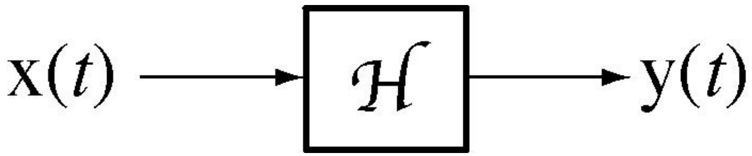
Systems

- ***Broadly speaking, a system is anything that responds when stimulated or excited***
- ***The systems most commonly analyzed by engineers are artificial systems designed by humans***
- ***Engineering system analysis is the application of mathematical methods to the design and analysis of systems***

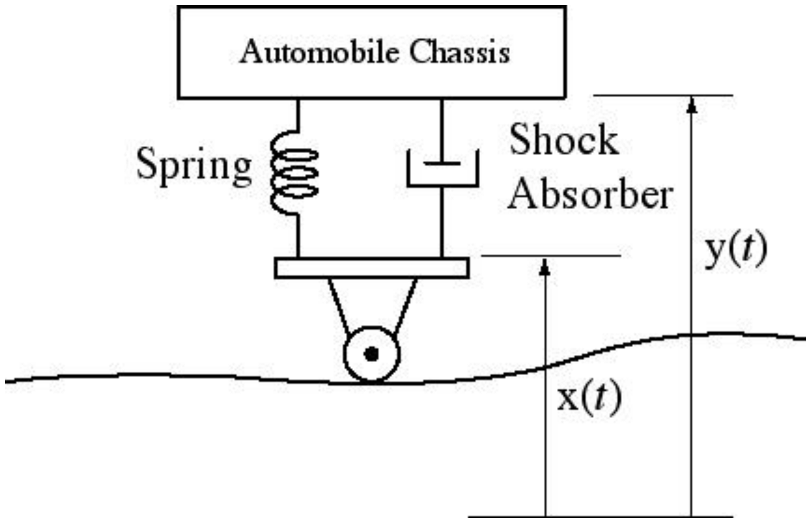
Systems

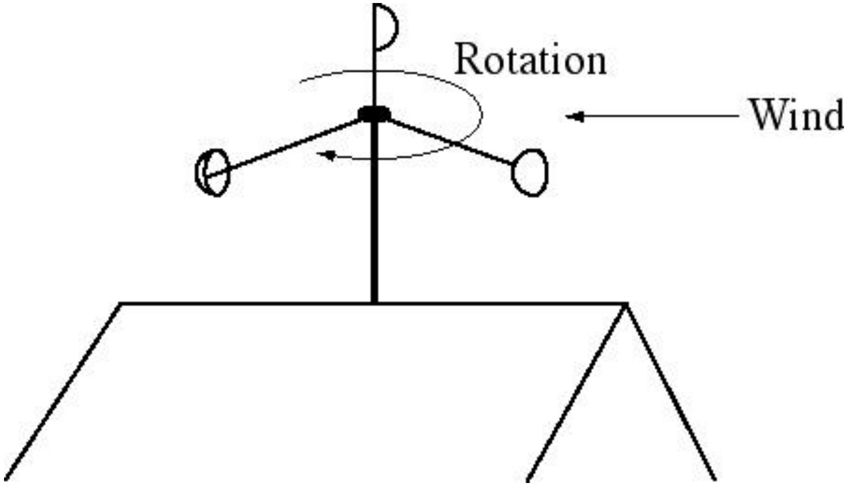
- ***Systems have inputs and outputs***
- ***Systems accept excitation signals at their inputs and produce response signals at their outputs***
- ***Systems are often usefully represented by block diagrams***

A single-input, single-output system block diagram

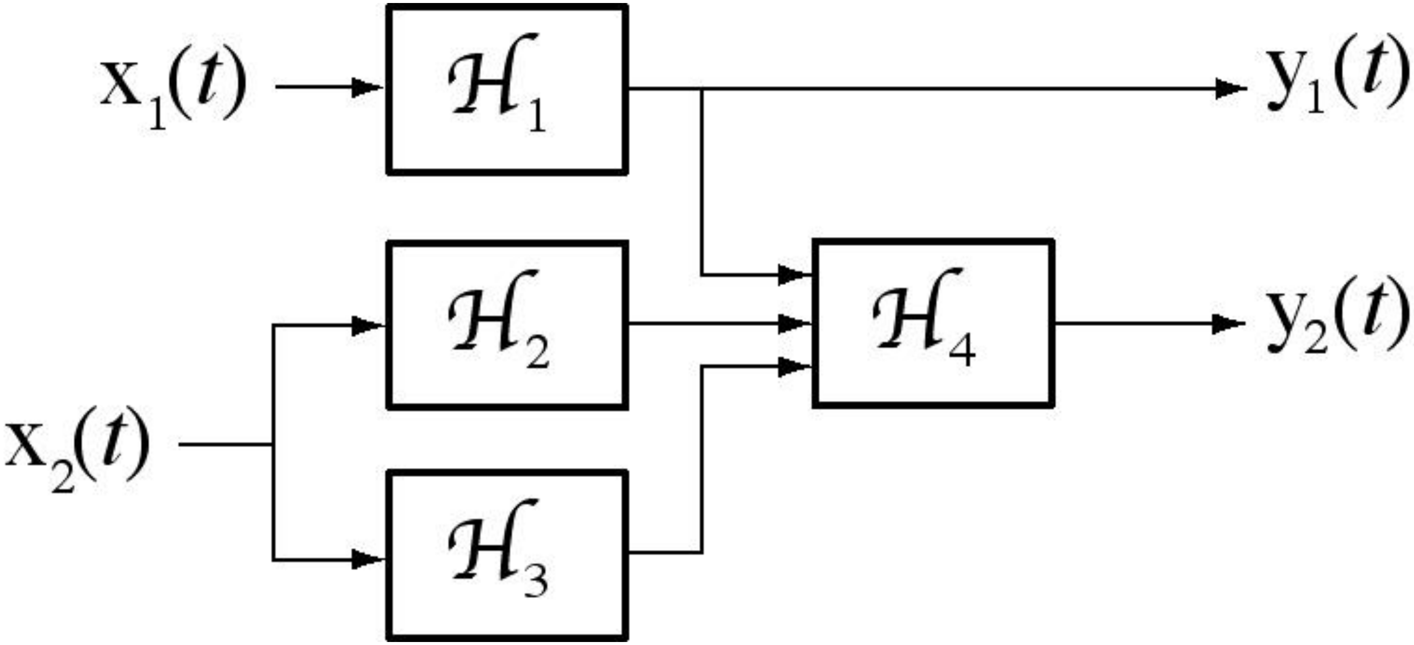


Some Examples of Systems



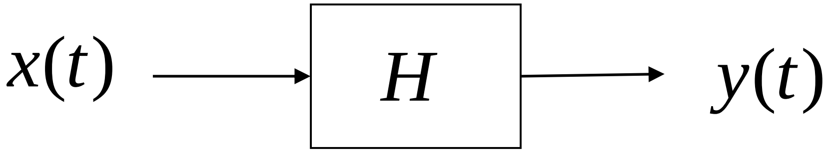


A Multiple-Input, Multiple-Output System Block Diagram



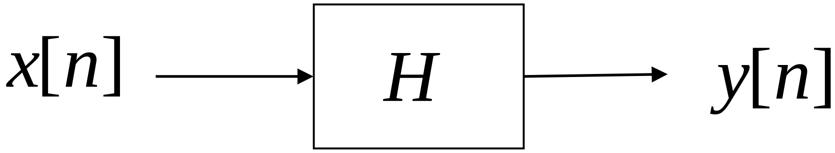
Continuous and Discrete Time Systems

Continuous Time Systems



Example: an RC circuit

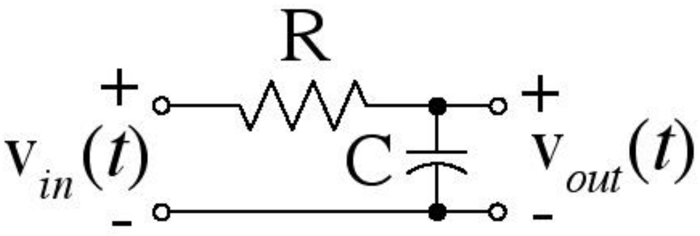
Discrete Time Systems

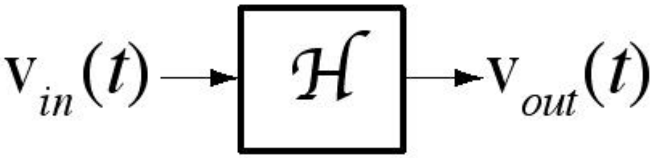


Example: a delayed adder

An Electrical Circuit Viewed as a System

1. *An RC lowpass filter is a simple electrical system*
2. *It is excited by a voltage,* $v_{in}(t)$ *, and responds with a voltage,* $v_{out}(t)$
3. *It can be viewed or modeled as a single-input, single-output system*





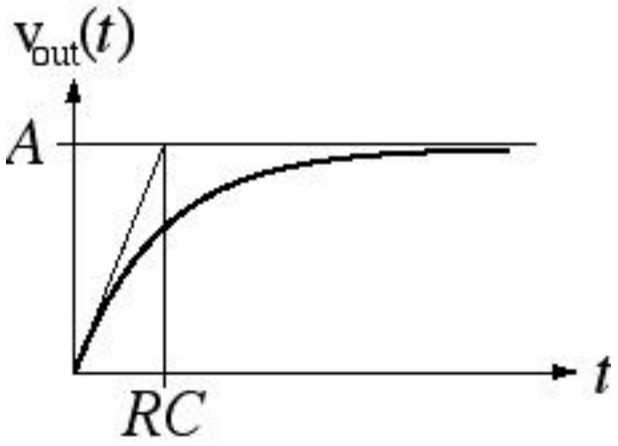
Response of an RC Lowpass Filter to a Step Excitation

If an RC lowpass filter is excited by a step of voltage,

$$v_{in}(t) = A\,u(t)$$

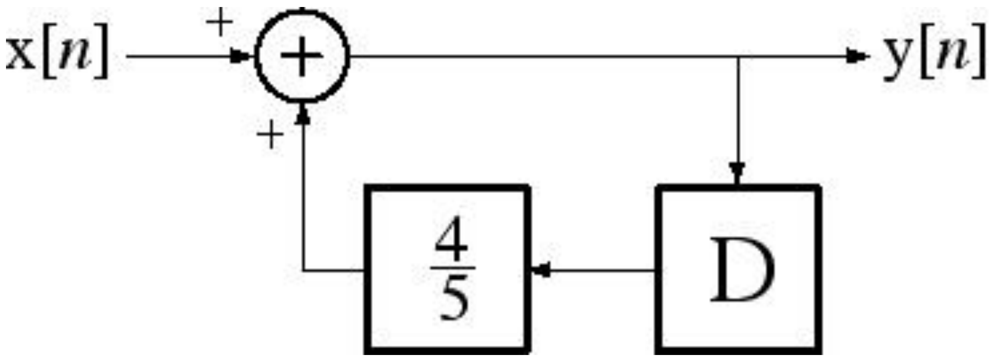
its response is

$$v_{out}(t) = A\left(1 - e^{-\frac{t}{RC}}\right)u(t)$$



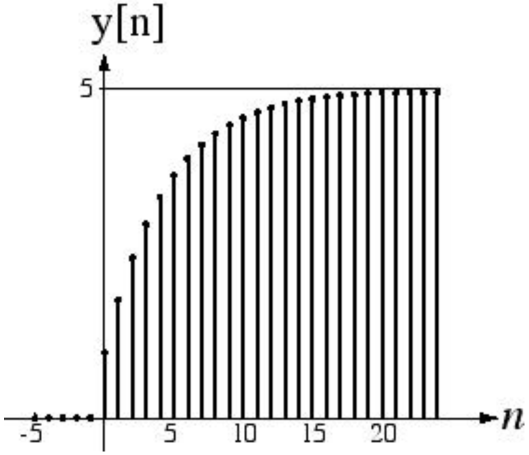
If the excitation is doubled, the response doubles.

A DT System



If the excitation, x[n], is the unit sequence, the response is

$$y[n] = \left[5 - 4\left(\frac{4}{5}\right)^n\right] u[n]$$



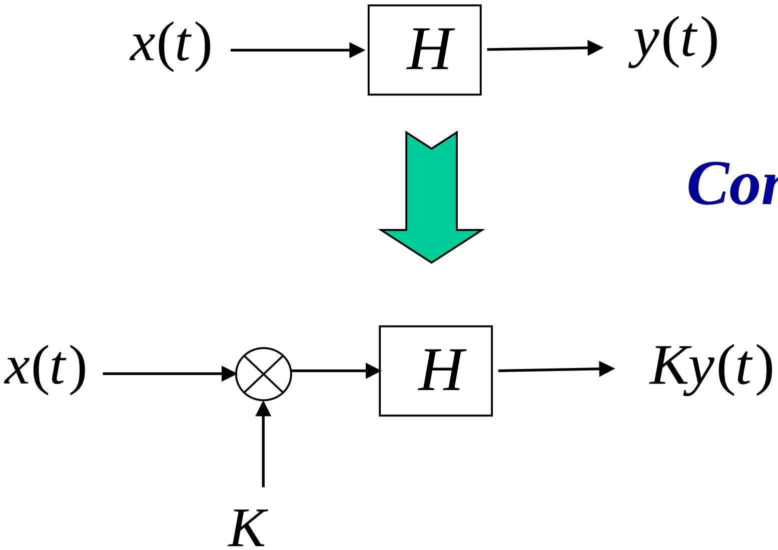
If the excitation is doubled, the response doubles.

Characteristics of a System

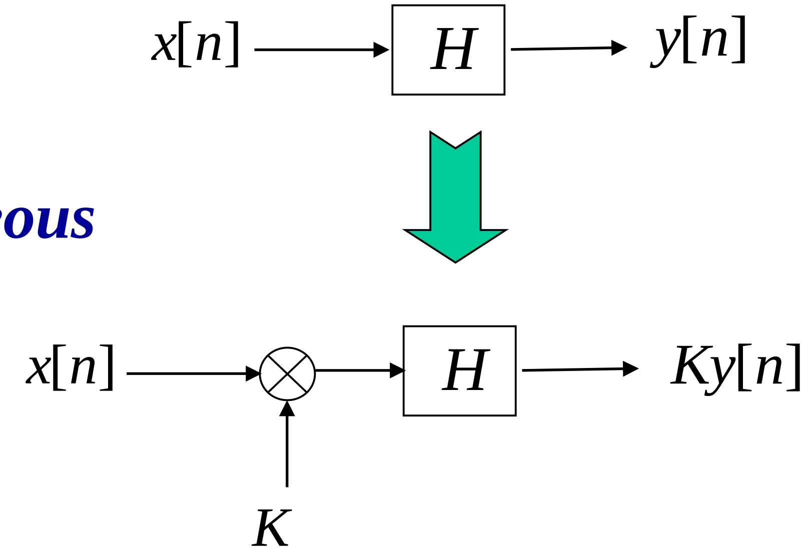
Linear Time-Invariant Systems (LTI Systems)

1. ***Homogeneity*** } ***Linearity***
2. ***Additivity*** }
3. ***Time Invariance***
4. ***Stabillity***
5. ***Causality***

Homogeneity

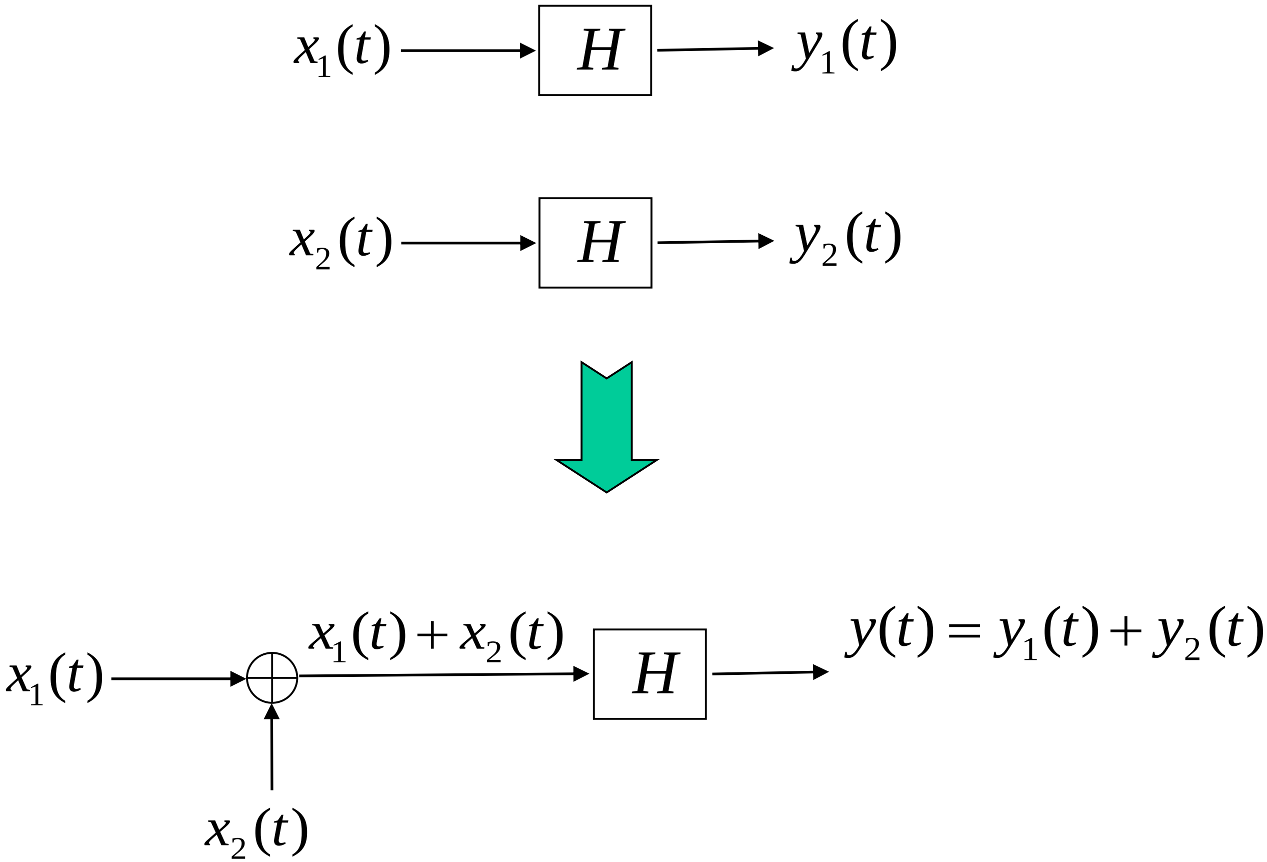


Continuous Time Homogeneous

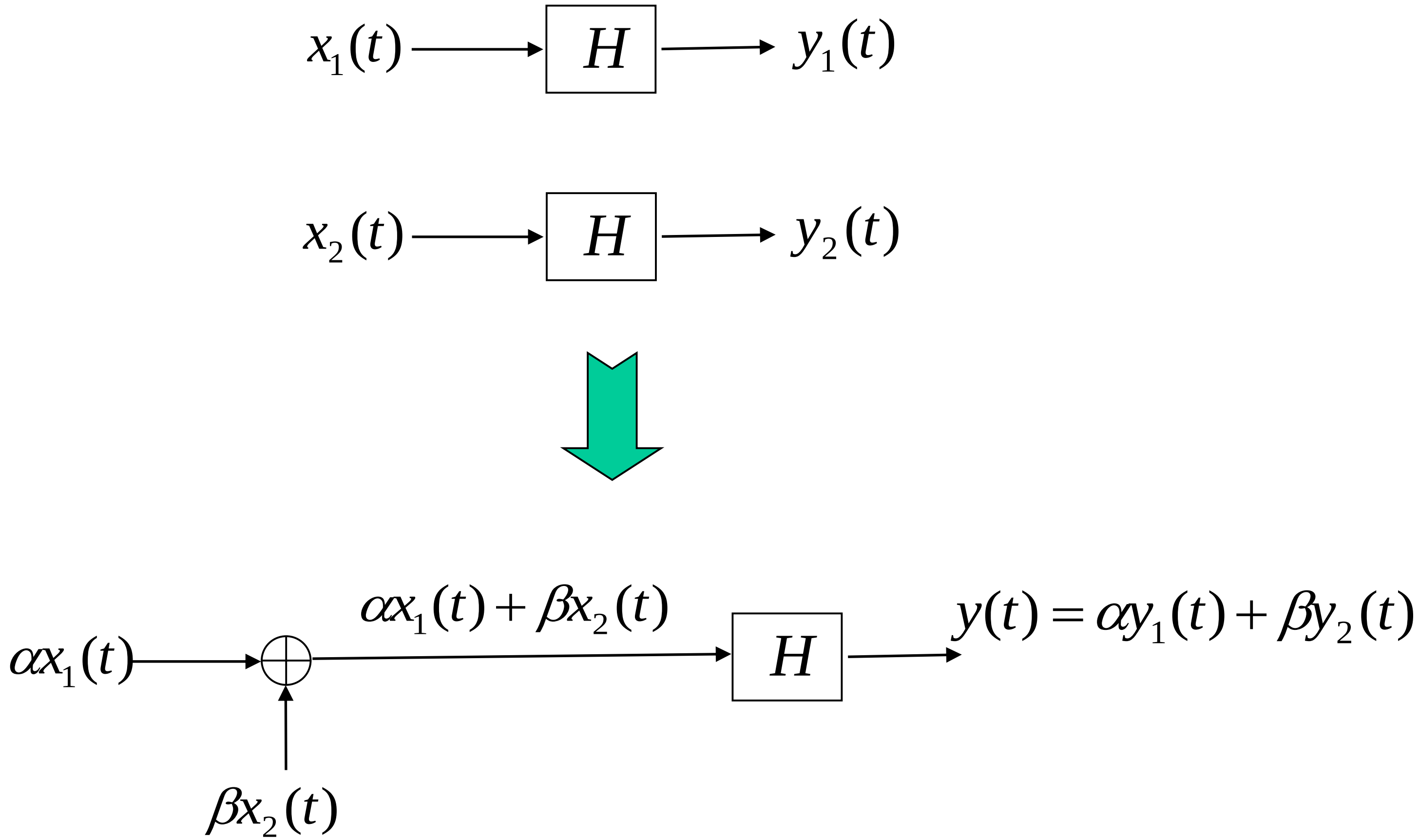


Discrete Time Homogeneous

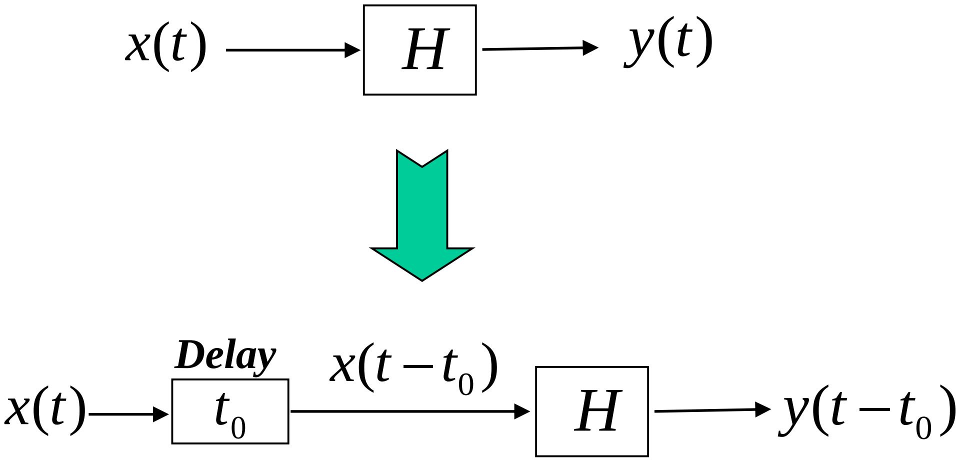
Additivity



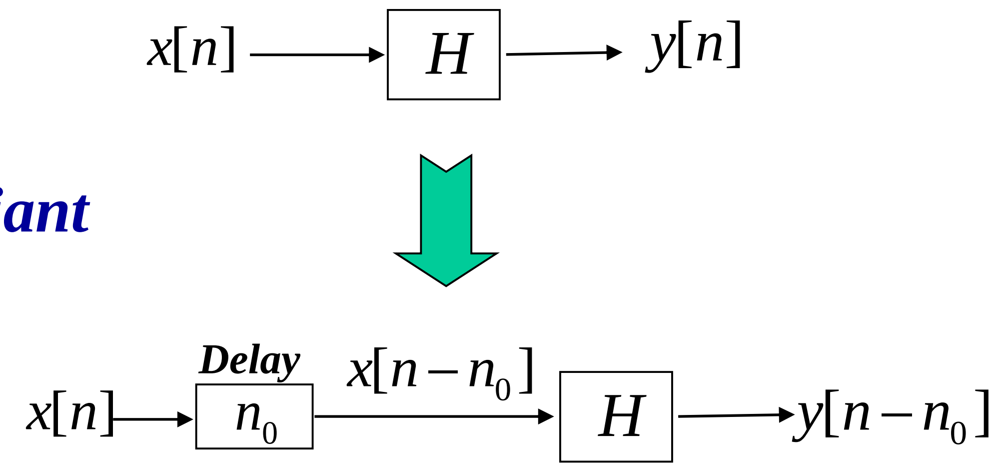
Linearity


$x_1(t)$
H
$y_1(t)$
$x_2(t)$
H
$y_2(t)$
$\alpha x_1(t)$
$\alpha x_1(t) + \beta x_2(t)$
H
$y(t) = \alpha y_1(t) + \beta y_2(t)$
$\beta x_2(t)$

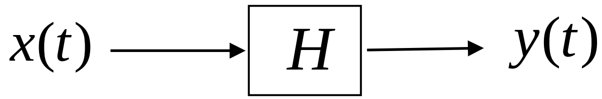
Time-Invariance



Discrete Time Time-Invariant



Stability



Stable Input ⟹ **Stable Output**

Stable Input means:

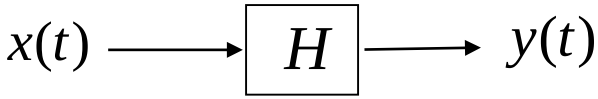
$$|x(t)| < \infty \qquad -\infty < t < \infty$$

Stable Output means:

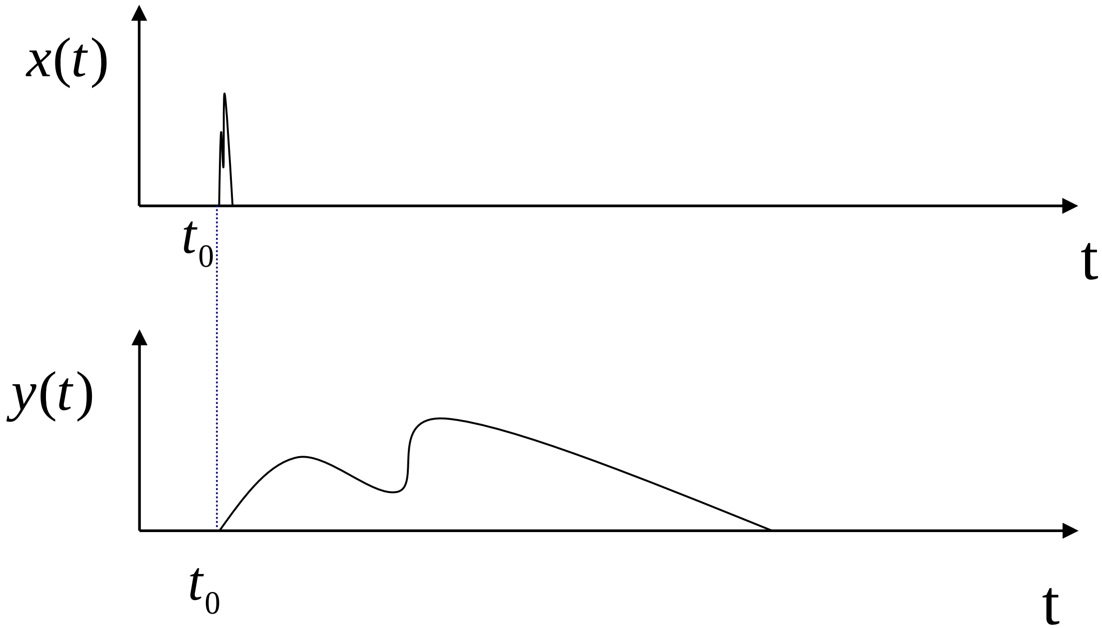
$$|y(t)| < \infty \qquad -\infty < t < \infty$$

also called BIBO Stable

Causality

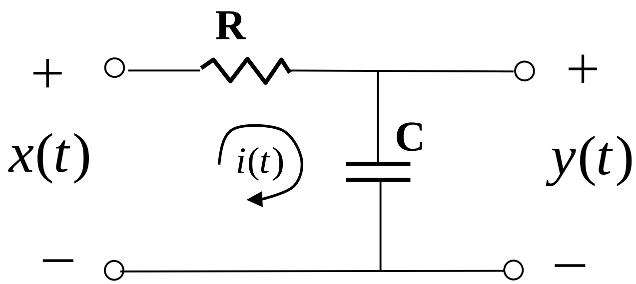


Output follows input and can not precede input.



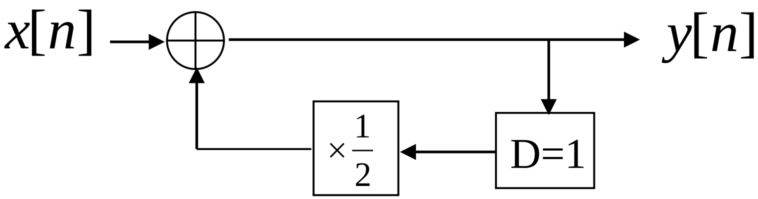
Let's look at Examples of LTI Systems

Continuous Time



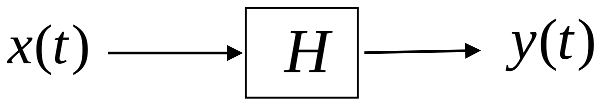
$$RC\frac{dy(t)}{dt}+y(t)=x(t)$$

Discrete Time

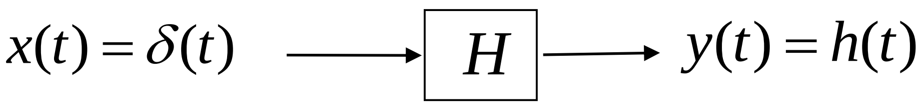


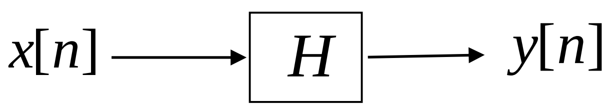
$$y[n]-\frac{1}{2}y[n-1]=x[n]$$

Idea of Unit Impulse Response

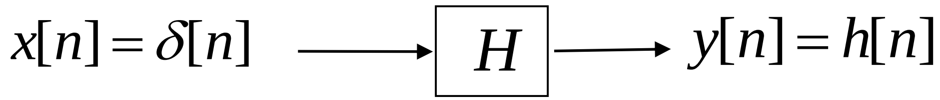


Continuous Time System





Discrete Time System



Higher Order Discrete System

$$a_n y[n] + a_{n-1} y[n-1] + \ldots.. + a_{n-D} y[n-D] = x[n]$$

$$x[n] = \delta[n]$$

$$\Rightarrow y[n] = h[n]$$

Impulse Response to System Response

$$a_n y[n] + a_{n-1} y[n-1] + ... + a_{n-D} y[n-D] = x[n]$$

$$x[n] = \delta[n] \qquad \Rightarrow y[n] = h[n]$$

Any Input x[n] can be written as

$$x[n] = \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] +$$

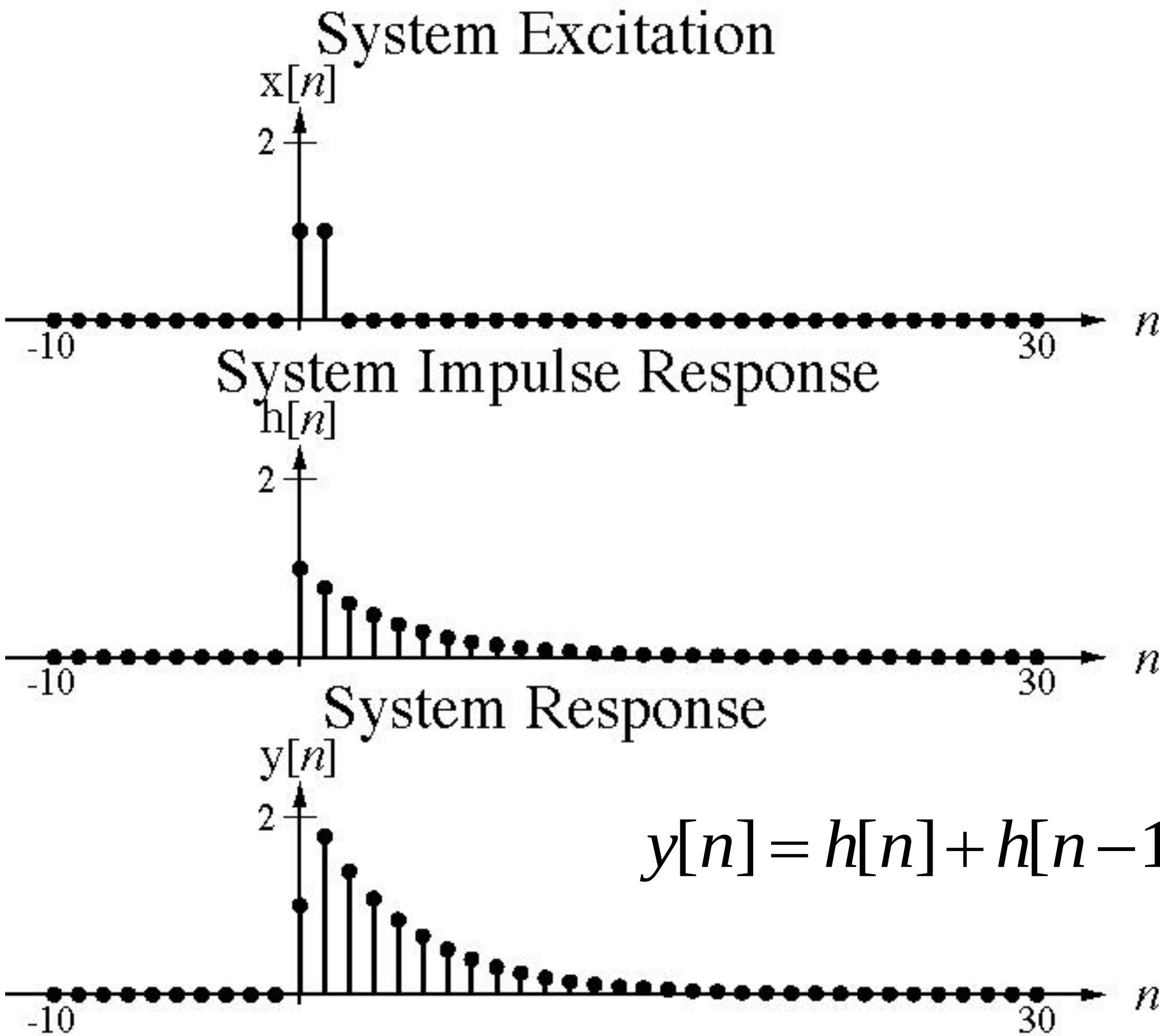
$$x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \cdots$$

This means system response, y[n] can be given by

$$y[n] = \cdots + x[-2]h[n+2] + x[-1]h[n+1] +$$

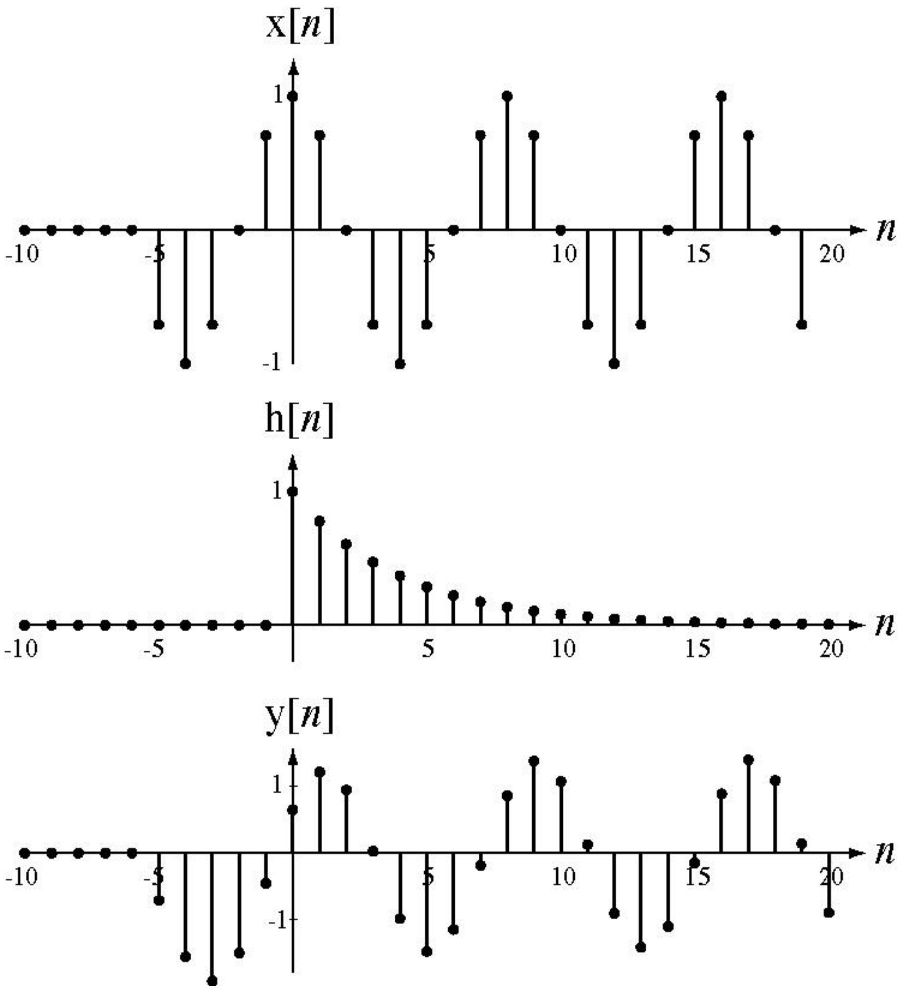
$$x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + \cdots$$

Simple System Response Example



$$y[n] = h[n] + h[n-1]$$

More Complicated System Response Example



Convolution Sum

$$y[n] = \cdots + x[-2]h[n+2] + x[-1]h[n+1]$$

$$+ x[0]h[n] + x[1]h[n-1] + x[2]h[n+2] + \cdots$$

$$y[n] = \sum_{m=-2}^{m=2} x[m]h[n-m]$$

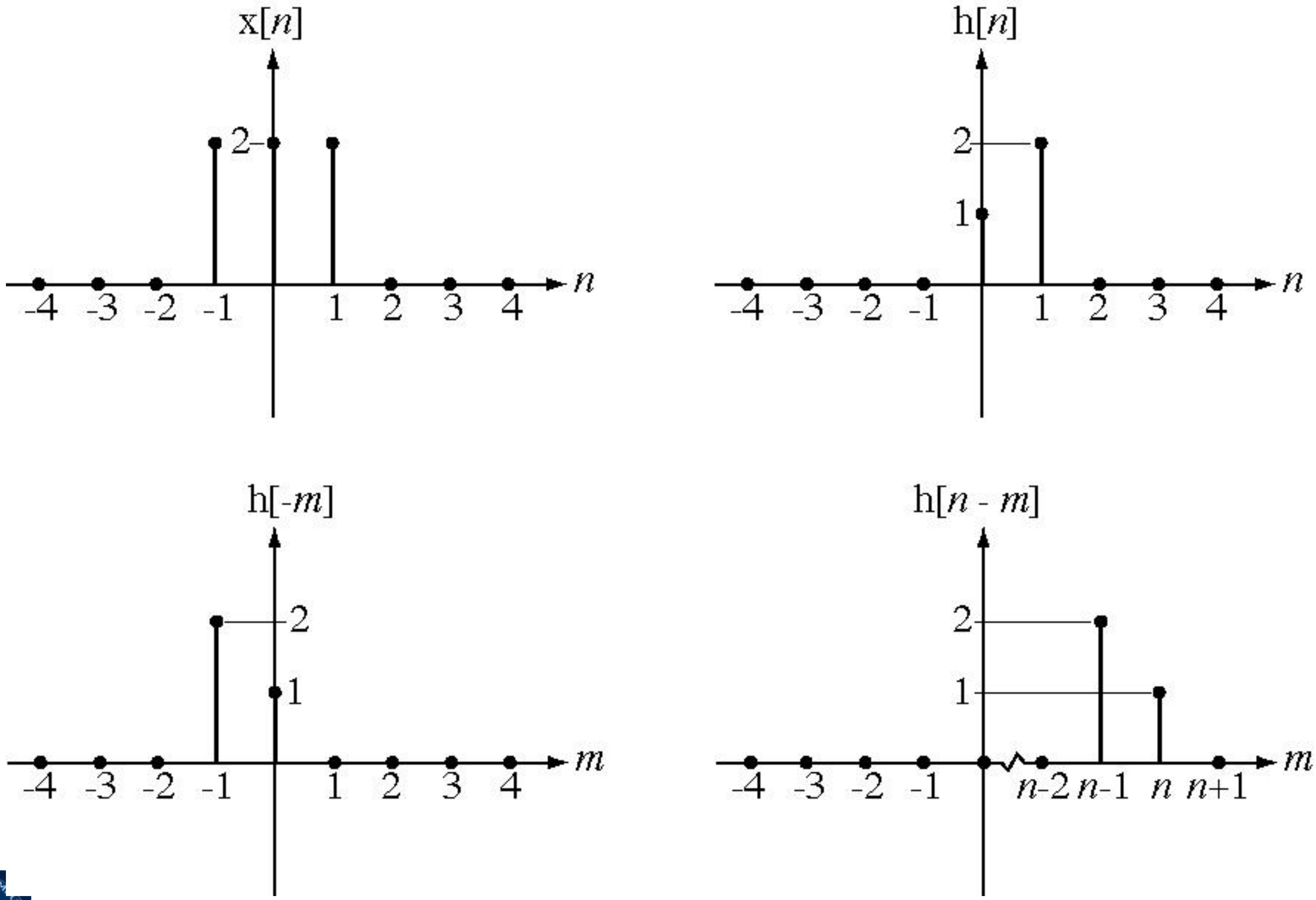
$$\boxed{y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]}$$

Convolution Sum

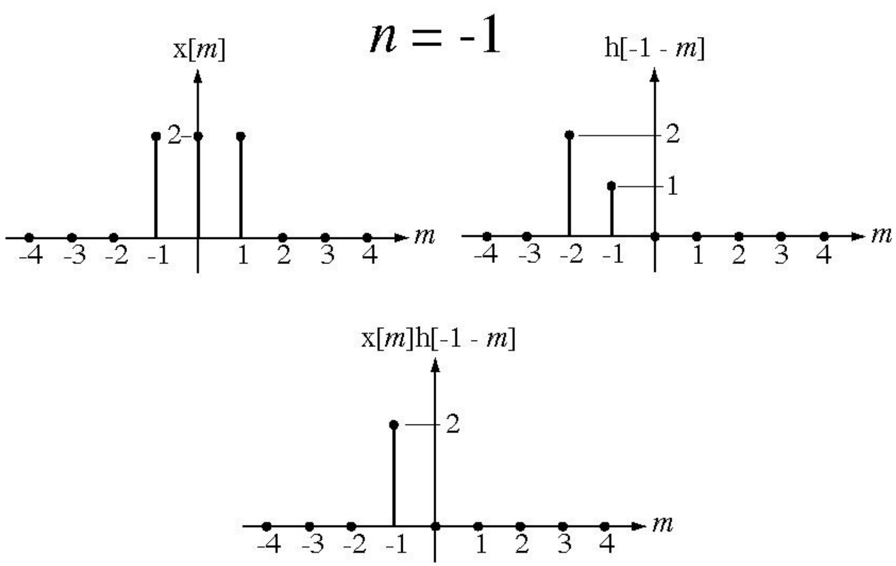
$$y[n] = x[n] * h[n]$$

Superposition of delayed and weighted impulse responses

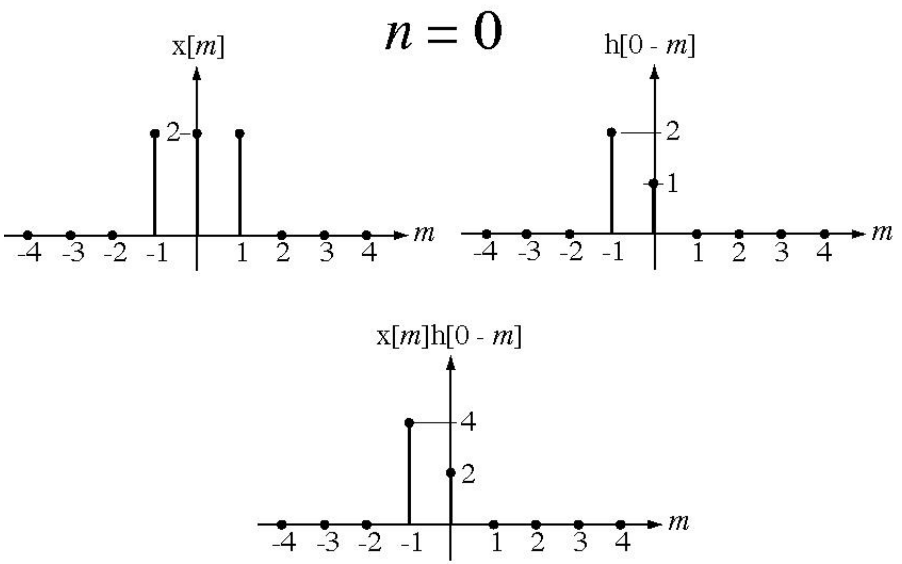
A Convolution Sum Example



A Convolution Sum Example



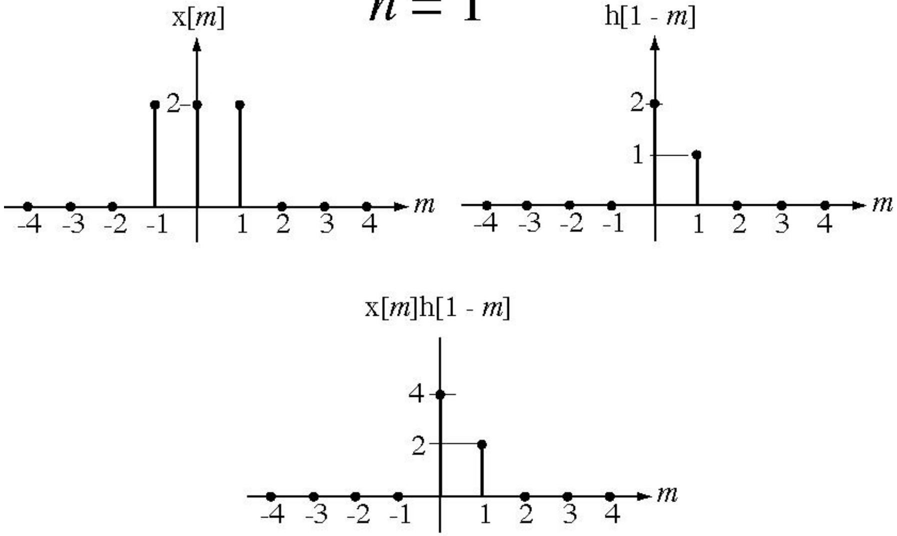
y[-1] = 2



y[0] = 6

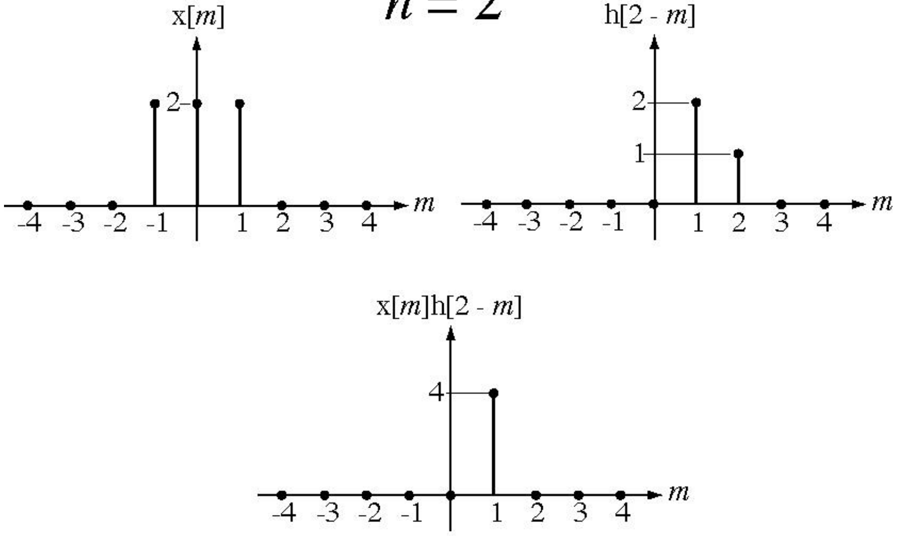
A Convolution Sum Example

$n = 1$



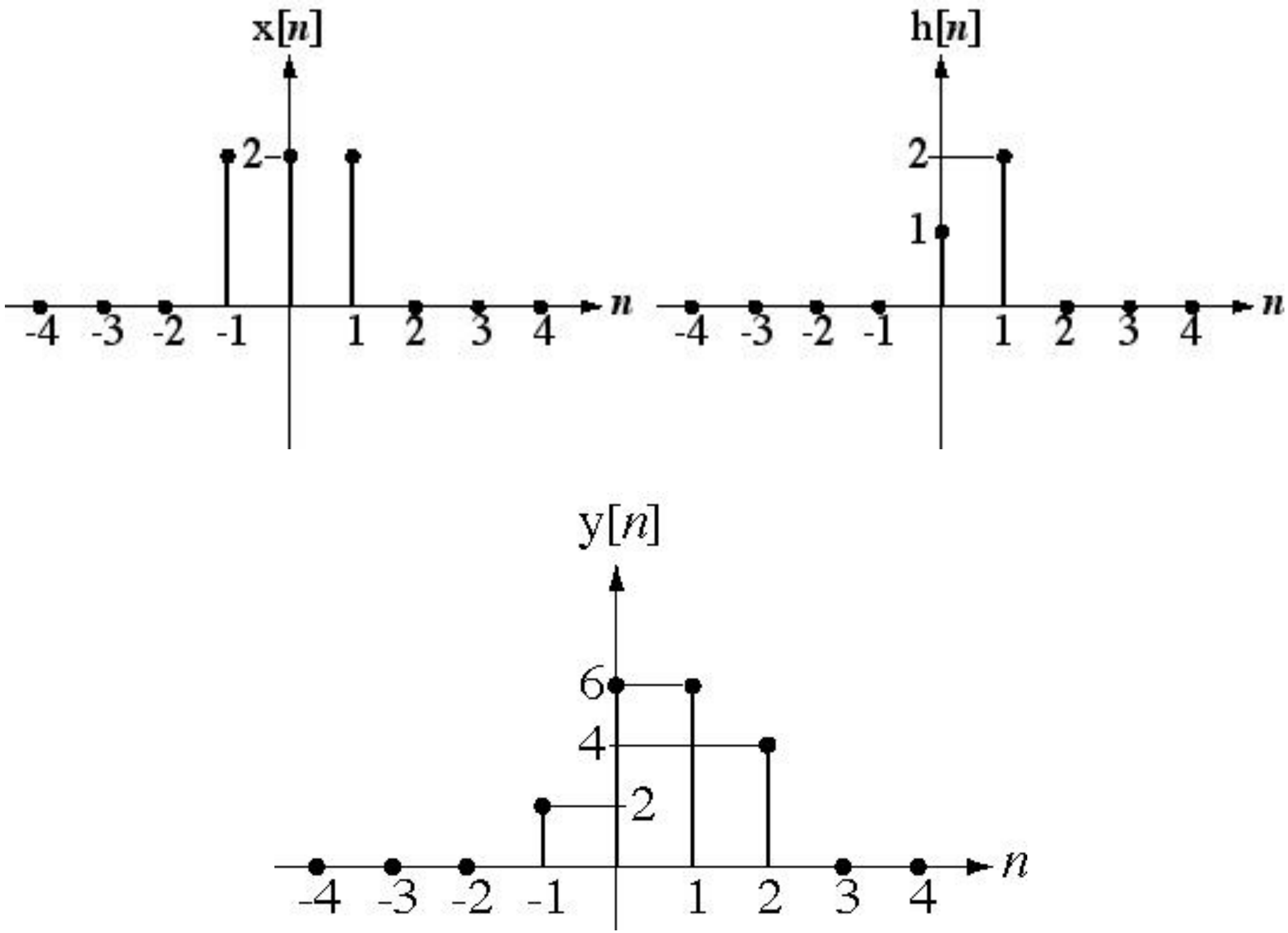
$y[1] = 6$

$n = 2$



$y[2] = 4$

A Convolution Sum Example



Convolution Integral in Continuous Time

$$x(t)=\delta(t) \longrightarrow \boxed{H} \longrightarrow y(t)=h(t)$$

$$y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y(t)=x(t)*h(t)$$

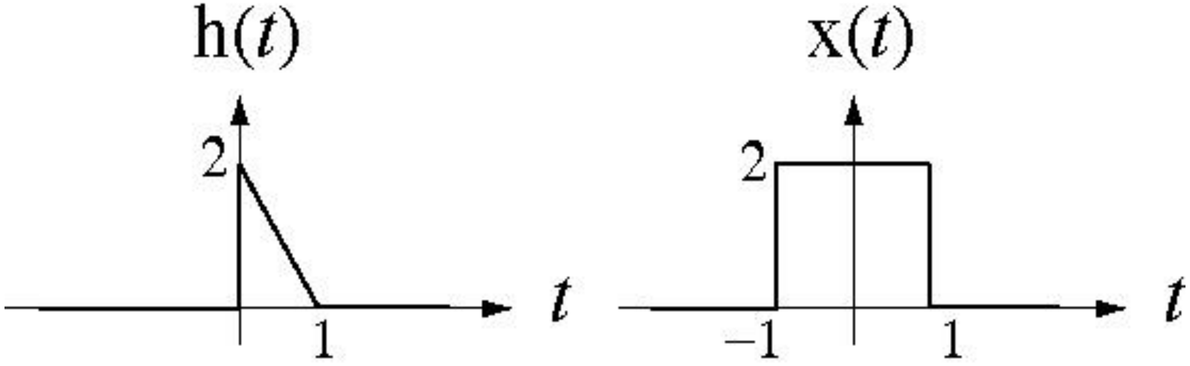
Superposition of delayed and weighted impulse responses

A Graphical Illustration of the Convolution Integral

The convolution integral is defined by

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

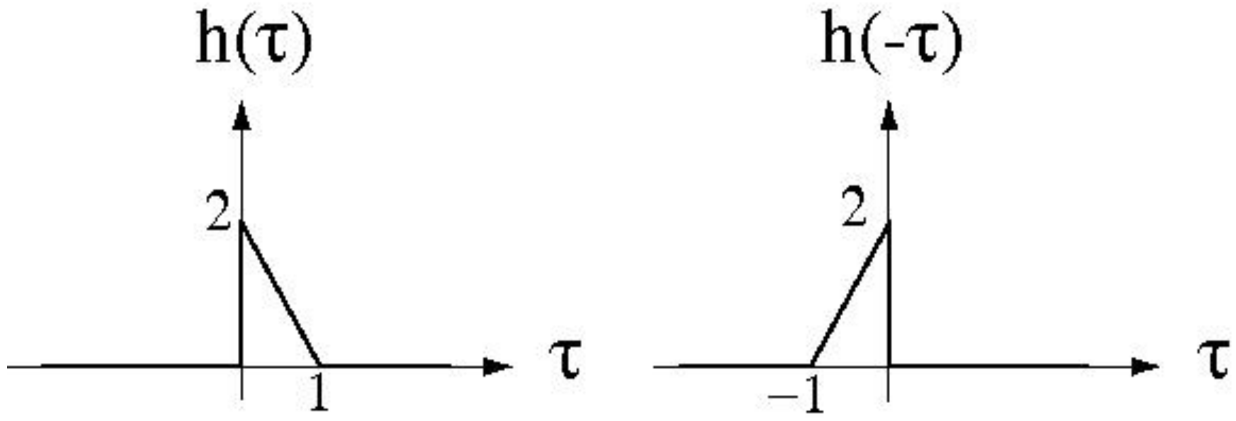
For illustration purposes let the excitation, x(*t*), and the impulse response, h(*t*), be the two functions below.



A Graphical Illustration of the Convolution Integral

In the convolution integral there is a factor, $h(t-\tau)$

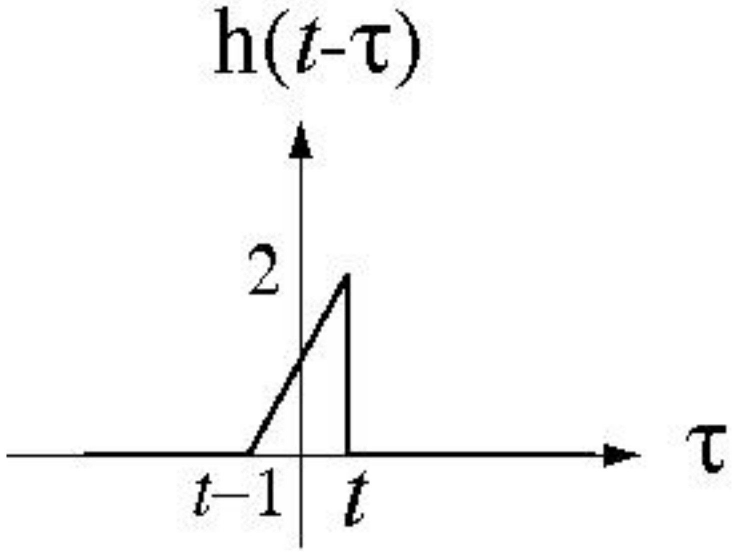
We can begin to visualize this quantity in the graphs below.



A Graphical Illustration of the Convolution Integral

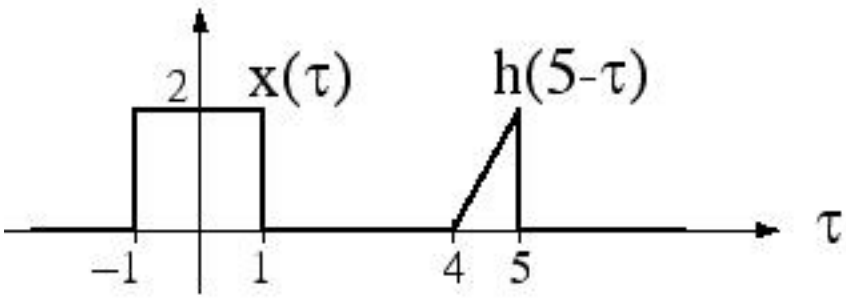
The functional transformation in going from h(τ) to h(*t* - τ) is

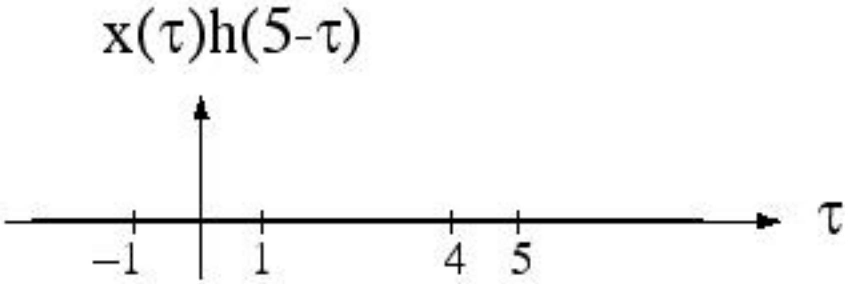
$$\mathrm{h}(\tau)\xrightarrow{\tau\to-\tau}\mathrm{h}(-\tau)\xrightarrow{\tau\to\tau-t}\mathrm{h}(-(\tau-t))=\mathrm{h}(t-\tau)$$



A Graphical Illustration of the Convolution Integral

The convolution value is the area under the product of x(t) and h(t - τ). This area depends on what t is. First, as an example, let t = 5.





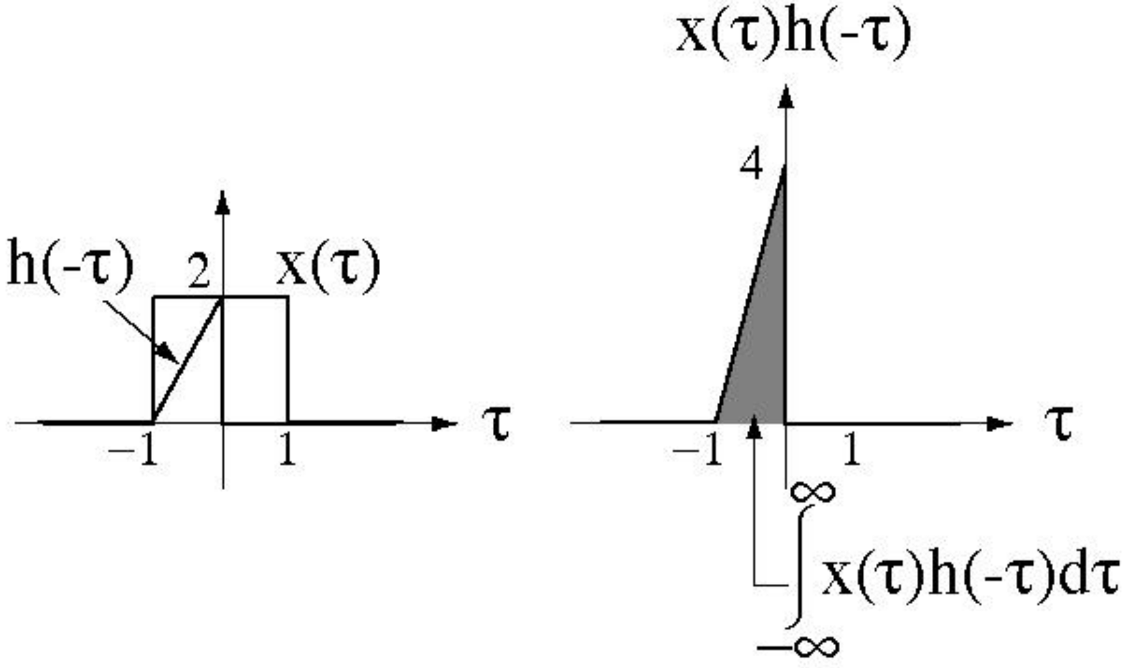
For this choice of t the area under the product is zero. If

$$y(t) = x(t) * h(t)$$

then y(5) = 0.

A Graphical Illustration of the Convolution Integral

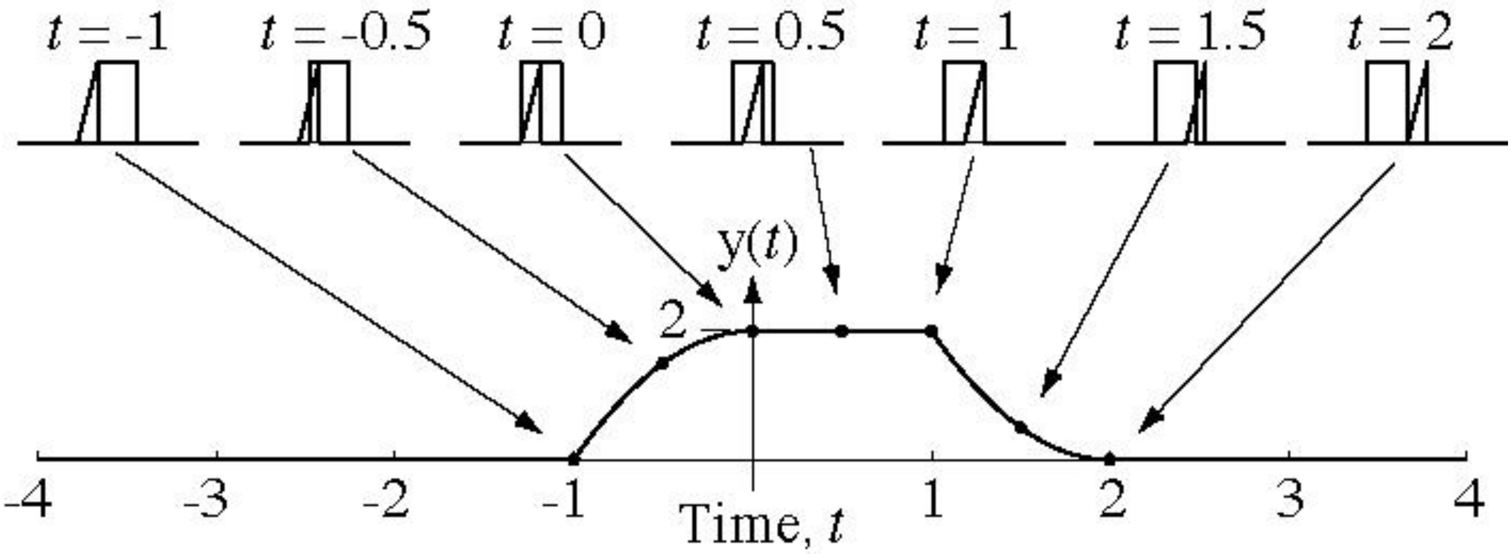
Now let $t = 0$.



Therefore y(0) = 2, the area under the product.

A Graphical Illustration of the Convolution Integral

The process of convolving to find y(t) is illustrated below.



Properties of Convolution

Continuous Time

$$h(t) = \int_{-\infty}^{\infty} \delta(\tau) h(t-\tau) d\tau$$

$$= \delta(t) * h(t)$$

Discrete Time

$$h[n] = \sum_{m=-\infty}^{\infty} \delta[m] h[n-m]$$

$$= \delta[n] * h[n]$$

Properties of Convolution … cont.

Continuous Time

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= x(t) * h(t)$$

$$= h(t) * x(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

Discrete Time

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

$$= x[n] * h[n]$$

$$= h[n] * x[n]$$

$$= \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$

Causality and Stability from Impulse Response

Continuous Time

Causality means for t < 0

$$h(t) = 0$$

Stability means

$$\int_{-\infty}^{\infty} h(t)dt < \infty$$

Example:

$$h(t) = e^{-t/RC}u(t)$$

Discrete Time

Causality means for n < 0

$$h[n] = 0$$

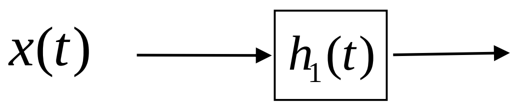
Stability means

$$\sum_{n=-\infty}^{\infty} h[n] < \infty$$

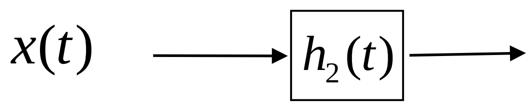
Example:

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Cascaded and Parallel Systems

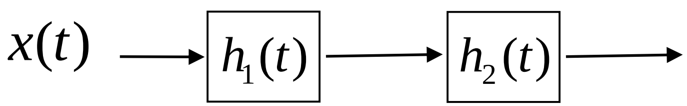


$$y(t) = x(t) * h_1(t)$$



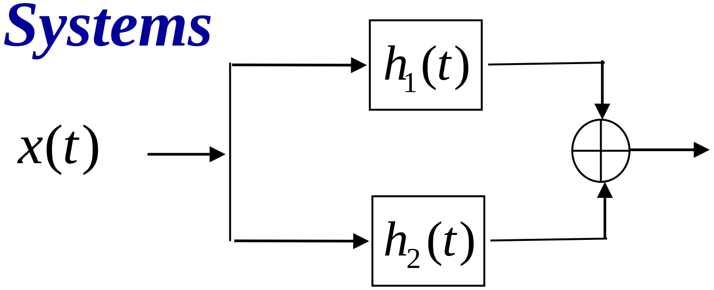
$$y(t) = x(t) * h_2(t)$$

Cascaded Systems



$$y(t) = x(t) * h_1(t) * h_2(t)$$

Parallel Systems



$$y(t) = x(t) * [h_1(t) + h_2(t)]$$

Responses to Standard Signals

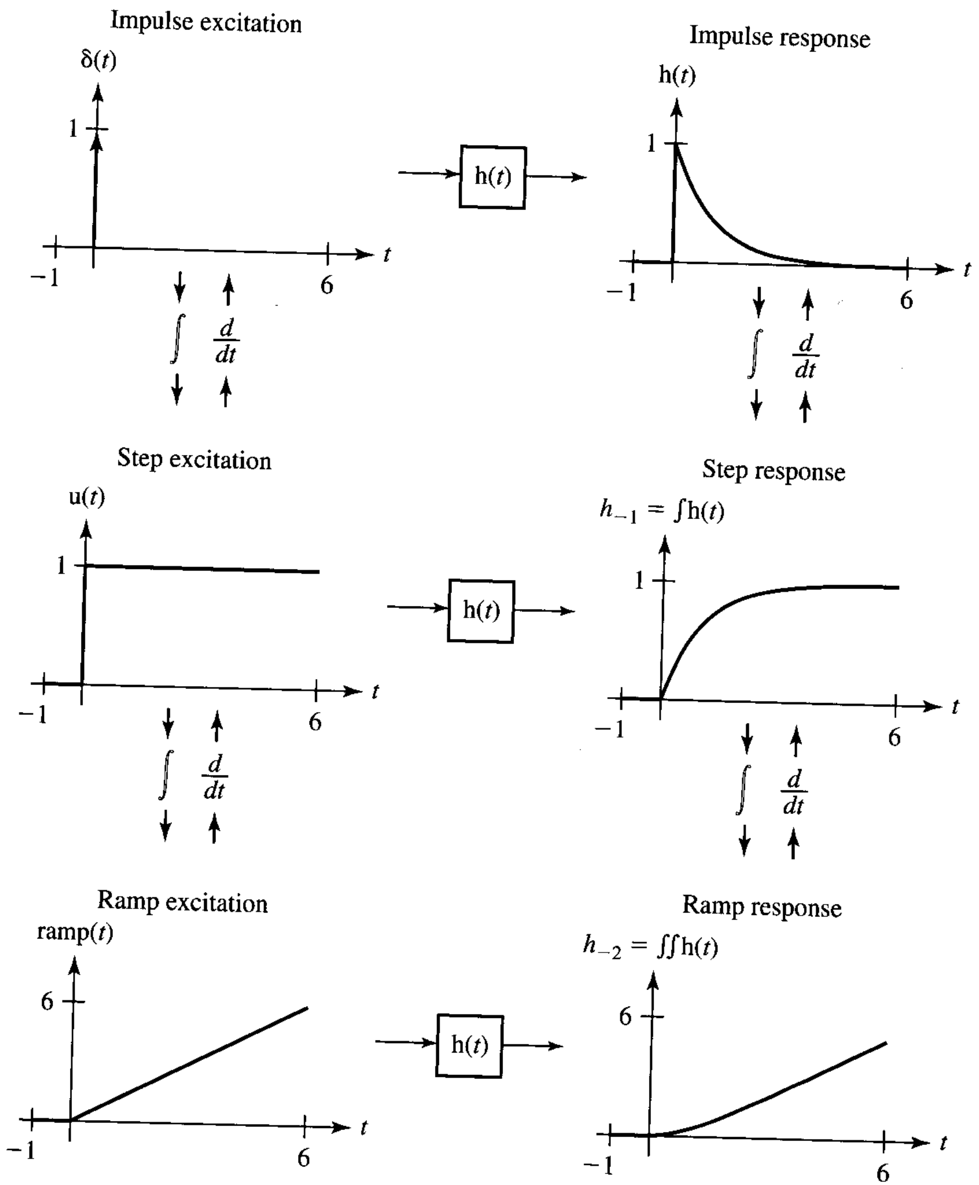
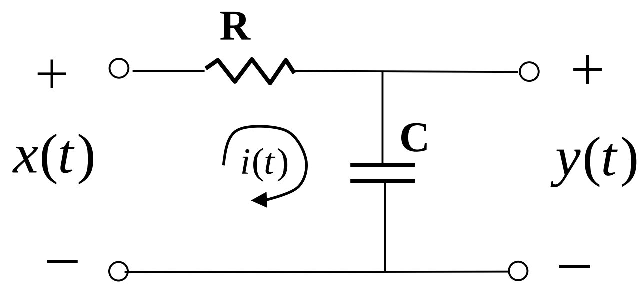


Figure 3.70
Relations between integrals and derivatives of excitations and responses for an LTI system.

Finding Impulse Response

Continuous Time

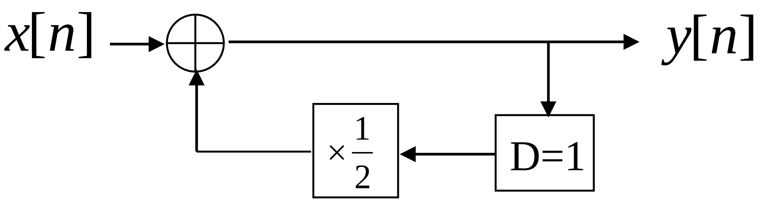


$$RC\frac{dy(t)}{dt}+y(t)=x(t)$$

$$\Rightarrow RC\frac{dh(t)}{dt}+h(t)=\delta(t)$$

$$\Rightarrow h(t)=e^{-t/RC}u(t)$$

Discrete Time

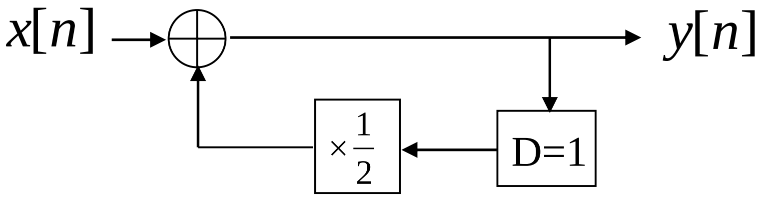


$$y[n]-\frac{1}{2}y[n-1]=x[n]$$

$$\Rightarrow h[n]-\frac{1}{2}h[n-1]=\delta[n]$$

$$\Rightarrow h[n]=\left(\frac{1}{2}\right)^{n}u[n]$$

Finding the Impulse Response by Recursive Method

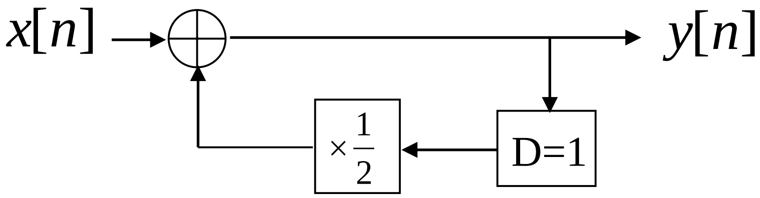


$$y[n]-\frac{1}{2}y[n-1]=x[n]$$

$$\Rightarrow y[n]=x[n]+\frac{1}{2}y[n-1]$$

n	Unit Impulse	y(n)	h(n)
-2	0	0	0
-1	0	0	0
0	1	1	1
1	0	1/2	1/2
2	0	1/4	1/4
3	0	1/8	1/8
4	0	1/16	1/16
5	0	1/32	1/32
6	0	1/64	1/64

Solving First Order Difference Equation



$$y[n]-\frac{1}{2}y[n-1]=x[n]$$

Homogeneous Solution

$$y[n]-\frac{1}{2}y[n-1]=0$$

$$y[n]=\frac{1}{2}y[n-1]$$

$$\frac{y[n]}{y[n-1]}=\frac{1}{2}$$

$$\Rightarrow y[n]=K\left(\frac{1}{2}\right)^n$$

Particular Solution

$$y[n]-\frac{1}{2}y[n-1]=\delta[n]$$

At n= 0

$$y[0]-\frac{1}{2}y[-1]=\delta[0]$$

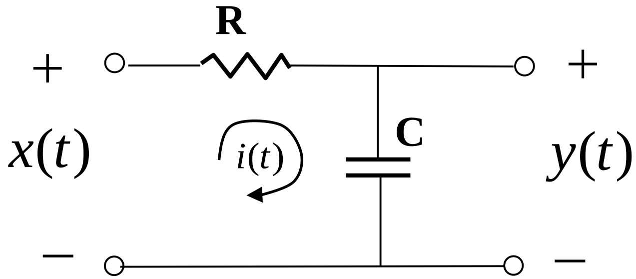
$$y[0]-0=1$$

$$K\left(\frac{1}{2}\right)^0=1$$

$$\Rightarrow K=1$$

$$\Rightarrow y[n]=\left(\frac{1}{2}\right)^n u[n]$$

Solving First Order Differential Equation



$$RC\frac{dy(t)}{dt}+y(t)=x(t)$$

Homogeneous Solution

$$RC\frac{dy(t)}{dt}+y(t)=0$$

$$\frac{dy(t)}{dt}=-\frac{1}{RC}y(t)$$

$$\Rightarrow y(t)=Ke^{-\frac{1}{RC}t}$$

Particular Solution

$$RC\frac{dy(t)}{dt}+y(t)=\delta(t)$$

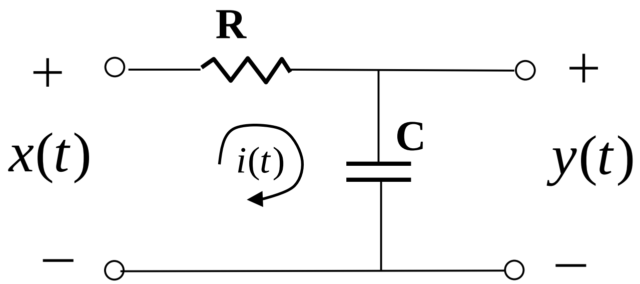
Integrating from t = 0⁻ to t = 0⁺

$$RC\int_{0^-}^{0^+}\frac{dy(t)}{dt}dt+\int_{0^-}^{0^+}y(t)dt=\int_{0^-}^{0^+}\delta(t)dt$$

$$RC[y(0^+)-y(0^-)]+\int_{0^-}^{0^+}y(t)dt=1$$

$$RC[y(0^+)-y(0^-)]+0=1$$

Solving First Order Differential Equation



$$RC\frac{dy(t)}{dt} + y(t) = x(t)$$

Homogeneous Solution

$$RC\frac{dy(t)}{dt} + y(t) = 0$$

$$\frac{dy(t)}{dt} = -\frac{1}{RC}y(t)$$

$$\Rightarrow y(t) = Ke^{-\frac{1}{RC}t}$$

Particular Solution … cont

$$RC[y(0^+) - y(0^-)] + 0 = 1$$

$$RC[y(0^+) - y(0^-)] = 1$$

$$RC[y(0^+) - 0] = 1$$

$$RCy(0^+) = 1$$

$$RCKe^{0^+} = 1 \qquad \Rightarrow K = \frac{1}{RC}$$

$$\Rightarrow y(t) = \frac{1}{RC}e^{-\frac{1}{RC}t}u(t)$$