Problem 8.7

(a) \[ A(s) = \frac{10(s + 200\pi)}{(s + 2000\pi)} = \frac{1 + s/200\pi}{1 + s/2000\pi} \]

\[ A(f) = \frac{1 + j(f/100)}{1 + j(f/1000)} \]

\[ 20\log |A(f)| \]

\[ 20\text{dB} \]

\[ \text{Phase} \quad 90^\circ \]

\[ -45^\circ \]

\[ -90^\circ \]

\[ 10 \quad 100 \quad 1000 \quad f \]

(b) \[ A(s) = \frac{s - 200\pi}{s + 200\pi} = \frac{1 - s/200\pi}{1 + s/200\pi} \]

\[ A(f) = \frac{1 - j(f/100)}{1 + j(f/100)} \]

\[ 20\log |A(f)| \]

\[ +20\text{ dB/decade} \]

\[ -20\text{ dB/decade} \]

\[ \text{Phase} \quad 90^\circ \]

\[ -70^\circ \]

\[ -180^\circ \]

\[ 10 \quad 100 \quad 1000 \quad 10^9 \quad f \]
(c) \[ A(s) = \frac{4\pi^2 10^9}{s^2 + (5\pi 10^4)s + 4\pi^2 10^8} \]

\[ = \frac{10}{[1 + s/(4\pi 10^4)][1 + s/(\pi 10^4)]} \]

\[ A(f) = \frac{10}{[1 + jf/(20 \times 10^3)](1 + jf/5000)} \]

![Graph showing the magnitude and phase of \( A(f) \) with dB and frequency on the axes.](image)
Problem 8.8

(a) Refer to the circuit shown in Figure P8.8a in the book.

\[ A(s) = \frac{V_o}{V_{in}} = \frac{sL}{R + sL} = \frac{s/(2 \times 10^8)}{1 + s/(2 \times 10^8)} \]

\[ A(f) = \frac{jf/(31.8 \times 10^6)}{1 + jf/(31.8 \times 10^6)} \]
Sketches of the magnitude and phase of $A(f)$ are:

(b) Refer to the circuit shown in Figure P8.8b in the book.

$$A(s) = \frac{V_o}{V_{in}} = \frac{R_2}{R_2 + sL R_1 / (sL + R_1)}$$

$$= \frac{sL/R_1 + 1}{sL(R_1 + R_2) / R_1 R_2 + 1}$$

$$A(f) = \frac{1 + j(f/f_z)}{1 + j(f/f_p)}$$

in which $f_z = R_1 / (2\pi L) = 143.2$ MHz and $f_p = R_1 R_2 / [2\pi L(R_1 + R_2)] = 14.32$ MHz.

Sketches of the magnitude and phase of $A(f)$ are shown on the next page.
Problem 8.15

(a) The small-signal equivalent circuit is:

```
V_{gs19} + \frac{1}{sC_{gs}} G_s + \frac{g_m V_{gs}}{sC_{gd}} G_d + R_{s1} s + R_{D} = 0
```

(b) Notice that $V_{gs} = -V_s$. Writing a current equation at the source node we obtain:
\[ \frac{V_s - V_{\text{sig}}}{R_{\text{sig}}} + sC_{gs} V_s + g_m V_s = 0 \]

Solving for \( V_s \) we obtain

\[ V_s = V_{\text{sig}} \frac{1}{sC_{gs} R_{\text{sig}} + 1 + g_m R_{\text{sig}}} \]  \hspace{1cm} (1)

Writing a current equation at the drain node we have

\[ g_m V_s = sC_{gd} V_0 + V_0 / R_L' \]

Solving for \( V_0 \) we obtain

\[ V_0 = V_s \frac{g_m R_L'}{sC_{gd} R_L' + 1} \]  \hspace{1cm} (2)

Using Equation (1) to substitute for \( V_s \) in Equation (2) and dividing both sides by \( V_{\text{sig}} \) we have

\[ A(s) = \frac{V_0}{V_{\text{sig}}} = \frac{g_m R_L'}{(sC_{gs} R_{\text{sig}} + 1 + g_m R_{\text{sig}})(sC_{gd} R_L' + 1)} \]

(c) The poles are the roots of the denominator of \( A(s) \).

\[ s_{p1} = -\left(1 + g_m R_{\text{sig}}\right) / \left(C_{gs} R_{\text{sig}}\right) \quad \text{and} \quad s_{p2} = -1 / \left(C_{gd} R_L'\right) \]

The break frequencies associated with these poles are

\[ f_{p1} = \frac{1 + g_m R_{\text{sig}}}{2\pi C_{gs} R_{\text{sig}}} \quad \text{and} \quad f_{p2} = 1 / \left(2\pi C_{gd} R_L'\right) \]

(d) The midband gain is obtained by evaluating \( A(s) \) for \( s = 0 \).

\[ A_{\text{mid}} = \frac{g_m R_L'}{1 + g_m R_{\text{sig}}} \]

(e) We have \( K = KF(W/L)/2 = 10^{-3} \); \( g_m = 2\sqrt{K f_D Q} = 3.16 \text{ mS}; \) \( r_d = \infty \) (because \( \lambda = 0 \)); \( R_L' = R_D || R_L = 1 \text{ k\Omega}; \) and \( A_{\text{mid}} = 2.40 \) which is
equivalent to 7.6 dB. The break frequencies are \( f_{p1} = 4.19 \) GHz and \( f_{p2} = 318 \) MHz. Because \( f_{p1} \) is considerably greater than \( f_{p2} \), the upper half-power frequency is approximately equal to \( f_{p2} \).

(f) The program is stored in the file named P8_15. The simulation results match the hand calculations.

**Problem 8.22**

The equivalent circuit is

\[ R_x \rightarrow \]

\[ U_x \]

\[ R_f = 5 \text{k}\Omega \]

\[ R_o = 1 \text{k}\Omega \]

\[ R_L = 5 \text{k}\Omega \]

First for the approximate analysis we use the Miller effect to replace \( R_f \) by \( R_{in, Miller} \) and \( R_{o, Miller} \) as shown below.

\[ R_x \rightarrow \]

\[ R_{in, Miller} \]

\[ U_x \]

\[ R_i \]

\[ R_o \]

\[ \approx 2.5 \text{k}\Omega \]

\[ R_L \]

\[ 5 \text{k}\Omega \]

\[ R_{o, Miller} \approx R_f = 5 \text{k}\Omega \]
\[ R_{\text{in, Miller}} = \frac{R_f}{1 - A_V} = 614 \, \Omega \]
\[ R_x = R_i \parallel R_{\text{in, Miller}} = 579 \, \Omega \]

For the exact analysis, we refer to the original equivalent circuit and write these circuit equations:

\[ i_x = \frac{v_x}{R_i} + \frac{v_x - v_o}{R_f} \quad (1) \]
\[ \frac{v_o}{R_L} + \frac{v_o - v_x}{R_f} + \frac{v_o - A_{vo}v_x}{R_o} = 0 \quad (2) \]

We solve Equation (2) for \( v_o \), substitute into Equation (1), and solve for \( R_x \):

\[ R_x = \frac{v_x}{i_x} = \frac{1}{\frac{1}{R_i} + \frac{1}{R_f} - \frac{1/R_f + A_{vo}/R_o}{1 + R_f/R_L + R_f/R_o}} \]

Evaluating we obtain \( R_x = 588 \, \Omega \) (compared to 579 \( \Omega \) for the approximate analysis).

Problem 8.23

Because \( R_o = 0 \), \( A_V = A_{vo} \). Thus we have \( C_{\text{in, Miller}} = C(1 - A_V) \).

\[ A_s = \frac{v_o}{v_s} = A_{vo} \frac{1/(j\omega C_{\text{in, Miller}})}{R_s + 1/(j\omega C_{\text{in, Miller}})} \]
\[ = \frac{A_{vo}}{1 + j(f/f_B)} \quad \text{in which} \quad f_B = 1/(2\pi R_s C_{\text{in, Miller}}) \]

For \( A_{vo} = -9 \) we have \( A_s = \frac{-9}{1 + jf/(15.9 \, \text{kHz})} \)

For \( A_{vo} = -99 \) we have \( A_s = \frac{-99}{1 + jf/(1.59 \, \text{kHz})} \)
The sketch of $|A_s|$ is

The equivalent circuits are shown in Figure 8.26 in the book, except that for the circuit under consideration we have $R'_L = R_D |r_d| R_L$. The midband voltage gain is $A_{mid} = -g_m R'_L$.

The bias current is $I_{DQ} = (V_{DD} - V_{DSQ})/(2 \text{ k}\Omega) = 2.5 \text{ mA}$. We have $K = KP/(W/L)/2 = 10^{-3}$. $g_m = 2|K| I_{DQ} = 3.16 \text{ mS}$. (This value for $g_m$ is approximate because the formula was derived assuming $\lambda = 0$.) Also we have $r_d \approx 1/(\lambda I_{DQ}) = 40 \text{ k}\Omega$.

$$R'_L = \frac{1}{1/r_d + 1/R_D + 1/R_L} = \frac{1}{1/40 + 1/2 + 1/1} = 656 \text{ \Omega}$$

$$A_{mid} = -g_m R'_L = -(3.16 \times 10^{-3}) \times 656 = -2.07$$

Then the Miller capacitance is

$$C_{Miller} = C_{gd}(1 - A_{mid}) = (0.5 \text{ pF})(1 + 2.07) = 1.54 \text{ pF}$$

$$C_{total} = C_{gs} + C_{Miller} = 2.04 \text{ pF}$$

$$f_b = \frac{1}{2\pi R_{sig} C_{total}} = 15.6 \text{ MHz}$$

In Problem 8.12 we did an exact analysis of this circuit and obtained $f_b = 15.1 \text{ MHz}$. (The simulation yielded $f_b = 14.8 \text{ MHz}$.)

Problem 8.27

(a) The equivalent circuit is shown on the next page.
In the midband, we have \( V_{\text{sig}} = V_{\text{in}} \). Also, we have
\[
V_{\text{in}} = V_{gs} + V_o \quad \text{and} \quad V_o = g_m V_{gs} R'_L
\]
From which we have
\[
A_{\text{vs}} = \frac{V_o}{V_{\text{sig}}} = A_V = \frac{V_o}{V_{\text{in}}} = \frac{g_m R'_L}{1 + g_m R'_L}
\]
(c) \( C_{\text{Miller, in}} = C_{gs} (1 - A_V) \)
\[
C_{\text{total}} = C_{gd} + C_{\text{Miller, in}}
\]
\[
f_b = \frac{1}{(2\pi R_{\text{sig}} C_{\text{total}})}
\]
(d) We have \( K = Kp (W/L)/2 = 10^{-3} \); \( g_m = 2 \sqrt{KID_Q} = 3.16 \text{ mS} \); \( r_d = \infty \) (because \( \lambda = 0 \)); \( R'_L = R_{\text{bias}} || R_L = 1 \text{ k\Omega} \); and \( A_V = 0.760 \) which is equivalent to \(-2.39 \text{ dB}\). \( C_{\text{Miller, in}} = C_{gs} (1 - A_V) = (0.5 \text{ pF})(1 - 0.760) = 0.120 \text{ pF}. \) \( C_{\text{total}} = C_{gd} + C_{\text{Miller, in}} = 0.620 \text{ pF}. \) \( f_b = 1/(2\pi R_{\text{sig}} C_{\text{total}}) = 25.7 \text{ MHz}. \)

In Problem 8.14, we did an exact analysis and determined that \( f_{3-\text{dB}} = 25.6 \text{ MHz}. \) Thus, the Miller approximation is very accurate in this case.