multilink flexible robots for high-speed applications. *IEEE Transactions on Robotics and Automation*, 20(2):269 – 282, April 2004.

[86] J. Cheong, Y. Youm, and W. K. Chung. Joint tracking controller for multi-link flexible robot using disturbance observer and parameter adaptation scheme. *Journal of Robotic Systems*, 19(8):401–417, 2002.

[87] C. Chevallereau. Time-scaling control for an underactuated biped robot. *IEEE Transactions on Robotics and Automation*, 19(2):362 – 368, Apr. 2003.

[88] J. S. Chiou. Design of controllers and observer-based controller for time-delay singularly perturbed systems via composite control. *Control Theory and Applications*, 2006.

[89] J. S. Chiou. Stability bound of discrete multiple time-delay singularly perturbed systems. *International Journal of Systems Science*, 37(14):1069–1076, Nov. 2006.

[90] J. S. Chiou. Design of controllers and observer-based controllers for time-delay singularly perturbed systems via composite control. *Journal of Applied Mathematics*, 2013.

[91] J. S. Chiou and C. J. Wang. An infinite e-bound stability criterion for a class of multi-parameter singularly perturbed time-delay systems. *International Journal of Systems Science*, 36(8):485–490, June 2005.

[92] H. L. Choi and J. T. Lim. Gain scheduling control of nonlinear singularly perturbed time-varying systems with derivative information. *International Journal of Systems Science*, 36(6):357–364, May 2005.

[93] H. L. Choi, J. W. Son, and J. T. Lim. Stability analysis and control of non-standard nonlinear singularly perturbed system. *IEE Proceedings Control Theory and Applications*, 153:703 – 708, 2006.

[94] J.H. Chow. *Time-Scale Modeling of Dynamic Networks with Applications to Power Systems*, volume 46 of *Lecture Notes in Control and Information Sciences*. Springer-Verlag, Berlin, Germany, 1982.

[95] J.D. Cole. *Perturbation methods in Applied Mathematics*. Blaisdell, Waltham, MA, 1968.

[96] M. N. Contou-Carrere, V. Sotiropoulos, Y. N. Kaznessis, and P. Daoutidis. Model reduction of multi-scale chemical langevin equations. *Systems and Control Letters*, 60:75 – 86, 2011.

[97] O. L. V. Costa and F. Dufour. Singular perturbation for the discounted continuous control of piecewise deterministic markov processes. In *49th IEEE Conference on Decision and Control (CDC)*, pages 1436 –1441, Dec. 2010.

[98] J.B. Dabney, F.H. Ghorbel, and Z. Y. Wang. Modeling closed kinematic chains via singular perturbations. *Proceedings of the American Control Conference,*, 5:4104 – 4110, 2002.

[99] J. H. Dai and E. Gu. Trajectory-tracking control of a multiple flexible joint robot based on singular perturbation. *Automation Congress, Proceedings of the 5th Biannual World*, 14:77 – 82, 2002.

[100] J. H. Dai and E. Gu. Singular perturbation method in control of multiple flexible-joint robots. *Intelligent Automation and Soft Computing*, 9(2):121–128, 2003.

[101] A.R. Danilin and O.O. Kovrizhnykh. Asymptotic representation of a solution to a singular perturbation linear time-optimal problem. *Proceedings of the Steklov Institute of Mathematics*, 281(1):22–35, 2013.

[102] P. Das and S. Natesan. Richardson Extrapolation Method for Singularly Perturbed Convection-Diffusion Problems on Adaptively Generated Mesh. *CMES: Computer Modeling in Engineering and Sciences*, 90(6), 463–485, 2013. Tech Science Press.

[103] P. Das and S. Natesan. A uniformly convergent hybrid scheme for singularly perturbed system of reaction-diffusion Robin type boundary value problems. *Journal of Applied Mathematics and Computing*, 41(1-2):447–471, 2013

[104] P. Das and S. Natesan. Higher order parameter uniform convergent schemes for Robin type reaction-diffusion problems using adaptively generated grid. *International Journal of Computational Methods*, 9(4), 2012

[105] K. B. Datta and A. R. Chaudhuri. H_2/H_∞ control of discrete singularly perturbed systems: the state feedback case. *Automatica*, 38:1791 – 1797, 2002.

[106] M. L. de Cristoforis. Asymptotic behaviour of the solutions of a non-linear transmission problem for the laplace operator in a domain with a small hole. a functional analytic approach. *Complex Variables and Elliptic Equations*, 55:269–303, 2010.

[107] M. L. de Cristoforis and P. Musolino. A singularly perturbed nonlinear robin problem in a periodically perforated domain: a functional analytic approach? *Complex Variables and Elliptic Equations*, 58(4):511–536, 2013.

[108] C. DeHoet, C. Kunkel, and A. Martin. Positive solutions to singular third-order boundary value problems on purely discrete time scales. *Involve*, 6(1):113–126, 2013.

[109] M.A. Demetriou and N. Kazantzis. Natural observers for singularly perturbed mechanical systems. In *American Control Conference*, pages 4802 – 4807, 2005.

[110] S. Djennoune and M. Bettayeb. Balancing for a class of non-linear singularly perturbed systems. *International Journal of Control*, 76(2):129–138, 2003.

[111] S. Djennoune and M. Bettayeb. On the structure of energy functions of singularly perturbed bilinear systems. *International Journal of Robust and Nonlinear Control*, 15(14):601–618, 2005.

[112] S. Djennoune and M. Bettayeb. Closed-loop balancing for a class of non-linear singularly perturbed systems. *International Journal of Systems Science*, 40(3):223–235, Mar. 2009.

[113] T. S. Doan, A. Kalauch, and S. Siegmund. Exponential stability of linear time-invariant systems on time scales. *Nonlinear Dynamics and Systems Theory*, 9:37–50, 2009.

[114] J. Dong and G.-H. Yang. Robust H_∞ control for standard discrete-time singularly perturbed systems. *IET Control Theory Applications*, 1:1141 –1148, 2007.

[115] J. X. Dong and G. H. Yang. H_∞ control for fast sampling discrete-time singularly perturbed systems. *Automatica*, 44:1385–1393, 2008.

[116] V. Dragan and A.Stoica. Robust stabilization of two-time scale systems with respect to the normalized coprime factorization. *International journal of control*, 75(1):1–10, 2002.

[117] N. H. Du and V. H. Linh. Implicit-system approach to the robust stability for a class of singularly perturbed linear systems. *Systems and Control Letters*, 54:33 – 41, 2005.

[118] C. Y. Duan, Y. X. Zhang, C. Y. Dong, and R. Sun. Adaptive sliding mode control for bank-to-turn missiles. In *9th International Conference on Electronic Measurement Instruments*, pages 3512–3517, Aug. 2009.

[119] M. Van Dyke. *Perturbation Methods in Fluid Mechanics.* Academic Press, New York, NY, 1964.

[120] W. Eckhaus. *Matched Asymptotic Expansions and Singular Perturbations.* North Holland Publishing Company, Amsterdam, The Netherlands, 1973.

[121] W. Eckhaus. *Asymptotic Analysis of Singular Perturbations.* North Holland Publishing Company, Amsterdam, The Netherlands, 1979.

[122] W. Eckhaus and E.M. de Jager, editors. *Theory and Applications of Singular Perturbations.* Lecture Notes in Mathematics, Vol. 942, Springer- Verlag, Berlin, Germany, 1982.

[123] V. Ejov, J. A. Filar, and J. Thredgold. Geometric interpretation of hamiltonian cycles problem via singularly perturbed markov decision processes. *Optimization*, 52:441–458, 2003.

[124] A. Elshabrawy and H.M. Schwartz. Fuzzy sliding mode control for a singularly perturbed systems. In *IEEE Annual Meeting of the Fuzzy Information*, volume 1, pages 238 – 241, 2004.

[125] L. Erbe, T. S. Hassan, A. Peterson, and S. H. Saker. Oscillation criteria for half-linear delay dynamic equations on time scales. *Nonlinear Dynamics and Systems Theory*, 9:51–68, 2009.

[126] A. Erdélyi. *Asymptotic Expansions.* Dover Publications, New York, New York, 1956.

[127] S. Esteban and D. Rivas. Singular perturbation control of the longitudinal flight dynamics of an uav. In *International Conference on Control*, pages 310 –315, Sep. 2012.

[128] A. Fakharian and T. Gustafsson. H_2 static state feedback control of linear singular perturbation systems: A new approach. In *24th Canadian Conference on Electrical and Computer Engineering (CCECE)*, pages 01 –06, May 2011.

[129] G. File and Y. N. Reddy. Computational method for solving singularly perturbed delay differential equations with negative shift. *International Journal of Applied Science and Engineering*, 11(1):101–113, 2013.

[130] A. Fradkov and B. Andrievsky. Singular perturbations of systems controlled by energy-speed-gradient method. In *43rd IEEE Conference on Decision and Control*, volume 4, pages 3441 – 3446, 2004.

[131] L.S. Frank, editor. *Singular Perturbations I: Spaces and Singular Perturbations on Manifolds without Boundary.* Elsevier Science Publishers, Amsterdam, The Netherlands, 1990.

[132] S J. Fraser. Slow manifold for a bimolecular association mechanism. *The Journal of chemical physics*, 120, 2004.

[133] E. Fridman. Effects of small delays on stability of singularly perturbed systems. *Automatica*, 38(5):897 – 902, 2002.

[134] E. Fridman. Robust sampled-data control of linear singularly perturbed systems. In *44th IEEE Conference on Decision and Control and European Control Conference.*, pages 4324 – 4329, Dec. 2005.

[135] E. Fridman. Robust sampled-data H_∞ control of linear singularly perturbed systems. *IEEE Transactions on Automatic Control*, 51(3):470 – 475, March 2006.

[136] E. Fridman, X. R. Han, and S. K. Spurgeon. A singular perturbation approach to sliding mode control in the presence of input delay. *Nonlinear Control Systems*, pages 1272–1277, 2010.

[137] L. Fridman. Slow periodic motions in variable structure systems. *International Journal of Systems Science*, 33(14):1145–1155, 2002.

[138] L. M. Fridman. Slow periodic motions with internal sliding modes in variable structure systems. *International Journal of Control*, 75(7):524–537, 2002.

[139] L. M. Fridman. Chattering analysis in sliding mode systems with inertial sensors. *International Journal of Control*, 76:906–912, 2003.

[140] D. F. Fu and Y. Xing. Study on linear dynamic model and analysis of operating characteristics of high-power vscf wind energy conversion system. *World Non-Grid-Connected Wind Power and Energy Conference*, pages 1–6, 2009.

[141] L. Fu, L. L. Wang, and J. H. Hu. Coning algorithm based on singular perturbation. *Aircraft Engineering and Aerospace Technology*, 85(3):178–185, 2013.

[142] V. Gaitsgory. On a representation of the limit occupational measures set of a control system with applications to singularly perturbed control systems. *SIAM journal on control and optimization*, 43:325–340, 2004.

[143] V. Gaitsgory and M. T. Nguyen. Multiscale singularly perturbed control systems: Limit occupational measures sets and averaging. *Society for Industrial and Applied Mathematics*, 2002.

[144] Z. Gajić and M. Lim. *Optimal Control of Singularly Perturbed Linear Systems and Applications: High- Accuracy Techniques.* Marcel Dekker, Inc., New York, New York, 2001.

[145] Z. Gajić and Myo-Taeg Lim. Optimal control of singularly perturbed linear systems and applications. *Automatica*, 39:369–372, 2003.

[146] Z. Gajić, D. Petrovski, and X. Shen. *Singularly Perturbed and Weakly Coupled Linear Control Systems: A Recursive Approach*, volume 140 of *Lecture Notes in Control and Information Sciences.* Springer-Verlag, New York, New York, 1990.

[147] Z. Gajić and X. Shen. *Parallel Algorithms for Optimal Control of Large Scale Systems.* Springer-Verlag, London, UK, 1993.

[148] M. Galli, M. Groppi, R. Riganti, and G. Spiga. Singular perturbation techniques in the study of a diatomic gas with reactions of dissociation and recombination. *Applied Mathematics and Computation*, pages 509–531, 2003.

[149] P. S. Gandhi and F. Ghorbel. High-speed precision tracking with harmonic drive systems using integral manifold control design. *International Journal of Control*, 78(2):112–121, 2005.

[150] S. Ganjefar. Adaptive wavenet controller design for teleoperation systems with variable time delays using singular perturbation method. *International Journal of Control, Automation and Systems*, 11(3):597–607, 2013.

[151] Y. Gao, G. Lu, and Z. Wang. Passive control for continuous singular systems with non-linear perturbations. *IET Control Theory Applications*, 4(11):2554 –2564, Nov. 2010.

[152] Y. B. Gao, B. Sun, and G. P. Lu. Passivity-based integral sliding-mode control of uncertain singularly perturbed systems. *IEEE Transactions on Circuits and System-II: Express Briefs*, 58:386–390, 2011.

[153] G. Garcia, J. Daafouz, and J. Bernussou. The infinite time near optimal decentralized regulator problem for singularly perturbed systems: a convex optimization approach. *Automatica*, 38:1397 – 1406, 2002.

[154] G. Garcia and S. Tarbouriech. Control of singularly perturbed systems by bounded control. *American Control Conference*, 5:4482 – 4487, 2003.

[155] A. Garijo, S. M. Marotta, and E. D. Russell. Singular perturbations in the quadratic family with multiple poles. *Journal of Difference Equations and Applications*, 19(1):124–145, 2013.

[156] R. Genesio and M. Milanese. A note on the derivation and the use of reduced-order models. *IEEE Transactions on Automatic Control*, AC-21:118–122, February 1976. (Comprehensive set of 137 references).

[157] Z.P. Gerdtzen, P. Daoutidis, and W. S. Hu. Non-linear model reduction for metabolic networks with multiple time-scales. In *IEEE International Symposium on Mediterrean Conference on Control and Automation Intelligent Control*, pages 519 –524, 2005.

[158] J. M. Ginoux, J. L., and L. O. Chua. Canards from chua's circuit. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, 2013.

[159] V. Y. Glizer. Controllability of nonstandard singularly perturbed systems with small state delay. *Automatic Control, IEEE Transactions on*, 48:1280 – 1285, 2003.

[160] V. Y. Glizer. Observability of singularly perturbed linear time-dependent differential systems with small delay. *Journal of Dynamical and Control Systems*, 10:329–363, 2004.

[161] A. L. Goldberger, L. A. N. Amaral, J. M. Hausdorff, P. Ch. Ivanov, C. K. Peng, and H. E. Stanley. Fractal dynamics in physiology: Alterations with disease and aging. *Proceedings of the National Academy of Sciences of the United States of America*, 99:2466–2472, 2002.

[162] F. Gong and K. Khorasani. Fault diagnosis of linear singularly perturbed systems. In *44th IEEE Conference on Decision and Control, and the European Control Conference*, pages 2415 – 2420, 2005.

[163] G. Gonzalez-A and N. Barrera-G. Quasy steady state model determination using bond graph for a singularly perturbed lti system. In *16th International Conference on Methods and Models in Automation and Robotics (MMAR)*, pages 194 –199, Aug. 2011.

[164] D. Gorinevsky, E. Nwadiogbu, and D. Mylaraswamy. Model-based diagnostics for small-scale turbomachines. *Proceedings of the 41st IEEE Conference on Decision and Control*, pages 1–5, 2002.

[165] G. A. Gottwald and J. Harlim. The role of additive and multiplicative noise in filtering complex dynamical systems. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, 469(2155), 2013.

[166] G. Grammel. On nonlinear control systems with multiple time scales. *Journal of Dynamical and Control Systems*, 10:11–28, 2004.

[167] G. Grammel. Robustness of exponential stability to singular perturbations and delays. *Systems and Control Letters*, 57(6):505 – 510, 2008.

[168] G. Grammel and A.V. Obukhovskii. On the reachable set of inflated singularly perturbed differential equations. *Systems and Control Letters*, 56:416 – 422, 2007.

[169] L.T. Grujić, A.A. Martynyuk, and M. Ribbens-Pavella. *Large Scale Systems Stability Under Structural and Singular Perturbations.* Springer-Verlag, Berlin, Germany, 1987.

[170] Z. H. Guan, J. Y., and D.J. Hill. Robust H_∞ control of singular impulsive systems with uncertain perturbations. *IEEE Transactions on Circuits and Systems II Express Briefs*, 52:293 – 298, 2005.

[171] J. Guckenheimer, K. Hoffman, and W. Weckesser. The forced van der pol equation I: The slow flow and its bifurcations. *Siam J. Applied Dynamical Systems*, pages 1–35, 2003.

[172] F. El Hachemi, M. Sigalotti, and J. Daafouz. Characterization of stability transitions and practical stability of planar singularly perturbed linear switched systems. In *50th IEEE Conference on Decision and Control and European Control Conference*, pages 423 –428, Dec. 2011.

[173] F. El Hachemi, M. Sigalotti, and J. Daafouz. Stability analysis of singularly perturbed switched linear systems. *IEEE Transactions on Automatic Control*, 57(8):2116 –2121, Aug. 2012.

[174] D. K. Han and P. H. Chang. A robust two-time-scale control design for a pneumatic vibration isolator. In *46th IEEE Conference on Decision and Control*, pages 1666 –1672, 2007.

[175] H. Han and Z. Y. Huang. Tailored finite point method based on exponential bases for convection-diffusion-reaction equation. *Mathematics of Computation*, 82(281):213–226, 2013.

[176] X. Han, E. Fridman, and S.K. Spurgeon. Sliding mode control in the presence of input delay: A singular perturbation approach. *Automatica*, 48(8):1904 – 1912, 2012.

[177] F. Haq, S. Islam, and I. Aziz. Numerical solution of singularly perturbed two-point bvps using nonuniform haar wave lets. *International Journal for Computational Methods in Engineering Science and Mechanics*, 12:168–175, 2011.

[178] J. Hasson and B. Z. Bobrovsky. Optimal design of an all-digital chip timing recovery loop for direct-sequence spreads pectrum systems. *European Transactions Telecommunications*, 2004.

[179] T. Heldt, J.L. Chang, J.J.S. Chen, G.C. Verghese, and R.G. Mark. Cycle-averaged dynamics of a periodically driven, closed-loop circulation model. *Control Engineering Practice*, 13(9):1163 – 1171, 2005.

[180] E. Hinch. *Perturbation Methods*. Cambridge University Bridge, Cambridge, UK, 1991.

[181] C. S. Holling. Cross-scale morphology, geometry, and dynamics of ecosystems. *Ecological Monographs*, 62(4):pp. 447–502, 1992.

[182] M. H. Holmes. *Introduction to Perturbation Methods*. Springer, second edition edition, 2013.

[183] J. W. Hong, J. H. Yeom, S. H. Song, and I. J. Ha. A singular perturbation-like method to compensate the effect of fin-actuator dynamics in nonlinear missile control. In *SICE-ICASE International Joint Conference*, pages 837 –841, 2006.

[184] Y. J. Hong, C. Y. Jung, and J. Laminie. Singularly perturbed reactioncdiffusion equations in a circle with numerical applications. *International Journal of Computer Mathematics*, pages 1–18, 2013.

[185] F. H. Hsiao. Robustness design of fuzzy controllers for nonlinear multiple time-delay singularly perturbed systems: using dither as auxiliary. *International Journal of Systems Science*, 44(3):416–431, Mar. 2013.

[186] F. H. Hsiao, J. D. Hwang, and S. T. Pan. D-stability problem of discrete singularly perturbed systems. *International Journal of Systems Science*, 34:227–236, 2003.

[187] Y. S. Huang, X. X. Chen, S. W. Zhou, L. L. Yu, and Z. W. Wang. Hgo-based decentralised indirect adaptive fuzzy control for a class of large-scale nonlinear systems. *International Journal of Systems Science*, 43(6):1133–1145, 2012.

[188] A. Huseynov. On solutions of a nonlinear boundary value problem on time scales. *Nonlinear Dynamics and Systems Theory*, 9:69–76, 2009.

[189] R.L. Williams II and D.A. Lawrence. *Linear State-Space Control Systems*. John Wiley & Sons, Inc., New York, NY, 2007.

[190] M. Innocenti, L. Greco, and L. Pollini. Sliding mode control for two-time scale systems: stability issues. *Automatica*, 39(2):273 – 280, 2003.

[191] P.A. Ioannou and P.V. Kokotović. *Adaptive Systems with Reduced Models*, volume 47 of *Lecture Notes in Control and Information Sciences*. Springer-Verlag, Berlin, Germany, 1983.

[192] A. F. Ivanov, M. A. Mammadov, and S. I. Trofimchuk. Global stabilization in nonlinear discrete systems with time-delay. *Journal of Global Optimization*, 56(2):251–263, 2013.

[193] S. Jagodzinski and M. Lachowicz. On two incompressible hydrodynamic limits of the boltzmanncenskog equation. I. formal derivations. *Transport Theory and Statistical Physics*, 33(2):157–181, 2004.

[194] S. Jagodzinski and M. Lachowicz. On two incompressible hydrodynamic limits of the boltzmanncenskog equation II: A rigorous result. *Transport Theory and Statistical Physics*, 34(6):447–474, 2004.

[195] S. Jaison, D.S. Naidu, and D. Zydek. Time scale analysis and synthesis of wind energy conversion systems. *Proceedings of the WSEAS-NAUN 4th International Conference on Circuits, Systems, Control, Signals(CSCS '13), Valencia,Spain*, pages 21–26, Aug. 2013.

[196] S. Jayanthi and D. Del Vecchio. Retroactivity attenuation in bio-molecular systems based on timescale separation. *IEEE Transactions on Automatic Control*, 56(4):748 –761, Apr. 2011.

[197] N. Jha. A fifth order accurate geometric mesh finite difference method for general nonlinear two point boundary value problems. *Applied Mathematics and Computation*, 219(16):8425 – 8434, 2013.

[198] S. S. Jogwar, M. Baldea, and P. Daoutidis. Dynamics and control of reactor- feed effluent heat exchanger networks. *American Control Conference*, pages 1481–1486, June 2008.

[199] V.I. Kachalov. On the algebraic fundamentals of singular perturbation theory. *Differential Equations*, 49(3):386–391, 2013.

[200] M. Kadalbajoo and K. Patidar. Variable mesh spline in compression for the numerical solution of singular perturbation problems. *International Journal of Computer Mathematics*, 80(1):83–93, 2003.

[201] M. K. Kadalbajoo and A. Awasthi. Crankcnicolson finite difference method based on a midpoint upwind scheme on a non-uniform mesh for time-dependent singularly perturbed convection-diffusion equations. *International Journal of Computer Mathematics*, 85(5):771–790, 2008.

[202] M. K. Kadalbajoo and A. Jha. Exponentially fitted cubic spline for two-parameter singularly perturbed boundary value problems. *International Journal of Computer Mathematics*, 89(6):836–850, April 2012.

[203] M. K. Kadalbajoo and K. C. Patidar. Numerical solution of singularly perturbed nonlinear two point boundary value problems by spline in compression. *International Journal of Computer Mathematics*, 79(2):271–288, 2002.

[204] M. K. Kadalbajoo and K. C. Patidar. A survey of numerical techniques for solving singularly perturbed ordinary differential equations. *Applied Mathematics and Computation*, 130(23):457 – 510, 2002.

[205] M. K. Kadalbajoo and K. C. Patidar. Tension spline for the solution of self-adjoint singular perturbation problems. *International Journal of Computer Mathematics*, 79(7):849–865, 2002.

[206] M. K. Kadalbajoo and K. C. Patidar. Singularly perturbed problems in partial differential equations: a survey. *Applied Mathematics and Computation*, 134:371 – 429, 2003.

[207] M. K. Kadalbajoo and K. K. Sharma. ϵ-uniform fitted mesh method for singularly perturbed differential-difference equations: Mixed type of shifts with layer behavior. *International Journal of Computer Mathematics*, 81(1):49–62, Jan. 2004.

[208] M. K. Kadalbajoo and K. K. Sharma. Numerical analysis of boundary-value problems for singularly perturbed differential-difference equations: small shifts of mixed type with rapid oscillations. *Communications In Numerical Methods In Engineering*, 20:167–182, 2004.

[209] M.K. Kadalbajoo and Y.N. Reddy. Asymptotic and numerical analysis of singular perturbation analysis: a survey. *Applied Mathematics and Computation*, 30:223–259, 1989. (Review article with 134 references).

[210] Ahmed R. Abd-Ellateef Kamar and T. Jankowski. Generalized quazilinearization for systems of degenerate singular perturbation problems. *Nonlinear Studies*, 20(2), 2013.

[211] M. Kamenski, P. Nistri, and M. Quincampoix. Sliding mode control of uncertain systems: a singular perturbation approach. *IMA Journal of Mathematical Control and Information*, 19:377–398, 2002.

[212] H. Kando and T. Iwazumi. Multirate digital control design of an optimal regulator via singular perturbation theory. *International Journal of Control*, 44:1555–1578, 1986.

[213] K. I. Kang, K. S. Park, and J. T. Lim. Exponential stability of singularly perturbed systems with time delay and uncertainties. *International Journal of Systems Science*, pages 1–10, 2013.

[214] R. R. Kao, D. M. Green, J. Johnson, and I. Z. Kiss. Disease dynamics over very different time-scales: foot-and-mouth disease and scrapie on the network of livestock movements in the UK. *Journal of the Royal Society Interface*, 4:907–916, 2007.

[215] H. G. Kaper and T. J. Kaper. Asymptotic analysis of two reduction methods for systems of chemical reactions. *Physica D: Nonlinear Phenomena*, 165:66 – 93, 2002.

[216] B V. Kapitonov and M. A. Raupp. Boundary observation and exact control of multilayered piezoelectric body. *Mathematical methods in the applied sciences*, 26:431–452, 2003.

[217] S. Kaplun. *Fluid Mechanics and Singular Perturbations.* Academic Press, New York, New York, 1967.

[218] G. Karali and C. Sourdis. Resonance phenomena in a singular perturbation problem in the case of exchange of stabilities. *Communications in Partial Differential Equations*, 37:1620–1667, 2012.

[219] H R. Karimi, P. J. Maralani, B. Moshiri, and B. Lohmann. Numerically efficient approximations to the optimal control of linear singularly perturbed systems based on haar wavelets. *International journal of computer mathematics*, 82(4):495–507, 2005.

[220] H.R. Karimi. Robust regulation with an H_∞ constrain for linear two-time scale systems. In *IEEE International Conference on Control Applications (CCA)*, pages 1975 –1980, Sep. 2010.

[221] M. Karow, E. Kokiopoulou, and D. Kressner. On the computation of structured singular values and pseudospectra. *Systems and Control Letters*, 59(2):122 – 129, 2010.

[222] V. Kecman and Z. Gajić. Optimal control and filtering for nonstandard singularly perturbed linear systems. *Journal of Guidance, Control and Dynamics*, 22:362–365, March-April 1999.

[223] K. J. Keesman. State and parameter estimation in biotechnical batch reactors. *Control Engineering Practice*, 10(2):219 – 225, 2002.

[224] K. J. Keesman, D. Peters, and L. J.S. Lukasse. Optimal climate control of a storage facility using local weather forecasts. *Control Engineering Practice*, 11:505–516, 2003.

[225] R. B. Kellogg and M. Stynes. A singularly perturbed convection-diffusion problem in a half-plane. *Applicable Analysis*, 85(12):1471–1485, 2006.

[226] J. Kevorkian and J.D. Cole. *Perturbation Methods in Mathematics*. Springer-Verlag, New York, New York, 1981.

[227] J.K. Kevorkian and J.D. Cole. *Multiple Scale and Singular Perturbation Methods*. Springer-Verlag, New York, New York, 1996.

[228] H.K. Khalil. Feedback control of nonstandard singularly perturbed systems. *IEEE Transactions on Automatic Control*, AC-34:1052–1060, 1989.

[229] H.K. Khalil. Two-time scale and averaging methods. In W.S. Levine, editor, *The Control Handbook*, pages 873–879. CRC Press, Boca Raton, FL, 1996.

[230] H.K. Khalil. *Nonlinear Systems, Third Edition*. Prentice Hall, Englewood Cliffs, NJ, 2002.

[231] B. S. Kim, Y. J. Kim, and M. T. Lim. LQG control for nonstandard singularly perturbed discrete time systems. In *Proceedings of the 41st IEEE Conference on Decision and Control*, volume 4, pages 3736 – 3741, Dec. 2002.

[232] Y.-J. Kim, B.-S. Kim, and M.-T. Lim. Composite control for singularly perturbed bilinear systems via successive galerkin approximation. *IEE Proceedings Control Theory and Applications*, 150:483–488, 2003.

[233] Y.-J. Kim, B.-S. Kim, and M.-T. Lim. Finite-time composite control for a class of singularly perturbed nonlinear systems via successive galerkin approximation. *IEE Proceedings Control Theory and Applications*, 152(5):507 – 512, Sep. 2005.

[234] J.W. Kimball and P.T. Krein. Singular perturbation theory for DC-DC converters and application to PFC converters. In *IEEE Power Electronics Specialists Conference*, pages 882 –887, June 2007.

[235] J.W. Kimball and P.T. Krein. Singular perturbation theory for DC-DC converters and application to PFC converters. *IEEE Transactions on Power Electronics*, 23(6):2970 –2981, Nov. 2008.

[236] P.V. Kokotović. Applications of singular perturbation techniques to control problems. *SIAM Review*, 26:501–550, 1984. (Review article with about 250 references).

[237] P.V. Kokotović. Recent trends in feedback design: an overview. *Automatica*, 21:225–236, 1985. (Review article with about 85 references).

[238] P.V. Kokotović, A. Bensoussan, and G. Blankenship, editors. *Singular Perturbations and Asymptotic Analysis in Control Systems*, volume 90 of *Lecture Notes in Control and Information Sciences*. Springer-Verlag, Berlin, Germany, 1987.

[239] P.V. Kokotović and H.K. Khalil, editors. *Singular Perturbations in Systems and Control*. IEEE Press, New York, NY, 1986. Reprint.

[240] P.V. Kokotović, H.K. Khalil, and J. O'Reilly. *Singular Perturbation Methods in Control: Analysis and Design*. Academic Press, London, UK, 1986. Republished as Volume 25 under *Classics in Applied Mathematics* by the Society of Industrial and Applied Mathematics (SIAM), Philadelphia, PA, in 1999.

[241] P.V. Kokotović, R.E. O'Malley, Jr., and P. Sannuti. Singular perturbations and order reduction in control theory-an overview. *Automatica*, 12:123–132, 1976. (Review article with about 125 references).

[242] P.V. Kokotović and W.R. Perkins, editors. *Singular Perturbations: Order Reduction in Control Systems Design*. American Society of Mechanical Engineers, New York, New York, 1972.

[243] P.V. Kokotović and P. Sannuti. Singular perturbation method for reducing model order in optimal control design. *IEEE Transactions on Automatic Control*, AC-13:377–384, 1968.

[244] V. Krishnamurthy, S. M. Monfared, and B. Cornell. Ion channel biosensors -part II: Dynamic modeling, analysis, and statistical signal processing. *IEEE Transactions on Nanotechnology*, 9(3):313 –321, May 2010.

[245] H. Krishnan and N.H. McClamroch. On the connection between nonlinear differential-algebraic equations and singularly perturbed systems in nonstandard form. *IEEE Transactions on Automatic Control*, 39:1079–1084, 1994.

[246] M. Kumar, P. Singh, and H. K. Mishra. An initial-value technique for singularly perturbed boundary value problems via cubic spline. *International Journal for Computational Methods in Engineering Science and Mechanics*, 8:419–427, 2007.

[247] M. Kumar, P. Singh, and H. K. Mishra. A recent survey on computational techniques for solving singularly perturbed boundary value problems. *International Journal of Computer Mathematics*, 84(10):1439–1463, 2007.

[248] M. Kumar and A. Srivastava. An elementary introduction to recently developed computational methods for solving singularly perturbed partial differential equations arising in science and engineering. *International Journal for Computational Methods in Engineering Science and Mechanics*, 14:45–60, 2012.

[249] M. Kumar and A. Srivastava. An elementary introduction to recently developed computational methods for solving singularly perturbed partial differential equations arising in science and engineering. *International Journal for Computational Methods in Engineering Science and Mechanics*, 14(1):45–60, 2013.

[250] M. Kumar and S. Tiwari. An initial-value technique to solve third-order reactioncdiffusion singularly perturbed boundary-value problems. *International Journal of Computer Mathematics*, 89(17):2345–2352, Nov. 2012.

[251] G.A. Kurina. Singular perturbations of control problems with equations of state not solved for the derivative (a survey). *Journal of Computer and System Sciences International*, 31(6):17–45, 1993. (135 references).

[252] G.A. Kurina. Asymptotic expansion of solutions of optimal control problems for discrete weakly controllable systems. *Journal of Applied Mathematics and Mechanics*, 66(2):201 – 213, 2002.

[253] H.J. Kushner. *Weak Convergence Methods and Singularly Perturbed Stochastic Control and Filtering Problems*. Birkhäuser, Boston, MA, 1990.

[254] P.A. Lagerstrom and R.G. Casten. Basic concepts underlying singular perturbation techniques. *SIAM Review*, 14:63–120, 1972. (survey with 25 references on the essential ideas and not on literature).

[255] S. H. Lam. Model reductions with special csp data. *Combustion and Flame*, 2013.

[256] X. Lamy and P. Mironescu. Existence of critical points with semi-stiff boundary conditions for singular perturbation problems in simply connected planar domains. *Mathematics Application*, pages 1–47, Mar. 2013.

[257] J. I. Lee and I. J. Ha. A novel approach to control of nonminimum-phase nonlinear systems. *IEEE Transactions on Automatic Control*, 47:1480 – 1486, Sep. 2002.

[258] K. Lee, K. H. Shim, and M. E. Sawan. Unified modeling for singular perturbed systems by delta operator: Pole assignment case. In Tag Kim, editor, *Artificial Intelligence and Simulation*, volume 3397, pages 24–32. Springer Berlin / Heidelberg, 2005.

[259] J. Li, K. Lu, and P. Bates. Normally hyperbolic invariant manifolds for random dynamical systems: Part 1-persistence. *Transactions of The American Mathematical Society*, pages 1–34, 2013.

[260] P. Li, B.H. Zhang, J. Shu, Z.Q. Bo, and A. Klimek. Research on order reduction of power system modeling for dynamic voltage stability analysis. In *IEEE PES Transmission and Distribution Conference and Exposition*, pages 1 –5, Apr. 2010.

[261] T. H.S. Li and K. J. Lin. Composite fuzzy control of nonlinear singularly perturbed systems. *IEEE Transactions on Fuzzy Systems*, 15:176 –187, 2007.

[262] X.Y. Li, X.H. Chen, and G.Q. Tang. Output feedback control of doubly-fed induction generator based on multi-time scale model. In *Third International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*, pages 2798 –2802, April 2008.

[263] Y. Li, J. L. Wang, and G. H. Yang. Reliable linear-quadratic control for singularly perturbed systems. *International Journal of Systems Science*, 33(12):949–958, 2002.

[264] Y. Li, S. J. Ye, and X. M. Wang. Guaranteed cost control for singularly perturbed uncertain systems. In *Sixth International Conference on Intelligent Systems Design and Applications*, volume 1, pages 1020 –1025, 2006.

[265] J. Lin. Hierarchical fuzzy logic controller for a flexible link robot arm performing constrained motion tasks. *IEE Proceedings Control Theory and Applications*, 150:355–364, 2003.

[266] J. Lin and F. L. Lewis. Fuzzy controller for flexible-link robot arm by reduced-order techniques. *IEE Proceedings Control Theory and Applications*, 149:177 –187, May 2002.

[267] K. J. Lin. Composite observer-based feedback design for singularly perturbed systems via lmi approach. In *Proceedings of SICE Annual Conference 2010*, pages 3056 –3061, Aug. 2010.

[268] K. J. Lin. Neural network based observer and adaptive control design for a class of singularly perturbed nonlinear systems. In *8th Asian Control Conference*, pages 1176 –1180, May 2011.

[269] K. J. Lin. Stabilisation of singularly perturbed nonlinear systems via neural network-based control and observer design. *International Journal of Systems Science*, 44(10):1925–1933, 2013.

[270] K. J. Lin and T.H.S. Li. Stabilization of uncertain singularly perturbed systems with pole-placement constraints. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 53:916 –920, 2006.

[271] Y. Y. Lin-Chen and D. P. Goodall. Stabilizing feedbacks for imperfectly known, singularly perturbed nonlinear systems with discrete and distributed delays. *International Journal of Systems Science*, 35(15):869–887, Dec. 2004.

[272] D. Liu, L. Liu, and Y. Yang. Control of discrete-time singularly perturbed systems via static output feedback. *Abstract and Applied Analysis*, 2013.

[273] H. P. Liu, F. C. Sun, and Z. Q. Sun. Stability analysis and synthesis of fuzzy singularly perturbed systems. *IEEE Transactions on Fuzzy Systems*, 13:273 – 284, 2005.

[274] I. Lizarraga and V. Etxebarria. Combined PD-H_∞ approach to control of flexible link manipulators using only directly measurable variables. *Cybernetics and Systems*, 34(1):19–31, 2003.

[275] F. Ma and L. J. Fu. Principle of multi-time scale order reduction and its application in AC/DC hybrid power systems. In *International Conference on Electrical Machines and Systems*, pages 3951 –3956, Oct. 2008.

[276] A. Machina, R. Edwards, and P. van den Driessche. Singular dynamics in gene network models. *SIAM Journal on Applied Dynamical Systems*, 12(1):95–125, 2013.

[277] M.S. Mahmoud and M.G. Singh. *Discrete Systems: Analysis, Control, and Optimization.* Springer-Verlag, Berlin, Germany, 1984.

[278] I. Malloci and J. Daafouz. Stabilisation of polytopic singularly perturbed linear systems. *International Journal of Control*, 85(2):135–142, Feb. 2012.

[279] I. Malloci, J. Daafouz, and C. Iung. Stabilization of continuous-time singularly perturbed switched systems. In *48th IEEE Conference on Decision and Control, held jointly with the 28th Chinese Control Conference*, pages 6371 –6376, Dec. 2009.

[280] I. Malloci, J. Daafouz, and C. Iung. Stability and stabilization of two time scale switched systems in discrete time. *IEEE Transactions on Automatic Control*, 55(6):1434 –1438, June 2010.

[281] B. Manhartsgruber. Stability analysis of the servohydraulic equations with linear state feedback and harmonic reference signal. *Nonlinear dynamics in engineering systems*, 2003.

[282] A. Massoum, M.K. Fellah, A. Meroufel, P. Wira, and A. Bendaoud. Neuro-fuzzy control of a singularly perturbed permanent magnet synchronous machine fed by a three levels inverter. *IEEE International Symposium on Industrial Electronics*, pages 1867 – 1872, 2008.

[283] S. Mastellone, D.M. Stipanovic, and M.W. Spong. Multi-agent formation control and trajectory tracking via singular perturbation. In *IEEE International Conference on Control Applications*, pages 557 –562, 2007.

[284] N.H. McClamroch and H. Krishnan. Nonstandard singularly perturbed control systems and differential-algebraic equations. *International Journal of Control*, 55:1239–1253, 1992.

[285] K. Mease. Multiple time-scales in nonlinear flight mechanics: diagnosis and modeling. *Applied Mathematics and Computation*, 164:627–648, 2005.

[286] J. Medanic. Design of nonlinear controls using structured representations. *42nd IEEE Conference on Decision and Control*, 4:4163 – 4168, 2003.

[287] J. Melenk, C. Xenophontos, and L. Oberbroeckling. Analytic regularity for a singularly perturbed system of reaction-diffusion equations with multiple scales. *Advances in Computational Mathematics*, pages 1–28, 2012.

[288] R. Meyer and S.V. Parter, editors. *Singular Perturbations and Asymptotics.* Academic Press, New York, NY, 1980.

[289] A. Meyer-Baese, S. Cappendijk, and E. Althaus. Global uniform stability analysis of biological networks with different time-scales under perturbations. In *International Joint Conference on Neural*, pages 2098 –2102, june 2009.

[290] A. Meyer-Baese, S. S. Pilyugin, and Y. Chen. Global exponential stability of competitive neural networks with different time scales. *IEEE Transactions on Neural Networks*, 14:716–719, 2003.

[291] A. Mezouar, M. K. Fellah, S. Hadjeri, and Y. Sahali. Adaptive speed sensorless vector control of induction motor using singularly perturbed sliding mode observer. *32nd Annual Conference on IEEE Industrial Electronics*, pages 932–939, 2006.

[292] E.F. Mishchenko and N.K. Rozo. *Differential Equations with Small Parameters and Relaxed Oscillations*. Plenum Publishing Company, New York, New York, 1980.

[293] H. Mishra and S. Saini. Numerical solution of singularly perturbed two-point boundary value problem via liouville-green transform. *American Journal of Computational Mathematics*, 3(1):1–5, 2013.

[294] H.B. Mogaver, A. Zare, and M.R. Tanhatalab. Singular perturbation method for robust control of nonlinear systems. In *International Conference on Energy, Power and Control*, pages 1 –6, Dec. 2010.

[295] R. K. Mohanty, D. J. Evans, and U. Arora. Convergent spline in tension methods for singularly perturbed two-point singular boundary value problems. *International Journal of Computer Mathematics*, 82(1):55–66, Jan 2005.

[296] R. K. Mohanty, N. Jha, and D. J. Evans. Spline in compression method for the numerical solution singularly perturbed two-point singular boundary-value problems. *International Journal of Computer Mathematics*, 81(5):615–627, May 2004.

[297] S.M. Monfared, V. Krishnamurthy, and B. Cornell. Stochastic modeling and signal processing of nano-scale protein-based biosensors. In *IEEE International Workshop on Genomic Signal Processing and Statistics*, pages 1–6, 2009.

[298] D. Moreira and L. H. Wang. Singular perturbation method for inhomogeneous nonlinear free boundary problems. *Calculus of Variations and Partial Differential Equations*, pages 1–25, 2013.

[299] H. Mukaidani. Nash games for multiparameter singularly perturbed systems with uncertain small singular perturbation parameters. *IEEE Transactions on Circuits and Systems II: Express Briefs,*, 52:586 – 590, 2005.

[300] H. Mukaidani. Recursive approach of optimal kalman filtering problem for multiparameter singularly perturbed systems. *International Journal of Systems Science*, 36(1):1–11, 2005.

[301] H. Mukaidani. Local uniqueness for nash solutions of multiparameter singularly perturbed systems. *IEEE Transactions on Circuits and Systems II: Express Briefs,*, 53:1103 –1107, 2006.

[302] H. Mukaidani, T. Shimomura, and H. Xu. Numerical computation of cross-coupled algebraic riccati equations related to H_2/H_∞ control problem for singularly perturbed systems. *International Journal of Robust and Nonlinear Control*, 14:697–717, 2004.

[303] H. Mukaidani, H. Xu, and K.Mizukami. A revised kleinman algorithm to solve algebraic Riccati equation of singularly perturbed systems. *Automatica*, 38:553–558, 2002.

[304] H. Mukaidani, H. Xu, and K. Mizukami. New results for near-optimal control of linear multiparameter singularly perturbed systems. *Automatica*, 39:2157 – 2167, 2003.

[305] B. Munsky, S. Peles, and M. Khammash. Stochastic analysis of gene regulatory networks using finite state projections and singular perturbation. *Proceedings of the 2007 American Control Conference*, pages 1323–1328, 2007.

[306] J.A. Murdock. *Perturbations: Theory and Methods*. John Wiley & Sons, New York, NY, 1991.

[307] D. S. Naidu. Analysis of non-dimensional forms of singular perturbation structures for hypersonic vehicles. *Acta Astronautica*, 66:577–586, 2010.

[308] D. S. Naidu. Singular perturbation analysis of a flexible beam used in underwater exploration. *International Journal of Systems Science*, 42(1):183–194, Jan. 2011.

[309] D.S. Naidu. *Singular Perturbation Methodology in Control Systems*, volume 34 of *IEE Control Engineering Series*. Peter Peregrinus Limited, Stevenage Herts, UK, 1988.

[310] D.S. Naidu. Guidance and control strategies for aeroassisted orbital transfer: status survey. In *Proceedings of the AIAA Atmospheric Flight Mechanics Conference*, Scottsdale, AZ, August 1-3 1994. (Review article with 20 important references).

[311] D.S. Naidu. Singular perturbations and time scales in aerospace systems: An overview. In S. Sivasundaram, editor, *Nonlinear Problems in Aviation and Aerospace*, pages 251–263. Gordon and Breach Science Publishers, UK, 2000. (Review article with 84 references).

[312] D.S. Naidu. Singular perturbations and time scales in control theory and applications: overview. *Dynamics of Continuous, Discrete and Impulsive Systems (DCDIS) Journal*, 9(2):233–278, June 2002. (Invited survey paper with 467 references in a Special Issue on Singularly Perturbed Dynamic Systems in Control Technology, edited by Z. Gajic).

[313] D.S. Naidu and A.J. Calise. Singular perturbations and time scales in control theory and applications: survey 1983-1989. In *IFAC Workshop on Singular Perturbations and Asymptotic Methods in Systems and Control*, Boston, MA, August 17-18 1989.

[314] D.S. Naidu and A.J. Calise. Singular perturbations and time scales in guidance, navigation and control of aerospace systems: survey. In *Proceedings of the AIAA Guidance, Navigation and Control Conference*, pages 1338–1362, Baltimore, MD, August 7-10 1995. (Invited Survey Paper with 246 references).

[315] D.S. Naidu and A.J. Calise. Singular perturbations and time scales in guidance and control of aerospace systems: survey. *AIAA Journal of Guidance, Control and Dynamics*, 24(6):1057–1078, November-December 2001. (Survey paper with 412 references).

[316] D.S. Naidu, D.B. Price, and J.L. Hibey. Singular perturbations and time scales in discrete control systems-an overview. In *Proceedings of The 26th IEEE Conference on Decision and Control*, pages 2096–2103, Los Angeles, CA, December 1987. (Invited survey paper with 121 references).

[317] D.S. Naidu and A.K. Rao. *Singular Perturbation Analysis of Discrete Control Systems*, volume 1154 of *Lecture Notes in Mathematics*. Springer-Verlag, New York, NY, 1985.

[318] K. Nakada, Y. D. Sato, and K. Matsuoka. Theoretical analysis of phase resetting on matsuoka oscillators. In *Advances in Cognitive Neurodynamics (III)*, pages 531–536. Springer Netherlands, 2013.

[319] Y. Nakayama. Holographic interpretation of renormalization group approach to singular perturbations in non-linear differential equations. *High Energy Physics - Theory*, 2013.

[320] A.H. Nayfeh. *Perturbation Methods*. Wiley-Interscience, New York, NY, 1973.

[321] A.H. Nayfeh. *Introduction to Perturbation Techniques*. John-Wiley & Sons, New York, NY, 1981.

[322] A.H. Nayfeh. *Problems in Perturbation*. John-Wiley & Sons, New York, NY, 1985.

[323] S.K. Nguang, W. Assawinchaichote, and P. Shi. Robust h1 control design for fuzzy singularly perturbed systems with markovian jumps: an lmi approach. *IET Control Theory Applications*, 1:893 –908, 2007.

[324] H. M. Nguyen and D. S. Naidu. Singular perturbation analysis and synthesis of wind energy conversion systems under stochastic environments. *Advances in Systems Theory, Signal Processing and Computational Science*, pages 283–288, 2012.

[325] H. M. Nguyen and D. S. Naidu. Time scale analysis and control of wind energy conversion systems. *5th International Symposium on Resilient Control Systems (ISRCS)*, pages 149–154, Agu. 2012.

[326] S. L. Nguyen and G. Yin. Asymptotic properties of markov-modulated random sequences with fast and slow timescales. *Stochastics: An International Journal of Probability and Stochastics Processes*, 82(5):445–474, Oct. 2010.

[327] T. Nguyen and Z. Gajic. Solving the singularly perturbed matrix differential Riccati equation: A Lyapunov equation approach. In *American Control Conference*, pages 782–787, July 2010.

[328] T. Nguyen, W. C. Su, and Z. Gajic. Singular perturbation analysis of discrete-time output feedback sliding mode control with disturbance attenuation. In *American Control Conference*, pages 757 –762, June 2009.

[329] T. Nguyen, W. C. Su, and Z. Gajic. Variable structure control for singularly perturbed linear continuous systems with matched disturbances. *IEEE Transactions on Automatic Control*, pages 777–783, Mar. 2012.

[330] L. Nie and Zhi D. Teng. Singular perturbation method for global stability of ratio-dependent predator-prey models with stage structure for the prey. *Electronic Journal of Differential Equations*, 2013(86):1–9, 2013.

[331] J. Niu, J. Zhao, Z. H. Xu, and J. X. Qian. A two-time scale decentralized model predictive controller based on input and out put model. *Journal of Automated Methods and Management in Chemistry*, pages 1–11, 2009.

[332] Touhami O., Mezouar A.E.K., Ibtiouen R., and Mekhtoub S. Sliding mode control and flux observer for a singularly perturbed model of an induction machine. In *IEEE International Conference on Industrial Technology*, volume 1, pages 108 – 114, 2004.

[333] J. B. Oliveira and A. D. Araujo. Design and stability analysis of an indirect variable structure model reference adaptive control. *International Journal of Control*, 81(12):1870–1877, 2008.

[334] H. Oloomi, M. Saif, and B. Shafai. On the well posedness of singularly perturbed fault detection filters. In *Proceedings of the 2004 American Control Conference*, volume 6, pages 5676 –5677, 2004.

[335] H. Oloomi and B. Shafai. Reduced-order controller design with low sensitivity to small feedback delays. *International Journal of Systems Science*, 35(9):547–555, July 2004.

[336] H. Oloomi and B. Shafai. Two-time-scale distributions and singular perturbations. *International Journal of Control*, 77(11):1040–1049, 2004.

[337] H. Oloomi and B. Shafai. Realization theory for two-time-scale distributions through approximation of markov parameters. *International Journal of Systems Science*, 39(2):127–138, Feb. 2008.

[338] R.E. O'Malley, Jr. Topics in singular perturbations. In *Advances in Mathematics*, volume 2, pages 356–470. Academic Press, New York, New York, 1968. (Review article with 183 references).

[339] R.E. O'Malley, Jr. *Introduction to Singular Perturbations.* Academic Press, New York, NY, 1974.

[340] R.E. O'Malley, Jr. *Singular Perturbation Methods for Ordinary Differential Equations.* Springer-Verlag, New York, NY, 1991.

[341] C. Ott, A. Albu-Schaffer, and G. Hirzinger. Comparison of adaptive and nonadaptive tracking control laws for a flexible joint manipulator. *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2:2018 – 2024, Oct. 2002.

[342] P. Padmaja and Y. N. Reddy. A numerical patching method for solving singular perturbation problems via pad approximates. *International Journal of Applied Science and Engineering*, 11(1):51–67, 2013.

[343] C. K. Pang, F. L. Lewis, S. S. Ge, G. X. Guo, B. M. Chen, and T. H. Lee. Singular perturbation control for vibration rejection in hdds using the PZT active suspension as fast subsystem observer. *IEEE Transactions on Industrial Electronics*, 54:1375 –1386, 2007.

[344] A. Pant, P. Seiler, and K. Hedrick. Mesh stability of look-ahead interconnected systems. *IEEE Transactions on Automatic Control*, 47(2):403 –407, Feb. 2002.

[345] K. S. Park and J. T. Lim. Exponential stabilisation of non-standard nonlinear singularly perturbed system. In *IET Control Theory and Applications*, pages 871–878, June 2010.

[346] K. S. Park and J. T. Lim. Time-scale separation of nonlinear singularly perturbed discrete systems. In *2010 International Conference on Control Automation and Systems (ICCAS)*, pages 892 –895, Oct. 2010.

[347] K. S. Park and J. T. Lim. Robust stability of non-standard nonlinear singularly perturbed discrete systems with uncertainties. *International Journal of Systems Science*, pages 1–9, 2012.

[348] No-Cheol Park, Hyun-Seok Yang, Hyung-Wug Park, and Young-Pil Park. Position/vibration control of two-degree-of-freedom arms having one flexible link with artificial pneumatic muscle actuators. *Robotics and Autonomous Systems*, 40(4):239 – 253, 2002.

[349] K. C. Patidar. On the use of nonstandard finite difference methods. *Journal of Difference Equations and Applications*, 11(8):735–758, July 2005.

[350] S.D. Pekarek, M.T. Lemanski, and E.A. Walters. On the use of singular perturbations to neglect the dynamic saliency of synchronous machines. *IEEE Transactions on Energy Conversion*, 17:385–391, 2002.

[351] S. T. Peng, J. J. Sheu, and C. C. Chang. A control scheme for automatic path tracking of vehicles subject to wheel slip constraint. In *Proceedings of the 2004 American Control Conference*, volume 1, pages 804 –809, 2004.

[352] A. Poliakovsky. On the γ-limit of singular perturbation problems with optimal profiles which are not one-dimensional. part i: The upper bound. *Differential Integral Equations*, 26(9):1179–1234, 2013.

[353] C. Potzsche. Slow and fast variables in non-autonomous difference equations. *Journal of Difference Equations and Applications*, 9(5):473–487, Dec. 2003.

[354] N. Prljaca and Z. Gajic. General transformation for block diagonalization of multitime-scale singularly perturbed linear systems. *IEEE Transactions on Automatic Control*, 53(5):1303 –1305, 2008.

[355] N. Prljaca and Z. Gajic. A method for optimal control and filtering of multitime-scale linear singularly-perturbed stochastic systems. *Automatica*, 44:2149 – 2156, 2008.

[356] M. Prochaska, F. A. Probst, and W. Mathis. Analysis of emitter-coupled multivibrators by singularly perturbed systems. *Mathematical and Computer Modelling of Dynamical Systems*, 13(6):531–543, Dec. 2007.

[357] Z. Q. Qi, G. D. Hu, Z. H. Yang, and F. E. Zhang. Flight guidance control using genetic algorithm combined with singular perturbation technique. *International Conference on Machine Learning and Cybernetics*, 2:1034–1038, 2003.

[358] Q. Q. Qiao and W. C. Chen. A near optimal midcourse guidance law for air-to-air missile. *Applied Mechanics and Materials*, pages 833–838, 2013.

[359] P. Rai and K. K. Sharma. Parameter uniform numerical method for singularly perturbed differential-difference equations with interior layers. *International Journal of Computer Mathematics*, 88(16):3416–3435, Nov. 2011.

[360] P. Rai and K.K. Sharma. The numerical study of singularly perturbed differential-difference turning point problems: Twin boundary layers. In *Numerical Mathematics and Advanced Applications*, pages 285–292. Springer Berlin Heidelberg, 2013.

[361] R. V. Ramnath. Multiple scales theory and aerospace applications. In *Multiple Scales Theory and Aerospace Applications*, 2010.

[362] H. Ramos and J. Vigo-Aguiar. A new algorithm appropriate for solving singular and singularly perturbed autonomous initial-value problems. *International Journal of Computer Mathematics*, 85(3-4):603–611, 2008.

[363] J. Rashidinia, R. Mohammadi, and S. H. Moatamedoshariati. Quintic spline methods for the solution of singularly perturbed boundary-value problems. *International Journal for Computational Methods in Engineering Science and Mechanics*, 11:247–257, 2010.

[364] B. Rasmussen, A. Alleyne, and R. Shah. Reduced order modeling of transcritical ac system dynamics using singular perturbation. In *Proceedings of the 2003 American Control Conference*, volume 3, pages 2264 – 2269, June 2003.

[365] C. Rattanakul, Y. Lenbury, N. Krishnamara, and D. J. Wollkind. Modeling of bone formation and resorption mediated by parathyroid hormone: response to estrogen/PTH therapy. *BioSystems*, 70:55–72, 2003.

[366] B. G. Rawn, P. W. Lehn, and M. Maggiore. Toward controlled wind farm output: adjustable power filtering. *IEEE Power Engineering Society General Meeting*, pages 1–6, 2006.

[367] B.G. Rawn, P.W. Lehn, and M. Maggiore. Control methodology to mitigate the grid impact of wind turbines. *IEEE Transactions on Energy Conversion*, 22(2):431 –438, June 2007.

[368] H. C. Renezeder, A. Steindl, and H. Troger. On the dynamics of axially moving strings. *Proc. Appl. Math. Mech*, 4(1):201–202, 2004.

[369] R. Riaza and P. J. Zufiria. Differential-algebraic equations and singular perturbation methods in recurrent neural learning. *Dynamical Systems*, 18(1):89–105, 2003.

[370] M. D. Riva. Stokes flow in a singularly perturbed exterior domain. *Complex Variables and Elliptic Equations*, 58(2):231–257, 2013.

[371] M. D. Riva and M. L. de Cristoforis. Microscopically weakly singularly perturbed loads for a nonlinear traction boundary value problem: a functional analytic approach. *Complex Variables and Elliptic Equations*, 55:771–794, 2010.

[372] D. Romeres, F. Dorfler, and F. Bullo. Novel results on slow coherency in consensus and power networks. *European Control Conference*, 2013.

[373] S. G. Ruan and D. M. Xiao. Stability of steady states and existence of travelling waves in a vector-disease model. *Proceedings of the Royal Society of Edinburgh*, pages 991–1011, 2004.

[374] J. Runge and B.R. Oswald. Modelling of a controlled doubly fed induction machine for the use in offshore wind power plants. *39th International Universities Power Engineering Conference*, pages 1155–1159, Sep. 2004.

[375] L. Rybarska-Rusinek and L. Socha. String stability of singularly perturbed stochastic systems. *Stochastic Analysis and Applications*, 25(4):719–737, 2007.

[376] M. Sagara, H. Mukaidani, and V. Dragan. Near-optimal control for multiparameter singularly perturbed stochastic systems. *Optimal Control Applications and Methods*, 32:113–125, 2011.

[377] V.R. Saksena, J. O'Reilly, and P.V. Kokotović. Singular perturbations and time-scale methods in control theory: survey 1976-1983. *Automatica*, 20:273–293, 1984. (Review article with about 350 references).

[378] A. A. Salama and S. A. Bakr. Optimal extended one-step schemes of exponential type for stiff initial-value problems. *International Journal of Computer Mathematics*, 81(11):1363–1379, Nov. 2004.

[379] R. Salinas and S. Drakunov. Higher-order optimal control design via singular perturbation. In *Proceedings of the 2005 American Control Conference*, pages 4631 – 4636, 2005.

[380] H. Salmasi, R. Fotouhi, and P. N. Nikiforuk. On the stability of a friction compensation strategy for flexible-joint manipulators. *Advanced Robotics*, 24(15):2059–2086, 2010.

[381] A.E. Sanchez-Orta, J. De Len-Morales, and E. Lopez-Toledo. Discrete-time nonlinear control scheme for small synchronous generator. *International Journal of Electrical Power and Energy Systems*, 24(9):751 – 764, 2002.

[382] S. Sancho, A. Suarez, and J. Chuan. General envelope-transient formulation of phase-locked loops using three time scales. *IEEE Transactions on Microwave Theory and Techniques*, 52:1310 – 1320, 2004.

[383] R. G. Sanfelice and A. R. Teel. On singular perturbations due to fast actuators in hybrid control systems. *Automatica*, 47(4):692 – 701, 2011.

[384] P. Sannuti. *Singular perturbation method in the theory of optimal control.* PhD thesis, University of Illinois, Urbana-Champaign, 1968.

[385] P. Sannuti and P.V. Kokotović. Near optimum design of linear systems by singular perturbation method. *IEEE Transactions on Automatic Control*, AC-14:15–22, 1969.

[386] A.R. Sarakhsi, S. Ashrafi, M. Jahanshahi, and M. Sarakhsi. Investigation of boundary layers in some singular perturbation problems including fourth order ordinary differential equations. *World Applied Sciences Journal*, 22(12):1695–1701, 2013.

[387] S. Sastry. *Nonlinear Systems: Analysis, Stability, and Control.* Springer-Verlag, New York, NY, 1999.

[388] N. Savva and S. Kalliadasis. Droplet motion on inclined heterogeneous substrates. *Journal of Fluid Mechanics*, pages 462–491, Jun. 2013.

[389] C. Schwartz and A.H. Haddad. Stability criteria for linear periodic switched systems. In *Proceedings of the 2003 American Control Conference*, volume 4, pages 3215 – 3219, 2003.

[390] B. Sedghi, B. Srinivasan, and R. Longchamp. Control of hybrid systems via dehybridization. *Proceedings of the 2002 American Control Conference*, 1:692 – 697, 2002.

[391] D. Selisteanu, E. Petre, M. Roman, and E. Bobasu. Structural properties and reduced order modeling of a class of bioprocesses. *Proceedings of the 2010 International Conference on Modeling, Identification and Control*, pages 88–93, 2010.

[392] V. Shanthi and N. Ramanujam. An asymptotic numerical method for fourth order singular perturbation problems with a discontinuous source term. *International Journal of Computer Mathematics*, 85(7):1147–1159, 2008.

[393] Z. Shao. Robust stability of singularly perturbed systems with state delays. *IEE Proceedings Control Theory and Applications*, 150:2 – 6, 2003.

[394] Z. H. Shao. Robust stability of two-time-scale systems with nonlinear uncertainties. *IEEE Transactions on Automatic Control*, 49:258–261, 2004.

[395] Z. Y. Shao and X. D. Zhang. Intelligent control of flexible-joint manipulator based on singular perturbation. In *IEEE International Conference on Automation and Logistics (ICAL)*, pages 243 –248, Aug. 2010.

[396] H. S. Shen and Y. Xiang. Postbuckling of nanotube-reinforced composite cylindrical shells under combined axial and radial mechanical loads in thermal environment. *Composites Part B: Engineering*, 52(0):311–322, 2013.

[397] K. H. Shim and M. E. Sawan. Rationalization of singularly perturbed continuous systems with time delays. *International Journal of Systems Science*, 34(4):263–268, Mar. 2003.

[398] K. H. Shim and M. E. Sawan. Approximate controller design for singularly perturbed aircraft systems. *Aircraft Engineering and Aerospace Technology: An International Journal*, pages 311–316, 2005.

[399] K. H. Shim and M. E. Sawan. Singularly perturbed unified time systems with low sensitivity to model reduction using delta operators. *International Journal of Systems Science*, 37(4):243–251, Mar. 2006.

[400] K. H. Shim and M. Edwin Sawan. Near-optimal state feedback design for singularly perturbed systems by unified approach. *International Journal of Systems Science*, 33(3):197–212, 2002.

[401] K. H. Shim and M.E. Sawan. Singularly perturbed unified systems with low sensitivity to model reduction. In *American Control Conference*, pages 2578–2583, 2005.

[402] S. R. Shimjith, A. P. Tiwari, and B. Bandyopadhyay. *Modeling and Control of a Large Nuclear Reactor.* Springer, 2013.

[403] G. I. Shishkin. Grid approximation of singularly perturbed parabolic reaction-diffusion equation with piecewise smooth initial-boundary conditions. *Mathematical Modelling and Analysis*, 12(2):235–254, 2007.

[404] L. Shishkina and G. Shishkin. Robust numerical method for a system of singularly perturbed parabolic reactioncdiffusion equations on a rectangle. *Mathematical Modelling and Analysis*, 13(2):251–261, 2008.

[405] B. Siciliano and L. Villani. A singular perturbation approach to control of flexible arms in compliant motion. In Laura Menini, Luca Zaccarian, and Chaouki T. Abdallah, editors,

Current Trends in Nonlinear Systems and Control, Systems and Control: Foundations and Applications, pages 253–269. Birkhauser Boston, 2006.

[406] A. Siddarth and J. Valasek. Kinetic state tracking for a class of singularly perturbed systems. *Journal of Guidance, Control and Dynamics*, 34:734–749, 2011.

[407] A. Siddarth and J. Valasek. Tracking control design for non-standard nonlinear singularly perturbed systems. *Proceedings of IFAC American Control Conference*, pages 220–225, 2012.

[408] Bernd Simeon. Stiff mechanical system. In *Computational Flexible Multibody Dynamics*, pages 159–172. Springer Berlin Heidelberg, 2013.

[409] M.G. Singh, editor. *Singular Perturbations*, volume 7, pages 4425–4443. Pergamon Press, New York, NY, 1987.

[410] L. A. Skinner. *Singular Perturbation Theory.* Springer, 2011.

[411] D.R. Smith. *Singular Perturbation Theory: An Introduction with Applications.* Cambridge University Press, Cambridge, MA, 1985.

[412] D.R. Smith. Decoupling and order reduction via the Riccati transformation. *SIAM Review*, 29:91–113, March 1987.

[413] W.R. Smith and H.J.J. Gramberg. Mathematical modelling of moisture-induced panel deformation. *Journal of Engineering Mathematics*, 43:347–366, 2002.

[414] J.-W. Son and J.-T. Lim. Robust stability of nonlinear singularly perturbed system with uncertainties. *IEE Proceedings Control Theory and Applications*, 153:104 – 110, 2006.

[415] J. W. Son and J. T. Lim. Stabilization of approximately feedback linearizable systems using singular perturbation. *IEEE Transactions on Automatic Control*, 53(6):1499 –1503, july 2008.

[416] B. J. Song, C. Castillo-Chavez, and J. P. Aparicio. Tuberculosis models with fast and slow dynamics: the role of close and casual contacts. *Mathematical Biosciences*, 2002.

[417] Z. Sorchini and P.T. Krein. Formal derivation of direct torque control for induction motors using the singular perturbation method. In *IEEE 36th Power Electronics Specialists Conference*, pages 2422 –2428, Jun. 2005.

[418] A. Soto-Cota, L. M. Fridman, A. G. Loukianov, and J. M. Canedo. Variable structure control of synchronous generator: singularly perturbed analysis. *International Journal of Control*, 79(1):1–13, Jan. 2006.

[419] A. Soto-Cota, L.M. Fridman, A.G. Loukianov, and J.M. Canedo. Power system singularly perturbed discontinuous control. In *2004. Proceedings of the 2004 American Control Conference*, volume 3, pages 2580–2585, 2004.

[420] A. Soto-Cota, A.G. Loukianov, J.M. Canedo, and L.M. Fridman. Variable structure control of synchronous generator: singularly perturbed analysis. In *Proceedings of the 42nd IEEE Conference on Decision and Control*, volume 4, pages 3513 – 3518, 2003.

[421] R.S. Srivastava. On the vorticity distribution over a normal diffracted shock for small and large bends. *Shock Waves*, pages 1–4, 2013.

[422] A. Stoica, S. Berbente, and A. Condurache. Optimal flight control system for the lateral motion of a fighter aircraft. *Revue Roumanie des Sciences Techniques, Serie de Mecanique Appliquee*, 45:277–293, 2002.

[423] B. Subudhi and A. S. Morris. Singular perturbation approach to trajectory tracking of flexible robot with joint elasticity. *International Journal of Systems Science*, 34(3):167–179, 2003.

[424] B. Subudhi and A.S. Morris. Dynamic modelling, simulation and control of a manipulator with flexible links and joints. *Robotics and Autonomous Systems*, 41(4):257 – 270, 2002.

[425] F. Q. Sun, L. N. Zhou, Q. L. Zhang, and Y. X. Shen. Stability bound analysis and synthesis for singularly perturbed systems with time-varying delay. *Mathematical Problems in Engineering*, 2013.

[426] M. J. Sun, G. G. Yan, and Y. J. Zhang. A design method of the complementary controller for multi- infeed hvdc transmission systems using singular perturbation theory. *Advanced Materials Research*, pages 766–770, 2013.

[427] P. Szmolyan and M. Wechselberger. Relaxation oscillations in r^3. *Journal of Differential Equations*, 200(1):69 – 104, 2004.

[428] H.D. Taghirad and M.A. Khosravi. Stability analysis and robust composite controller synthesis for flexible joint robots. In *IEEE/RSJ International Conference on Intelligent Robots and Systems*, volume 3, pages 2073 – 2078, 2002.

[429] A. Tamilselvan and N. Ramanujam. A parameter uniform numerical method for a system of singularly perturbed convectioncdiffusion equations with discontinuous convection coefficients. *International Journal of Computer Mathematics*, 87(6):1374–1388, 2010.

[430] A.R. Teel, L. Moreau, and D. Nesic. A unified framework for input-to-state stability in systems with two time scales. *IEEE Transactions on Automatic Control*, 48:1526 – 1544, 2003.

[431] A. Tellili, M. N. Andelkrim, and M. Benrejeb. Reliable H_∞ control of multiple time scales singularly perturbed systems with sensor failure. *International Journal of Control*, 80(5):659–665, 2007.

[432] A.N. Tikhonov and A.B. Vasil'eva. *Differential Equations.* Springer-Verlag, Berlin, Germany, 1984.

[433] A.N. Tikhonov, A.B. Vasil'eva, and A.G. Sveshnikov. *Differential Equations.* Springer-Verlag, Berlin, Germany, 1980.

[434] K. M. Tsang and J. Wang. Design of second order sliding mode controller for synchronous generators based on singular perturbation method. *International Conference on Power System Technology*, 1:333 – 338, 2002.

[435] M. Vakil, R. Fotouhi, and P. N. Nikiforuk. End-effector trajectory tracking of a flexible link manipulator using integral manifold concept. *International Journal of Systems Science*, 42(12):2057–2069, Dec. 2011.

[436] T. Valanarasu and N. Ramanujam. An asymptotic numerical method for singularly perturbed third-order ordinary differential equations with a weak interior layer. *International Journal of Computer Mathematics*, 84(3):333–346, 2007.

[437] S. Valarmathi and N. Ramanujam. Boundary value technique for finding numerical solution to boundary value problems for third order singularly perturbed ordinary differential equations. *International Journal of Computer Mathematics*, 79(6):747–763, 2002.

[438] A.B. Vasil'eva. Asymptotic behavior of solutions to certain problems involving nonlinear ordinary differential equations containing a small parameter multiplying the highest derivatives. *Russian Mathematical Surveys*, 18:13–84, 1963.

[439] A.B. Vasil'eva. The development of the theory of ordinary differential equations with a small parameter multiplying the highest derivatives in the years 1966-1976. *Russian Mathematical Surveys*, 31:109–131, 1976.

[440] A.B. Vasil'eva. On the development of singular perturbation theory at Moscow State University and elsewhere. *SIAM Review*, 36:440–452, 1994.

[441] A.B. Vasil'eva and V.F. Butuzov. *Asymptotic Expansions of Solutions of Singularly Perturbed Differential Equations.* Izadat. Nauka, Moscow, Russia, 1973.

[442] A.B. Vasil'eva, V.F. Butuzov, and L.V. Kalachev. *The Boundary Function Method for Singular Perturbation Problems.* SIAM Studies in Applied Mathematics, Philadelphia, PA, 1995.

[443] R. Vazquez and M. Krstic. Thermal convection loop control by continuous backstepping and singular perturbations. *American Control Conference*, pages 3882–3887, 2005.

[444] D. Del Vecchio and J.J. Slotine. A contraction theory approach to singularly perturbed systems. *IEEE Transactions on Automatic Control*, 58(3):752–757, 2013.

[445] D. Del Vecchio and J.J. Slotine. A contraction theory approach to singularly perturbed systems with application to retroactivity attenuation *2011 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, 5831–5836, 2011

[446] D. Del Vecchio and Richard M. Murray. Biomolecular Feedback Systems California Institute of Technology, USA, 2012.

[447] M. Vishik. *Asymptotic Behavior of Solutions of Evolution Equations.* Cambridge University Press, Cambridge, UK, 1992.

[448] R. Vrabel. On the approximation of the boundary layers for the controllability problem of nonlinear singularly perturbed systems. *Systems and Control Letters*, 61(3):422 – 426, 2012.

[449] R. Vulanovic and L. Teofanov. A modification of the shishkin discretization mesh for one-dimensional reactioncdiffusion problems. *Applied Mathematics and Computation*, 220(0):104 – 116, 2013.

[450] J. Waldmann. Forced singular perturbations as theoretical background to a split-coordinate frame multirate strapdown terrestrial navigation algorithm. *4th International Conference on Control and Automation*, pages 28 – 32, 2003.

[451] N. Wand and M. K. Ni. The interior layer phenomena for a class of singularly perturbed delay-differential equations. *Acta Mathematica Scientia*, 33(2):532 – 542, 2013.

[452] J. Wang and H. Li. Nonlinear pi control of a class of nonlinear singularly perturbed systems. *IEE Proceedings Control Theory and Applications*, 152(5):560 – 566, sept. 2005.

[453] L. D. Wang, S. C., J. W. Zhang, and Y. Hu. Control of a redundantly actuated power line inspection robot based on a singular perturbation model. In *IEEE International Conference on Robotics and Biomimetics*, pages 198 –203, Feb. 2009.

[454] L. J. Wang, H. Mukaidani, X. M. Shen, and X. Z. Liu. Delay-dependent stability analysis for large-scale multiple-bottleneck systems using singular perturbation approach. In *IEEE Global Telecommunications Conference*, pages 1–5, 2008.

[455] L. M. Wang and E.D. Sontag. A remark on singular perturbations of strongly monotone systems. In *45th IEEE Conference on Decision and Control*, pages 989 –994, 2006.

[456] W. Wang, A. R. Teel, and D. Nesic. Averaging tools for singularly perturbed hybrid systems. *Australian Control Conference*, pages 88–93, Nov. 2011.

[457] W. Wang, A. R. Teel, and D. Nesic. Analysis for a class of singularly perturbed hybrid systems via averaging. *Automatica*, pages 1057–1068, 2012.

[458] Z. M. Wang and W. Liu. Output feedback networked control of singular perturbation. In *9th World Congress on Intelligent Control and Automation*, pages 645 –650, Jun. 2011.

[459] Z. M. Wang, W. Liu, H. H. Dai, and D.S. Naidu. Robust stabilization of model-based uncertain singularly perturbed systems with networked time-delay. In *Proceedings of the 48th IEEE Conference on Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference*, pages 7917 –7922, Dec. 2009.

[460] Z. Y. Wang and F. H. Ghorbel. Control of closed kinematic chains: A comparative study. *Proceedings of the 2006 American Control Conference*, pages 2498 – 2503, 2006.

[461] Z. Y. Wang and F.H. Ghorbel. Control of closed kinematic chains using a singularly perturbed dynamic model. In *43rd IEEE Conference on Decision and Control*, volume 1, pages 317 – 322, 2004.

[462] Z. Y. Wang, F.H. Ghorbel, and J.B. Dabney. On the domain and error characterization in the singular perturbation modeling of closed kinematic chains. In *Proceedings of the 2004 American Control Conference*, volume 1, pages 493 –498, 2004.

[463] W. Wasow. *Asymptotic Expansions for Ordinary Differential Equations*. Wiley-Interscience, New York, NY, 1965. Unabridged and unaltered publication by Dover Publications, Inc., New York, NY, in 1987 of the corrected and slightly enlarged publication by Robert E. Krieger Publishing Company, Huntington, NY, in 1976.

[464] M. Wechselberger. Extending melnikov theory to invariant manifolds on non-compact domains. *Dynamical Systems*, 17(3):215–233, 2002.

[465] S. P. Wen, Z. G. Zeng, and T. W. Huang. Passivity and passification for a class of singularly perturbed nonlinear systems via neural networks. In *The 2012 International Joint Conference on Neural Networks*, pages 1–6, Jun. 2012.

[466] S. P. Wen, Z. G. Zeng, T. W. Huang, and G. Bao. Passivity and passification for a class of singularly perturbed nonlinear systems with time-varying delays and polytopic uncertainties via neural networks. *Circuits System Signal Process*, Sep. 2012.

[467] Widowati, R. Bambang, R. Saragih, and S.M. Nababan. Model reduction for unstable lpv systems based on coprime factorizations and singular perturbation. In *2004 5th Asian Control Conference*, volume 2, pages 963 –970, 2004.

[468] M. Wogrin and L. Glielmo. An MPC scheme with guaranteed stability for linear singularly perturbed systems. *49th IEEE Conference on Decision and Control*, pages 5289–5295, Dec. 2010.

[469] J. C. Wu and S. L. Wo. Non-fragile robust control for singular systems with nonlinear perturbation and time-delay. In *2010 Chinese Control and Decision Conference*, pages 3443 –3447, May 2010.

[470] Q. H. Wu, L. Jiang, and J. Y. Wen. Decentralized adaptive control of interconnected nonlinear systems using high gain observer. *International Journal of Control*, 77(8):703 – 712, 2004.

[471] C. Xenophontos. Optimal mesh design for the finite element approximation of reaction-diffusion problems. *International Journal for Numerical Methods in Engineering*, 53(4):929–943, 2002.

[472] J. K. Xia and M. Xin. Bifurcation analysis for power system voltage stability based on singular perturbation method. In *International Conference on Electrical Machines and Systems*, pages 1811–1814, 2007.

[473] R. Xie, X. M. Wang, and Y. Li. Neural network adaptive inversion control law design for a supermaneuverable aircraft. In *The 4th IEEE Conference on Industrial Electronics and Applications*, pages 3434 –3437, May 2009.

[474] H. Xin, D. Gan, M. Huang, and K. Wang. Estimating the stability region of singular perturbation power systems with saturation nonlinearities: an linear matrix inequality based method. *IET Control Theory Applications*, 4(3):351 –361, Mar. 2010.

[475] G.S. Yablonsky, I.M.Y. Mareels, and M. Lazman. The principle of critical simplification in chemical kinetics. *Chemical Engineering Science*, 58(21):4833 – 4842, 2003.

[476] F. J. Yan and J. M. Wang. Control of diesel engine dual-loop EGR air-path systems by a singular perturbation method. *Control Engineering Practice*, 21(7):981 – 988, 2013.

[477] C. Y. Yang and Q. L. Zhang. Multiobjective control for t-s fuzzy singularly perturbed systems. *IEEE Transactions on Fuzzy Systems*, 17:104–115, 2009.

[478] C. Y. Yang, Q. L. Zhang, J. H. Chou, and Y. W. Zhang. Absolute stability of lur'e singularly perturbed systems with multiple nonlinearities. In *American Control Conference*, pages 2677 –2681, 2010.

[479] C. Y. Yang, Q. L. Zhang, J. S., and T. Y. Chai. Lur'e lyapunov function and absolute stability criterion for lur'e singularly perturbed systems. *IEEE Transactions on Automatic Control,*, 56(11):2666 –2671, Nov. 2011.

[480] C. Y. Yang, Q. L. Zhang, and L. N. Zhou. Multi-objective control for t-s fuzzy singularly perturbed systems. In *Stability Analysis and Design for Nonlinear Singular Systems*, volume 435, pages 175–195. Springer Berlin Heidelberg, 2013.

[481] C. Y. Yang, L. N. Zhou, and Q. L. Zhang. *Stability Analysis and Design for Nonlinear Singular Systems*. Spinger, 2012.

[482] G. H. Yang and J. X. Dong. Control synthesis of singularly perturbed fuzzy systems. *IEEE Transactions on Fuzzy Systems*, 16:615 –629, Jun. 2008.

[483] G. H. Yang and J. X. Dong. H_∞ filtering for fuzzy singularly perturbed systems. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 38:1371 –1389, 2008.

[484] X. J. Yang and J. J. Zhu. A generalization of chang transformation for linear time-varying systems. In *49th IEEE Conference on Decision and Control*, pages 6863 –6869, Dec. 2010.

[485] X. J. Yang and J. Jim Zhu. Singular perturbation margin for nonlinear time-invariant systems. In *American Control Conference (ACC)*, pages 3309 –3315, Jun. 2012.

[486] K. C. Yao. Robust sub-optimal pole assignment of decentralized actuator type large-scale systems by the aggregation matrix with robustness test. In *American Control Conference*, volume 5, pages 4506 – 4511, 2003.

[487] K. C. Yao. An iteration method to sub-optimal output feedback computer control of decentralized singularly-perturbed systems. In *International Conference on Innovative Computing,Information and Control Second*, 2007.

[488] S. Yao, S. Mei, and G. W. Hua. One novel variable-speed wind energy system based on pmsg and super sparse matrix converter. *International Conference on Electrical Machines and Systems*, pages 2384–2389, Oct. 2008.

[489] G. Yin and H. Q. Zhang. Discrete-time markov chains with two-time scales and a countable state space: limit results and queueing applications. *Stochastics: An International Journal of Probability and Stochastics Processes*, 80(4):339–369, Aug. 2008.

[490] G. Yin and J. F. Zhang. Hybrid singular systems of differential equations. *Science in China Series F: Information Sciences*, 45:241–258, 2002.

[491] G.George Yin and Q. Zhang. Asymptotic expansions of solutions for forward equations. In *Continuous-Time Markov Chains and Applications*, volume 37, pages 59–140. Springer New York, 2013.

[492] Z. Yoshida and P. J. Morrison. Unfreezing casimir invariants: singular perturbations giving rise to forbidden instabilities. *ArXiv e-prints*, Mar 2013.

[493] H. W. Yu, G. P. Lu, and Y. F. Zheng. On the model-based networked control for singularly perturbed systems with nonlinear uncertainties. *Systems and Control Letters*, 60(9):739 – 746, 2011.

[494] H. W. Yu, Z. M. WANG, and Y. F. Zheng. On the model-based networked control for singularly perturbed systems. *Journal of Control Theory Applications*, pages 153–162, 2008.

[495] H. W. Yu and B. S. Zhang. Stabilizability of a class of singularly perturbed systems via switched output feedback. In *Proceedings of 2013 Chinese Intelligent Automation Conference*, volume 254, pages 729–734. Springer Berlin Heidelberg, 2013.

[496] H. W. Yu, X. M. Zhang, G. P. Lu, and Y. F. Zheng. On the model-based networked control for singularly perturbed systems with nonlinear uncertainties. In *Proceedings of the 48th IEEE Conference on Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference*, pages 684 –689, Dec. 2009.

[497] W. C. Yu and G. J. Wang. Discrete sliding mode controller design based on the lqr suboptimal approach with application on ac servo motor. *Journal of the Chinese Institute of Engineers*, 29(5):873–882, 2006.

[498] Y. Yuan, F. C. Sun, and Y. N. Hu. Decentralized multi-objective robust control of interconnected fuzzy singular perturbed model with multiple perturbation parameters. In *IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pages 1–8, Jun. 2012.

[499] Y. Yuan, F. C. Sun, H. P. Liu, and B. Xu. Robust control for fuzzy singular perturbed unified model. *American Control Conference*, pages 5628–5633, Jun. 2012.

[500] V.D. Yurkevich. Bifurcations caused by non-affinity of singularly perturbed control systems. In *Third International Forum on Strategic Technologies*, pages 482 –486, Jun. 2008.

[501] V.D. Yurkevich. Adaptive gain tuning in nonlinear control systems designed via singular perturbation technique. In *IEEE Intelligent Control on Control Applications*, pages 37–42, Jul. 2009.

[502] V.D. Yurkevich. Adaptive output feedback control of nonlinear siso systems via singular perturbation technique. In *Proceedings of the 48th IEEE Conference on Decision and Control*, pages 3495 –3500, Dec. 2009.

[503] A.I. Zadorin and S.V. Tikhovskaya. Solving a second-order nonlinear singular perturbation ordinary differential equation by a samarskii scheme. *Numerical Analysis and Applications*, 6(1):9–23, 2013.

[504] B. L. Zhang, D. X. Gao, Q. Lu, and F. L. Cao. Approximation design of composite control for singularly perturbed time-delay systems via delay compensation. In *2010 29th Chinese Control Conference (CCC)*, pages 1676 –1680, Jul. 2010.

[505] J. M. Zhang. Existence of travelling waves in a modified vector-disease model. *Applied Mathematical Modelling*, 33(2):626 – 632, 2009.

[506] R. J. Zhang and Y. B. Chen. Dual-loop feedback control of servo motor systems using singular perturbation method. *Proceedings of the 2003 American Control Conference*, 6:4657–4662, 2003.

[507] W. Q. Zhang. Numerical solutions of singular perturbation problems with multiple boundary layers and interior layers. *International Journal of Pure and Applied Mathematics*, 83(4):549–558, 2013.

[508] W. Z. Zhang and P. Y. Huang. Precise integration method for a class of singular two-point boundary value problems. *Acta Mechanica Sinica*, 29(2):233–240, 2013.

[509] Y. Zhang, H. Nguyen, D.S. Naidu, Yun Zou, and Chenxiao Cai. Time scale analysis and synthesis for model predictive control. *Proceedings of the WSEAS-NAUN 4th International Conference on Circuits, Systems, Control, Signals (CSCS '13)*, pages 27–32, Aug. 2013.

[510] L. Zhou and G. P. Lu. Robust stability of singularly perturbed descriptor systems with nonlinear perturbation. *IEEE Transactions on Automatic Control*, 56(4):858 –863, Apr. 2011.

[511] L. N. Zhou and C. Y. Yang. Passivity analysis of singularly perturbed systems with nonlinear uncertainties. In *25th Chinese Control and Decision Conference*, pages 1429–1434, 2013.

[512] I.J. Zhu, D.A. Lawrence, J. Fisher, and Shtessel Y.B. Direct fault tolerant rlv attitude control-a singular perturbation approach. In *Proceedings of the Thirty-Fourth Southeastern Symposium on System Theory*, pages 86 – 91, 2002.

[513] J.J. Zhu, X. J. Yang, and A.S. Hodel. A singular perturbation approach for time-domain assessment of phase margin. In *American Control Conference*, pages 315 –322, Jul. 2010.

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