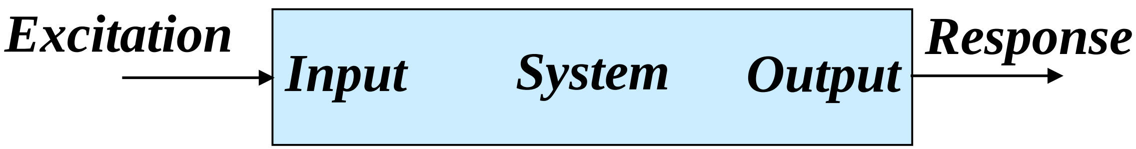
Signal – A function of time

System – Processes input signal (excitation) and produces output signal (response)



1. ***Types of signals***
2. ***Going from analog to digital world***
3. ***An example of a system***
4. ***Mathematical representation of signals***

Types of Signals

	Time	***Value***
1	*Continuous*	*Continuous*
2	*Continuous*	*Discrete*
3	*Discrete*	*Continuous*
5	*Discrete*	*Discrete*

← ***Analog*** (row 1)

← ***Digital*** (row 2)

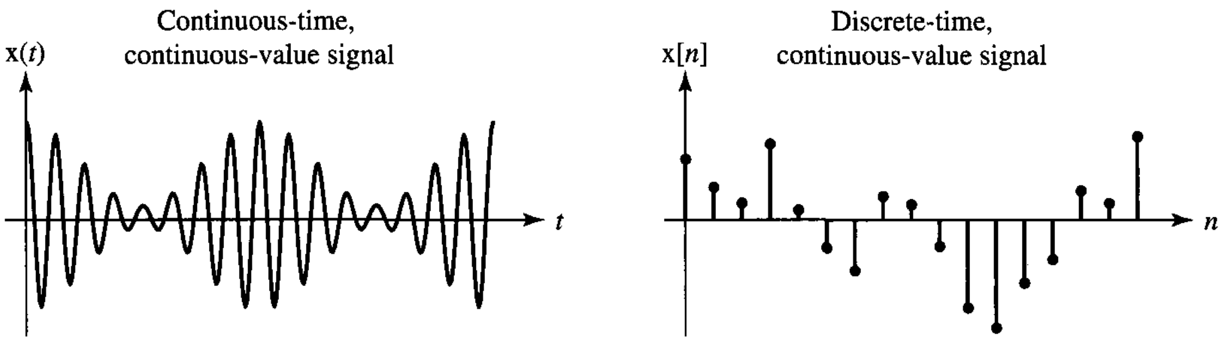


Figure 1.3
Examples of continuous-time and discrete-time signals.

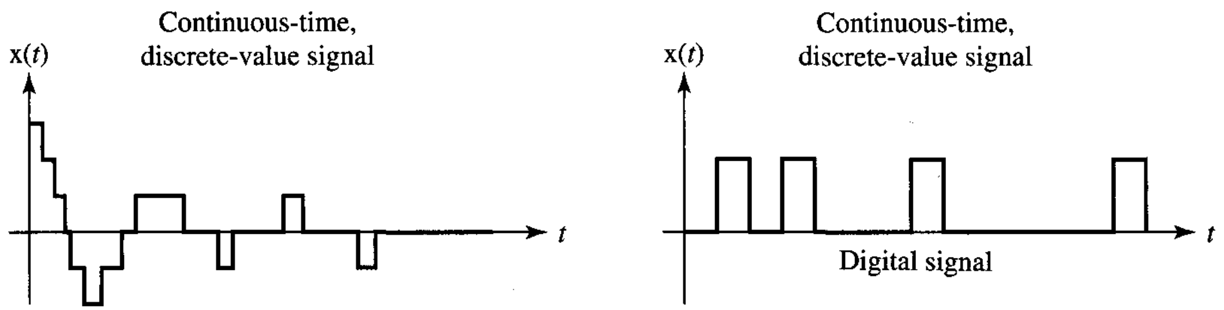


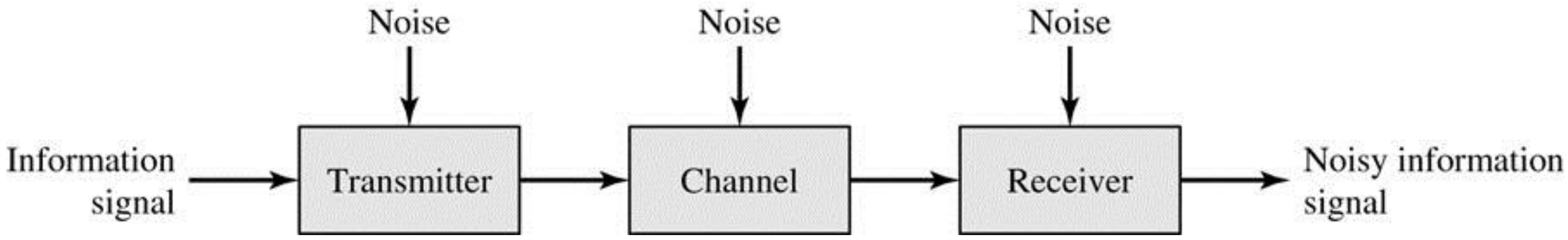
Figure 1.4
Examples of continuous-time and digital signals.

Types of Signals

	Time	***Value***
1	*Continuous*	*Continuous*
2	*Continuous*	*Discrete*
3	*Discrete*	*Continuous*
5	*Discrete*	*Discrete*

← ***Analog***

← ***Digital***



Role of Noise!

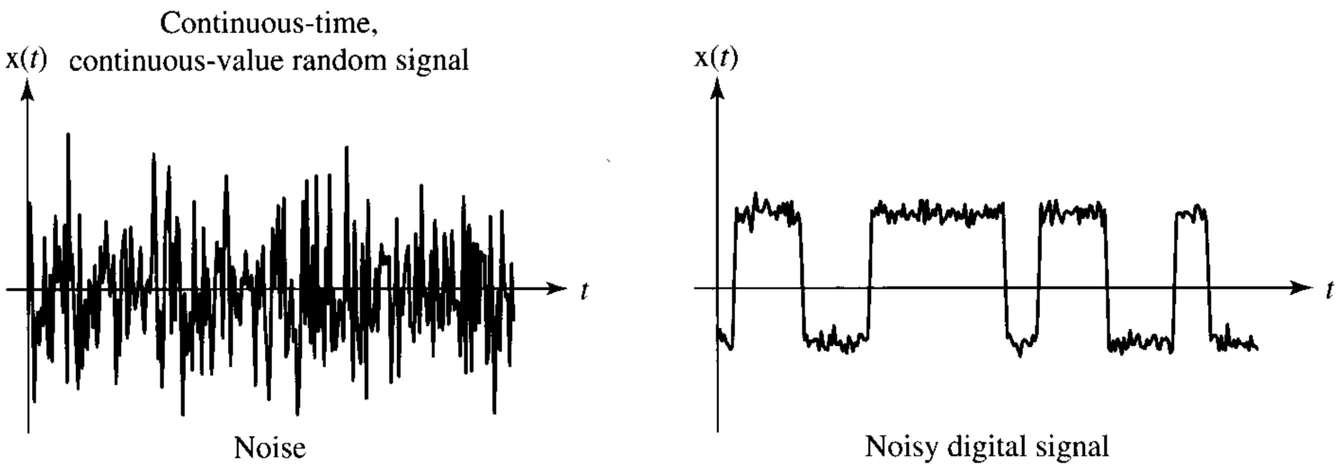
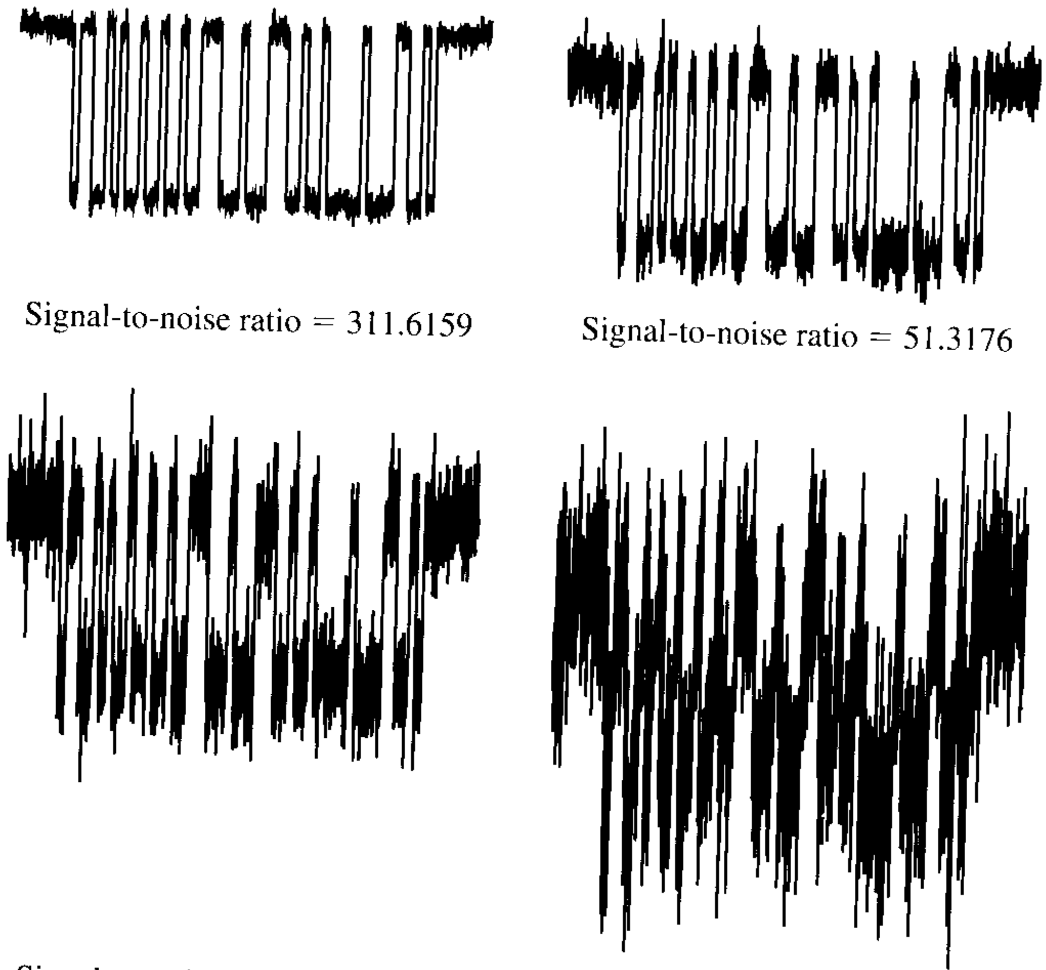


Figure 1.5
Examples of noise and a noisy digital signal.



Signal-to-noise ratio = 311.6159

Signal-to-noise ratio = 51.3176

Signal-to-noise ratio = 12.6983

Signal-to-noise ratio = 3.2081

Figure 1.8
Noisy digital ASCII signal.

Advantage of Digital World

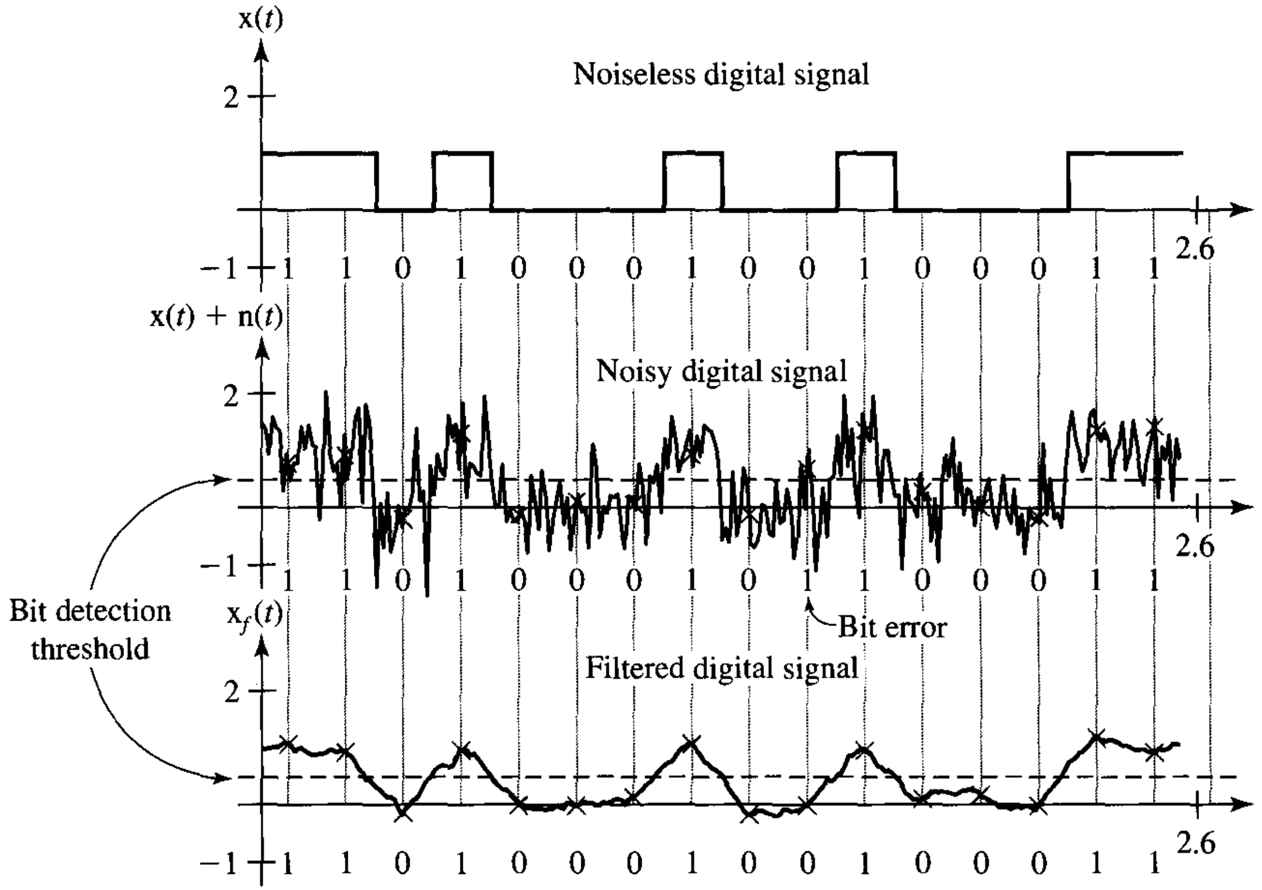


Figure 1.9
Use of a filter to reduce bit error rate in a digital signal.

Going from Analog to Digital World

Three Step Process

Sampling

↓

Quantization

↓

Encoding

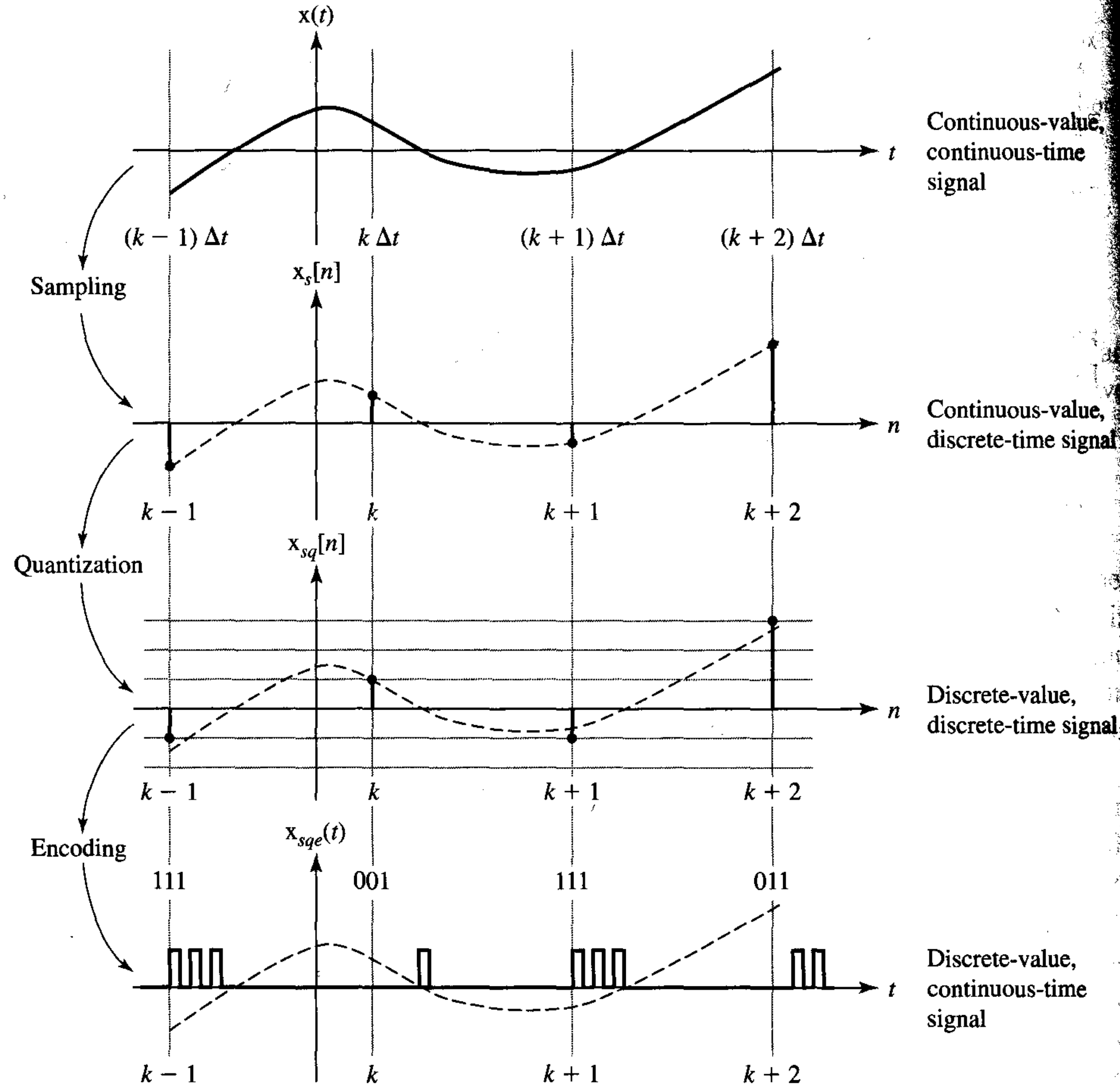


Figure 1.6
Sampling, quantization, and encoding of a signal to illustrate various signal types.

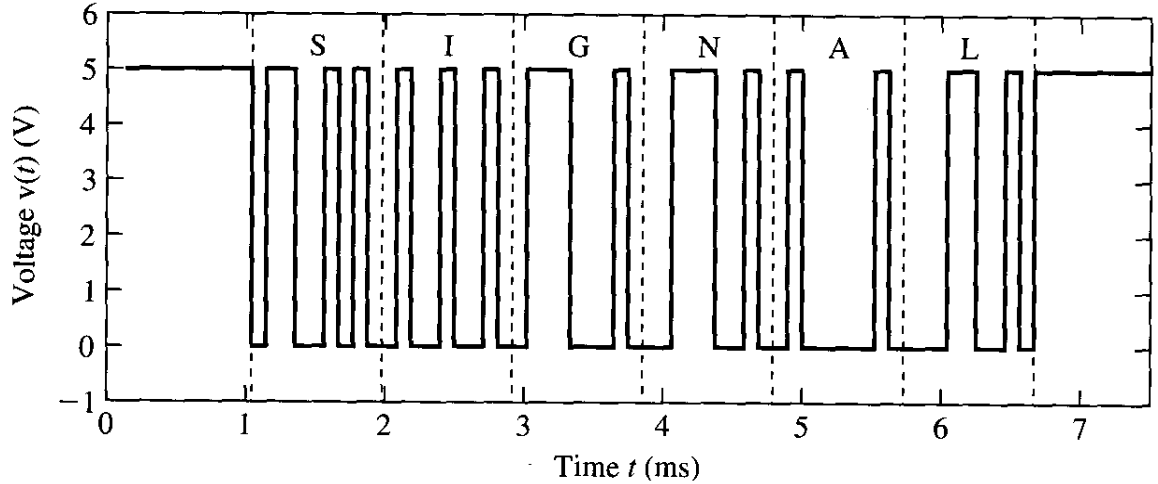
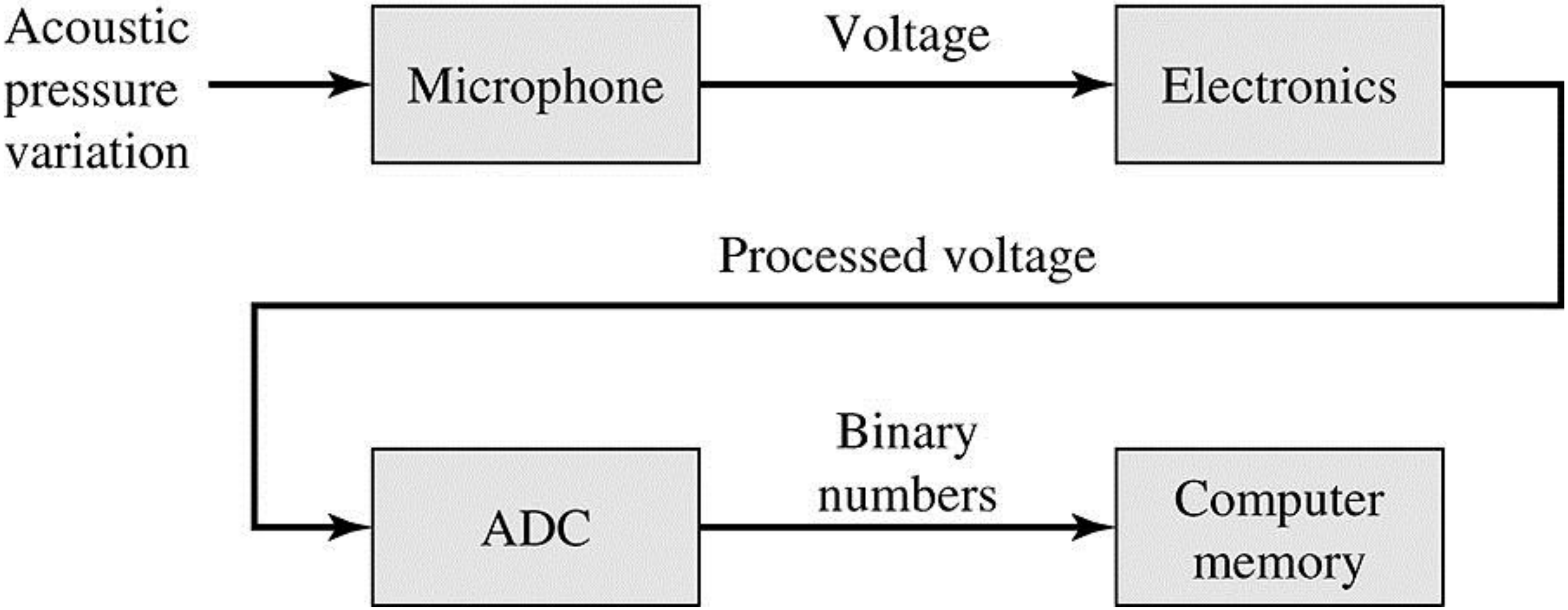


Figure 1.7
Asynchronous serial binary ASCII-encoded voltage signal for the message "SIGNAL."

Example of System

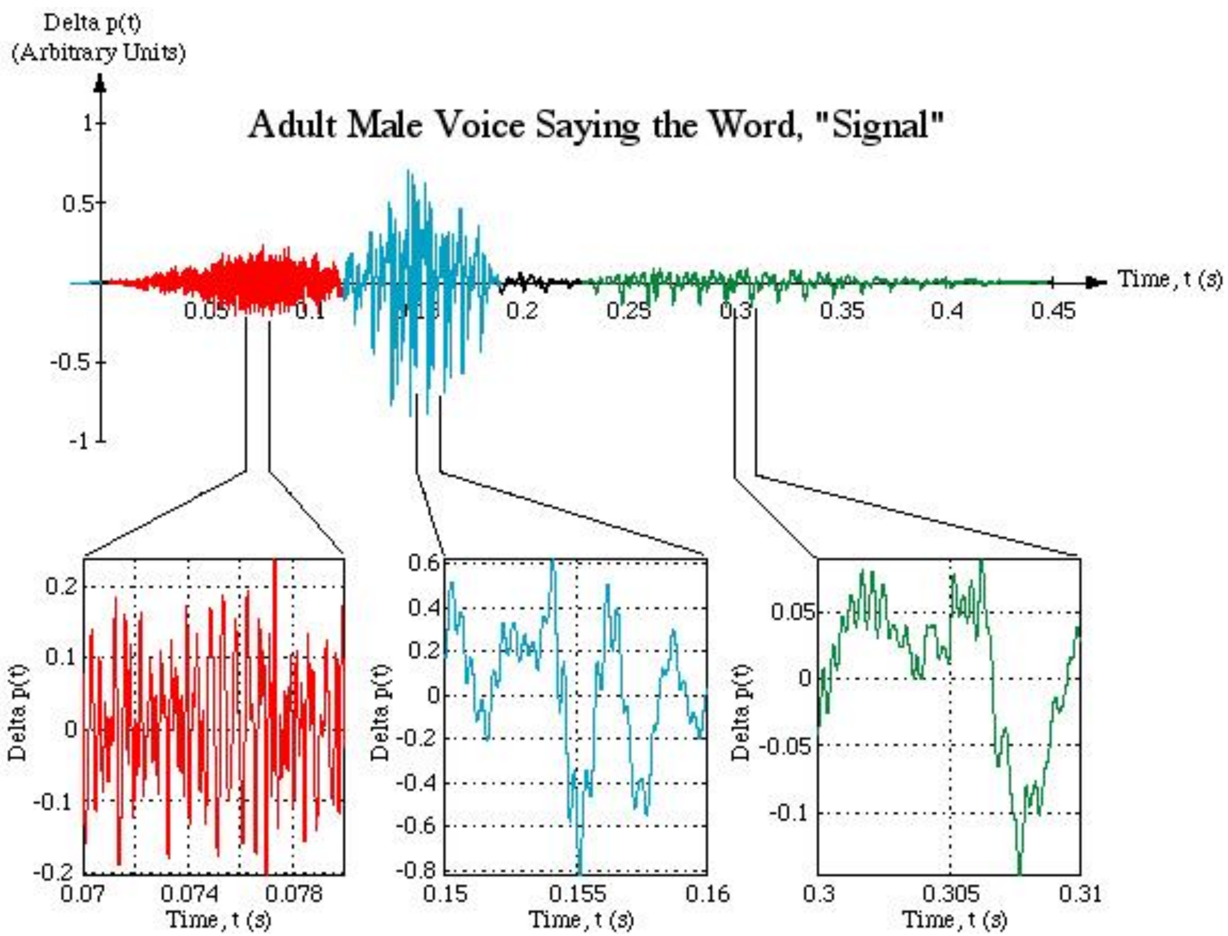
A simple system example – Sound Recording System

What constitutes a sound recording system?



Recorded Sound as a Signal Example

- “s” “i” “gn” “al”



Representation of Signals

Time Domain

Frequency Domain

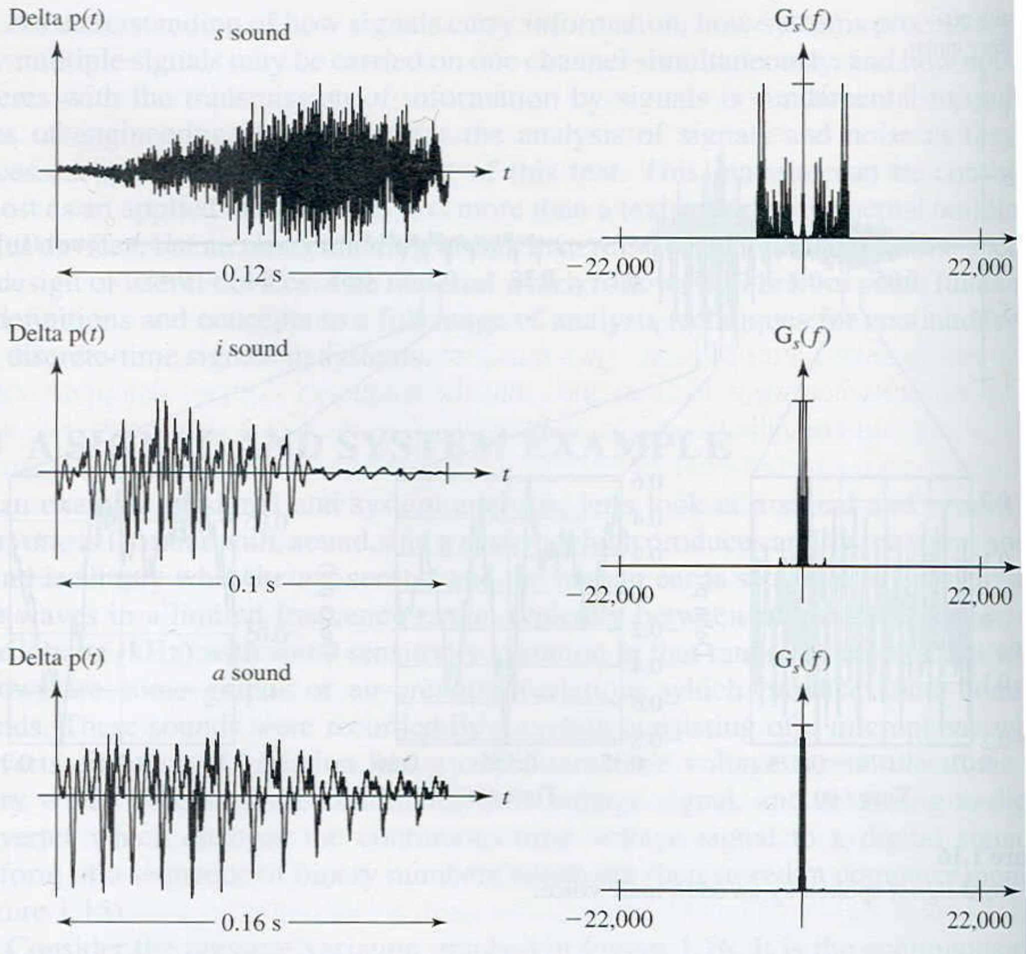


Figure 1.17
Three sounds in the word *signal* and their associated power spectral densities.

Another Example of System

A very complex system example – Human Brain

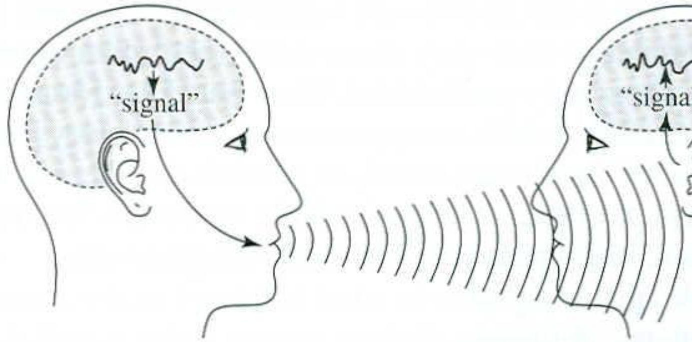


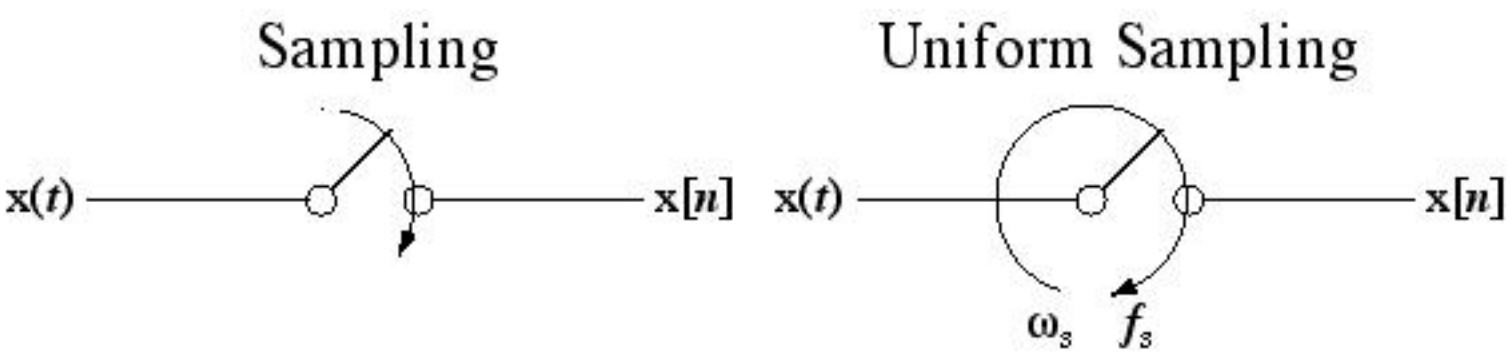
Figure 1.18
Communication between two people involving signals and signal processing by systems.

Sampling a CT Signal to Create a DT Signal

- ***Sampling is acquiring the values of a CT signal at discrete points in time***

- ***x(t) is a CT signal --- x[n] is a DT signal***

$x[n] = x(nT_s)$ ***where*** T_s ***is the time between samples***



Mathematical Representation of Signals

Continuous Time

$$x(t) = A\sin(\omega_0 t)$$

$$= A\sin(2\pi f_0 t)$$

Discrete Time

$$x[nT_s] = A\sin[\omega_o nT_s]$$

$$= A\sin[2\pi f_o nT_s]$$

Continuous Time Signals

Sinusoidal Signal:

$$g(t) = A\sin(\omega_o t)$$

$$= A\sin(2\pi f_o t)$$

$$= A\sin(\frac{2\pi t}{T_o})$$

General Form:

$$g(t) = C\cos(2\pi f_o t + \theta)$$

Review of Euler's Identity – Complex valued sinusoidal signals

Euler's Identity

$$e^{j2\pi f_F t} = \cos(2\pi f_F t) + j\sin(2\pi f_F t)$$

Also

$$e^{-j2\pi f_F t} = \cos(2\pi f_F t) - j\sin(2\pi f_F t)$$

$$\Rightarrow \cos(2\pi f_F t) = \frac{e^{j2\pi f_F t} + e^{-j2\pi f_F t}}{2}$$

and

$$\Rightarrow \sin(2\pi f_F t) = \frac{e^{j2\pi f_F t} - e^{-j2\pi f_F t}}{2j}$$

Review of Euler's Identity – Complex valued sinusoidal signals

Euler's Identity

$$Ce^{j(2\pi f_F t+\theta)} = C\cos(2\pi f_F t+\theta) + jC\sin(2\pi f_F t+\theta)$$

Also

$$Ce^{-j(2\pi f_F t+\theta)} = C\cos(2\pi f_F t+\theta) - jC\sin(2\pi f_F t+\theta)$$

$$\Rightarrow C\cos(2\pi f_F t+\theta) = \frac{Ce^{j(2\pi f_F t+\theta)} + Ce^{-j(2\pi f_F t+\theta)}}{2}$$

and

$$\Rightarrow C\cos(2\pi f_F t+\theta) = \frac{Ce^{j\theta}}{2}e^{j2\pi f_F t} + \frac{Ce^{-j\theta}}{2}e^{-j2\pi f_F t}$$

Three ways to represent a sinusoidal frequency f_F

Method 1:

$$\frac{Ce^{j\theta}}{2}e^{j2\pi f_F t}+\frac{Ce^{-j\theta}}{2}e^{-j2\pi f_F t}$$

Method 2:

$$C\cos(2\pi f_F t+\theta)$$

Method 3:

$$A\cos(2\pi f_F t)+B\sin(2\pi f_F t)$$

Where

$$C=\sqrt{A^2+B^2} \qquad \theta=\tan^{-1}-\frac{B}{A}$$

Exponential Functions

$$g(t) = Ae^{\sigma t}$$

$$g(t) = Ae^{-\sigma t}$$

Complex valued exponential signal:

$$g(t) = Ae^{(\sigma + j\omega)t} = Ae^{\sigma t}[\cos \omega t + j \sin \omega t]$$

Where do these functions occur in real life?

Discontinuity of a function

Definition:

$$\lim_{\varepsilon \to 0} g(t+\varepsilon) \neq \lim_{\varepsilon \to 0} g(t-\varepsilon)$$

Simple words:

If the value of function is different at time t_0 when approached at t_0 by decreasing and increasing time, then the function is discontinuous at time t_0

Examples:

Unit Step Function

Definition:

$$u(t) = \begin{cases} 1 & t > 0 \\ 1/2 & t = 0 \\ 0 & t < 0 \end{cases}$$

Real Physical Phenomenon: ***Switching***

Signum Function

Definition:

$$\operatorname{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

$$= 2u(t) - 1$$

Ramp Function

Definition:

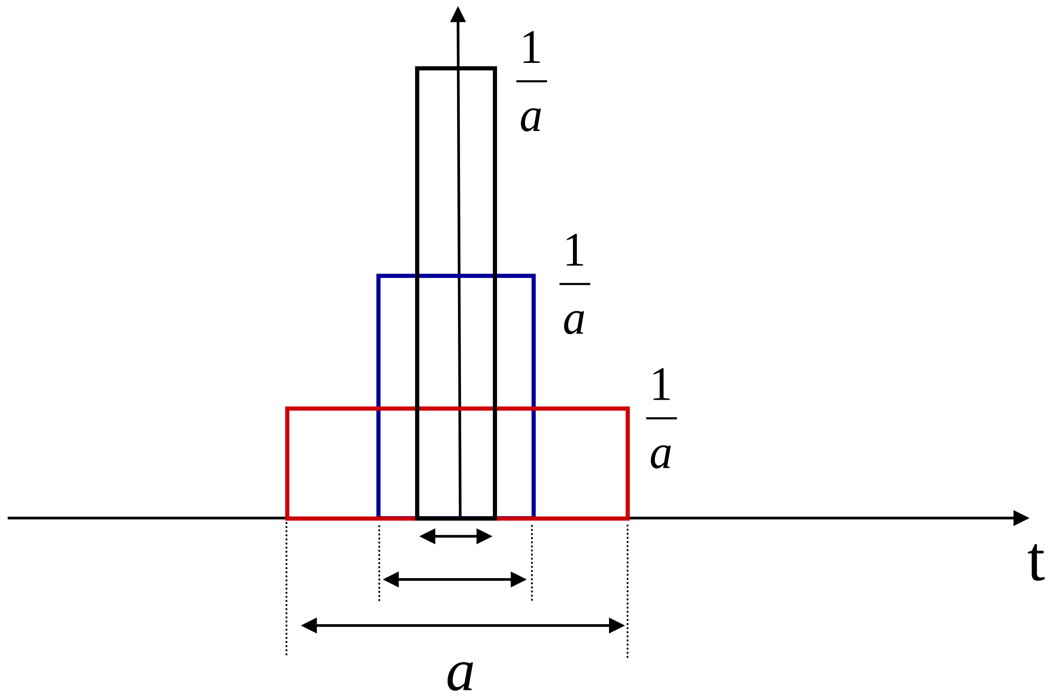
$$ramp(t) = \begin{cases} t & t > 0 \\ 0 & t \leq 0 \end{cases}$$

$$= tu(t)$$

$$= \int_{-\infty}^{t} u(x)dx$$

Can you generate this function?

Unit Impulse Function



$$\lim_{a \to 0} Area = \frac{1}{a}(a) = 1$$

Definition:

$$\begin{cases} \delta(t) = 0 \quad t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)dt = 1 \end{cases}$$

Can you represent u(t) in terms of unit impulse function?

$$u(t) = \int_{-\infty}^{t} \delta(x)dx$$

Another Important Fact about Unit Impulse Function!

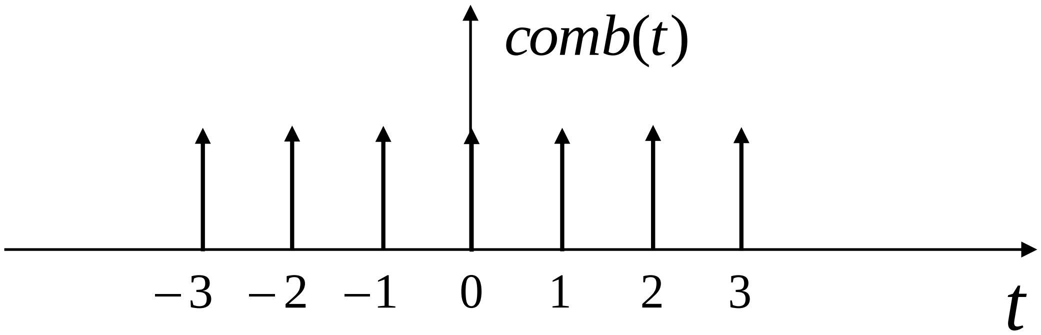
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} g(t)\delta(t)dt = g(0)$$

Isn't it Sampling?

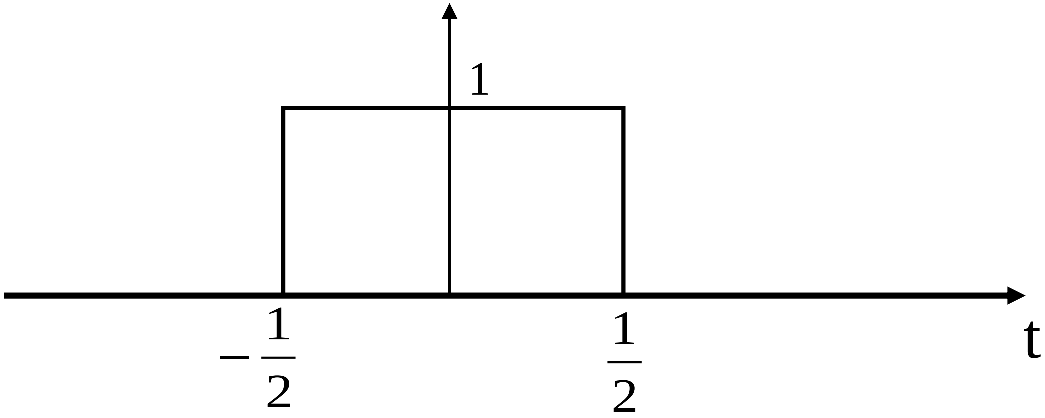
Unit Comb

$$comb(t) = \sum_{n=-\infty}^{n=\infty} \delta(t-n)$$



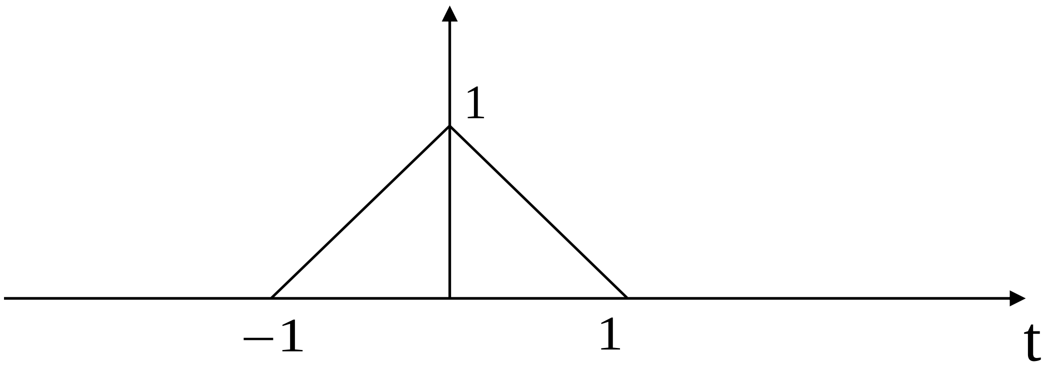
Rectangular Function

$$rect(t) = \begin{cases} 1 & |t| < 1/2 \\ 1/2 & |t| = 1/2 \\ 0 & |t| > 1/2 \end{cases}$$



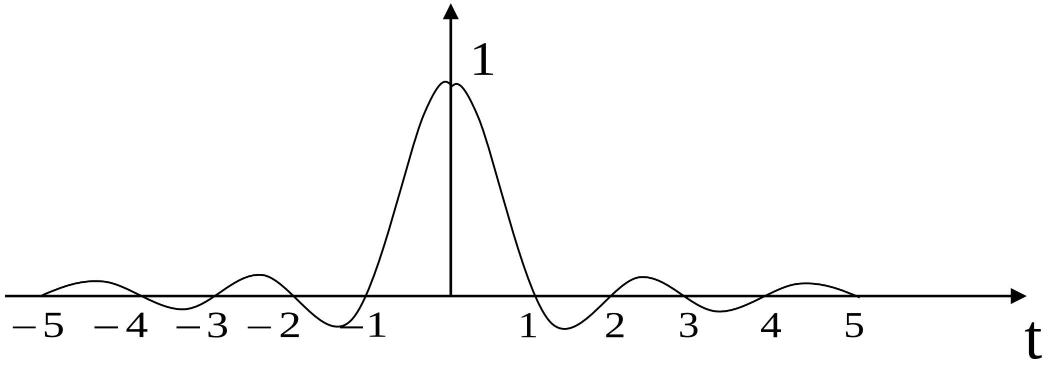
Triangular Function

$$tri(t) = \begin{cases} 1-|t| & |t|<1 \\ 0 & |t|\geq 1 \end{cases}$$



Unit Sinc Function

$$\sin c(t) = \frac{\sin(\pi t)}{\pi t}$$



Combinations of Functions

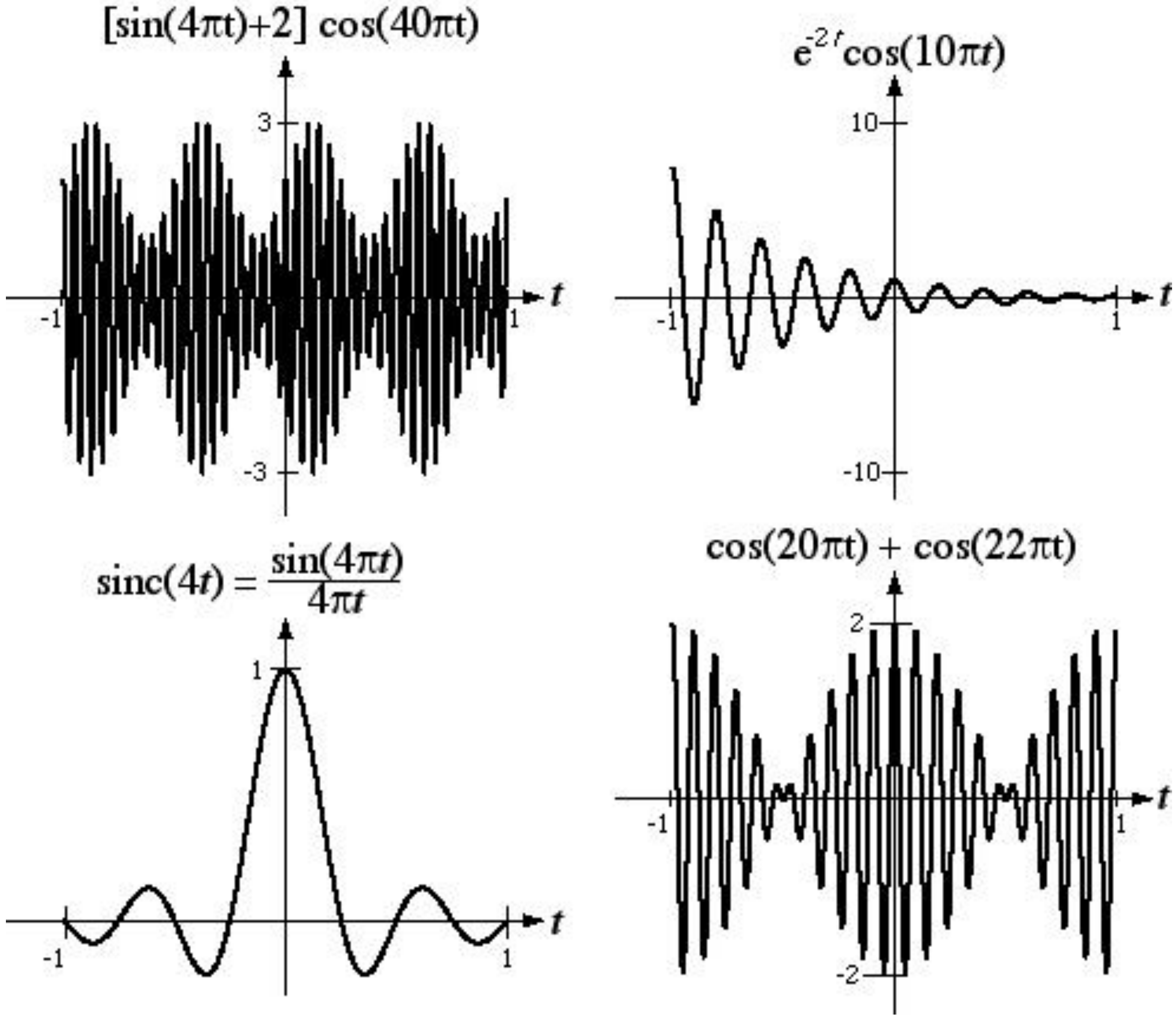
$$g(t) = \sin c(t)\cos(20\pi t)$$

$$g(t) = Ae^{-t}\cos 20\pi t$$

$$g(t) = u(t) + ramp(t)$$

$$g(t) = \mathrm{sgn}(t)\sin(2\pi t)$$

Some More Examples



Amplitude Transformations of Functions

Amplitude Shifting

$$g(t) \to A + g(t)$$

Amplitude Scaling

$$g(t) \to Ag(t)$$

Time Transformations of Functions

Time Shifting

$$g(t) \to g(t-a)$$

Time Scaling

$$g(t) \to g(\frac{t}{a})$$

Multiple Transformations

Case 1

$$g(t) \rightarrow Ag(\frac{t-t_o}{a})$$

$$g(t) \xrightarrow[]{\text{Amplitude Scaling}} Ag(t) \xrightarrow[]{\text{Time Scaling}} Ag(\frac{t}{a}) \xrightarrow[]{\text{Time Shifting}} Ag(\frac{t-t_o}{a})$$

Case 2

$$g(t) \rightarrow Ag(bt-t_o)$$

$$g(t) \xrightarrow[]{\text{Amplitude Scaling}} Ag(t) \xrightarrow[]{\text{Time Shifting}} Ag(t-t_o) \xrightarrow[]{\text{Time Scaling}} Ag(bt-t_o)$$

Example of Case 1

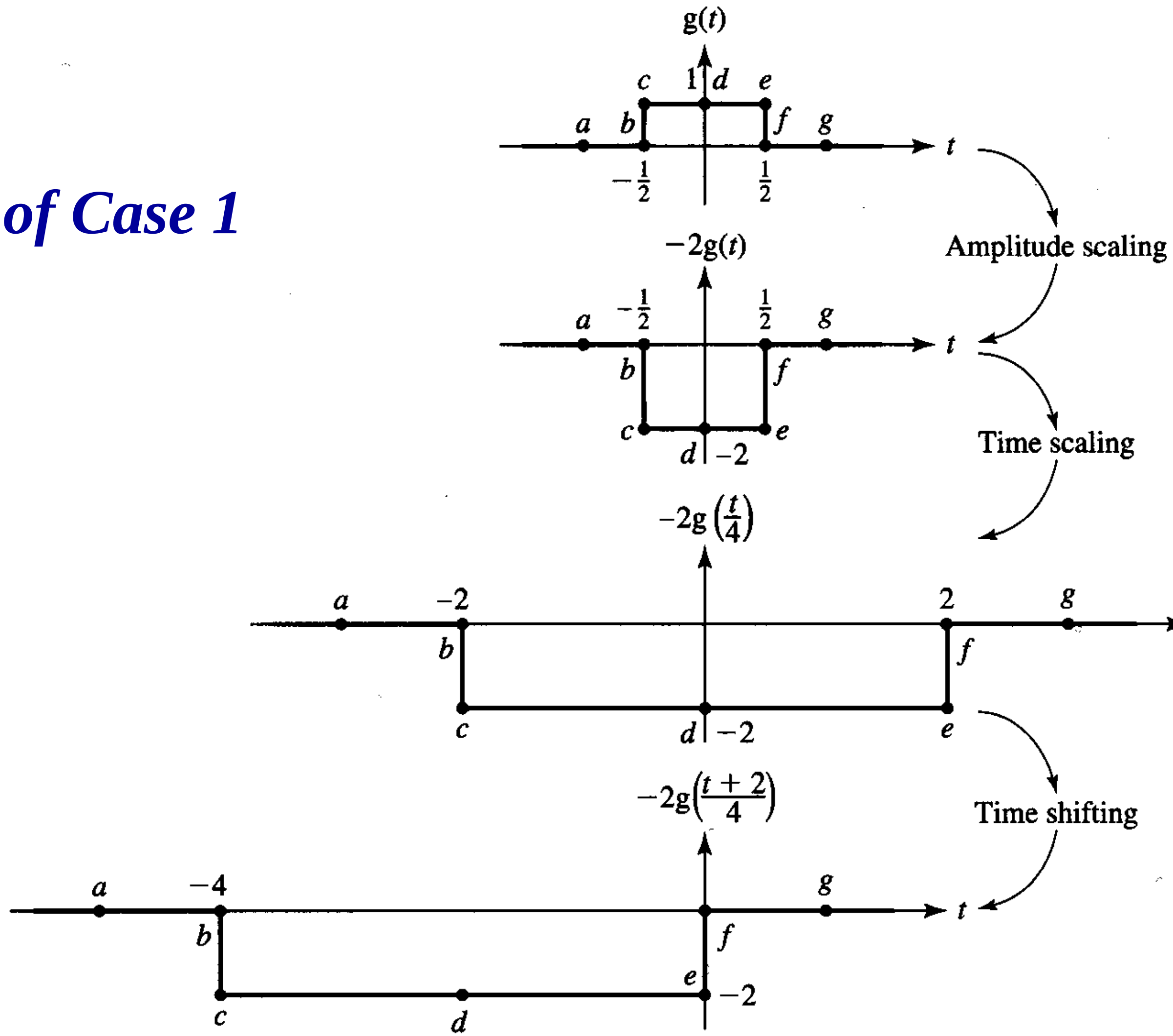


Figure 2.35
A sequence of amplitude scaling, time scaling, and time shifting a function.

Example of Case 2

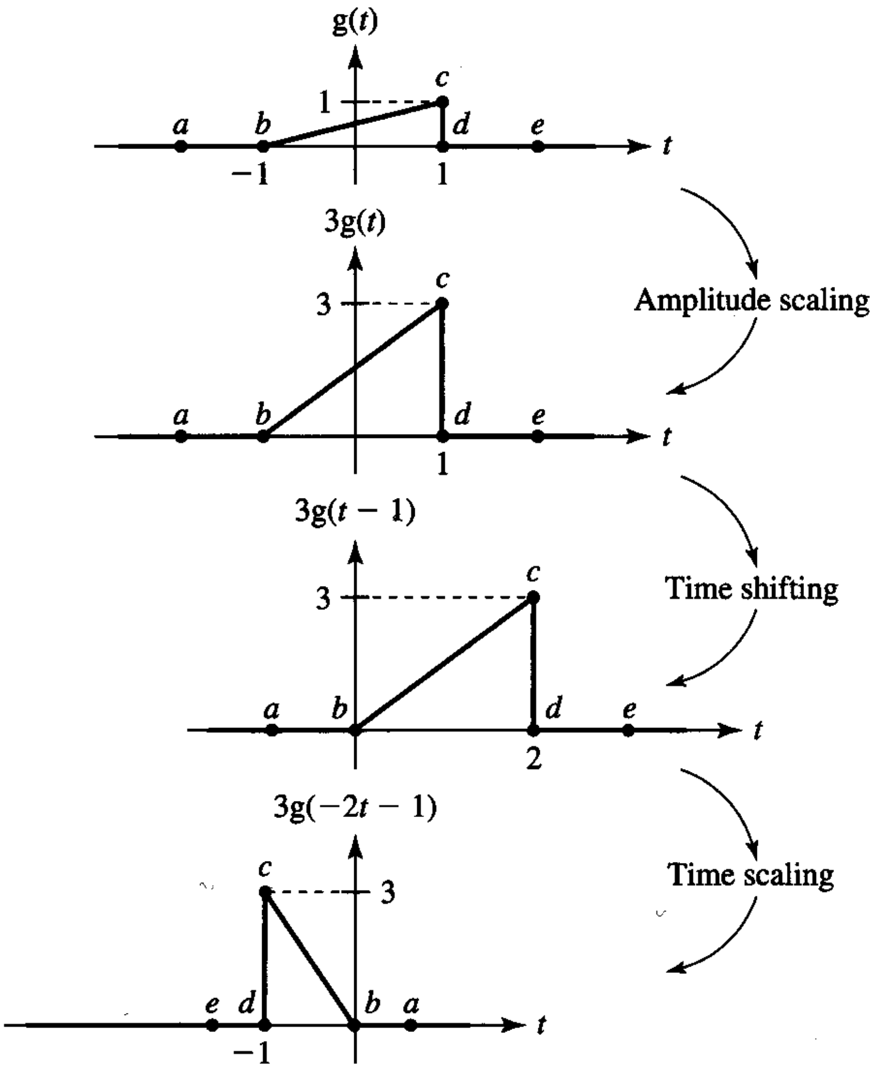
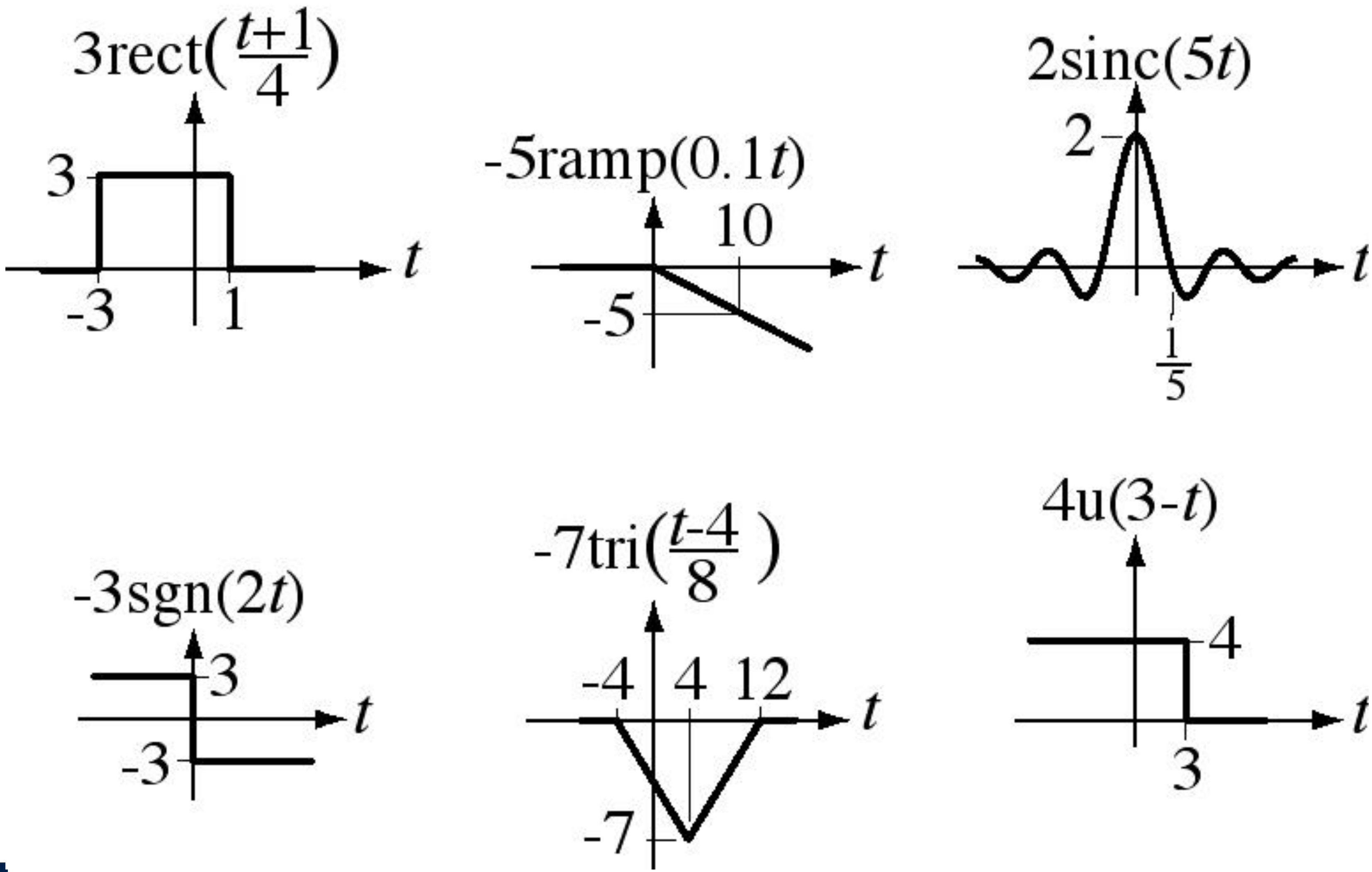


Figure 2.36
A sequence of amplitude scaling, time shifting, and time scaling a function.

Some More Examples



Differentiation and Integration of Functions

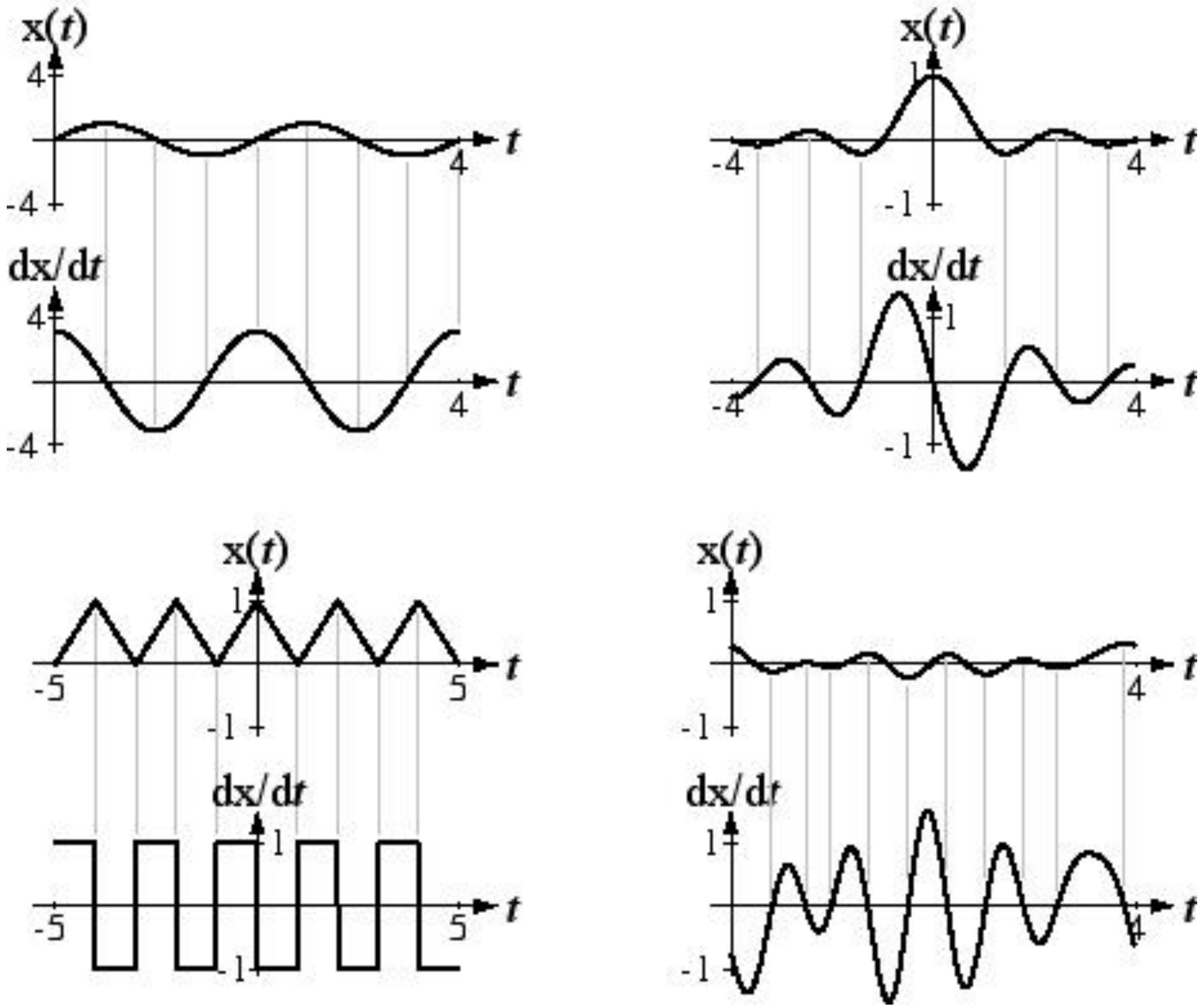
Differentiation: ***Slope of the function at time t***

$$\frac{dg(t)}{dt}$$

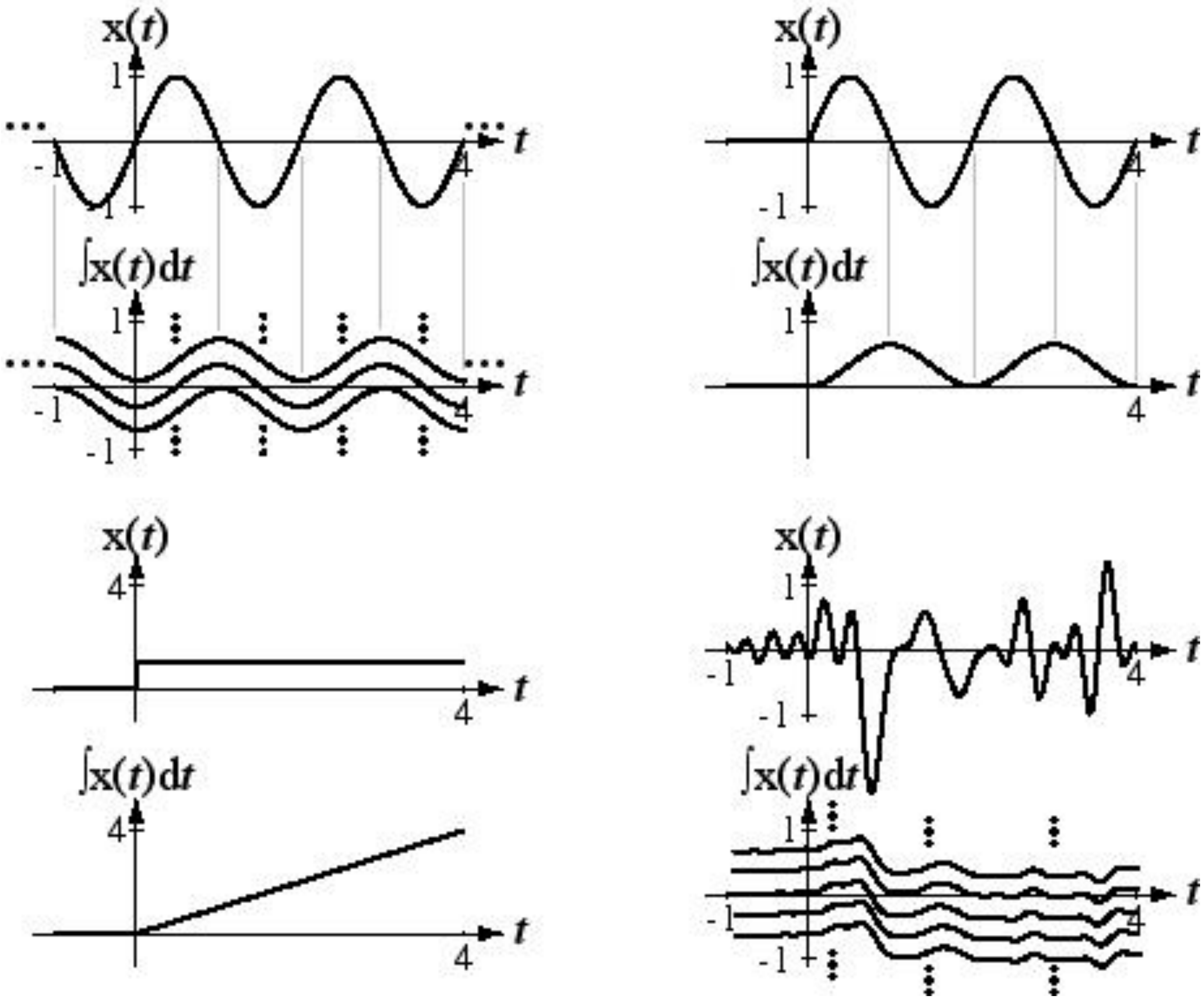
Integration: ***Accumulative area under the curve***

$$\int_{-\infty}^{t} g(x)dx$$

Differentiation – A kind of Transformation of a Signal



Integration – A kind of Transformation of a Signal



Even and Odd Functions

Function is Even if $g(t) = g(-t)$

Example: $\cos(\omega t)$

Function is Odd if $g(t) = -g(-t)$

Example: $\sin(\omega t)$

Even and Odd Components of a Function

If function is neither even nor odd, then

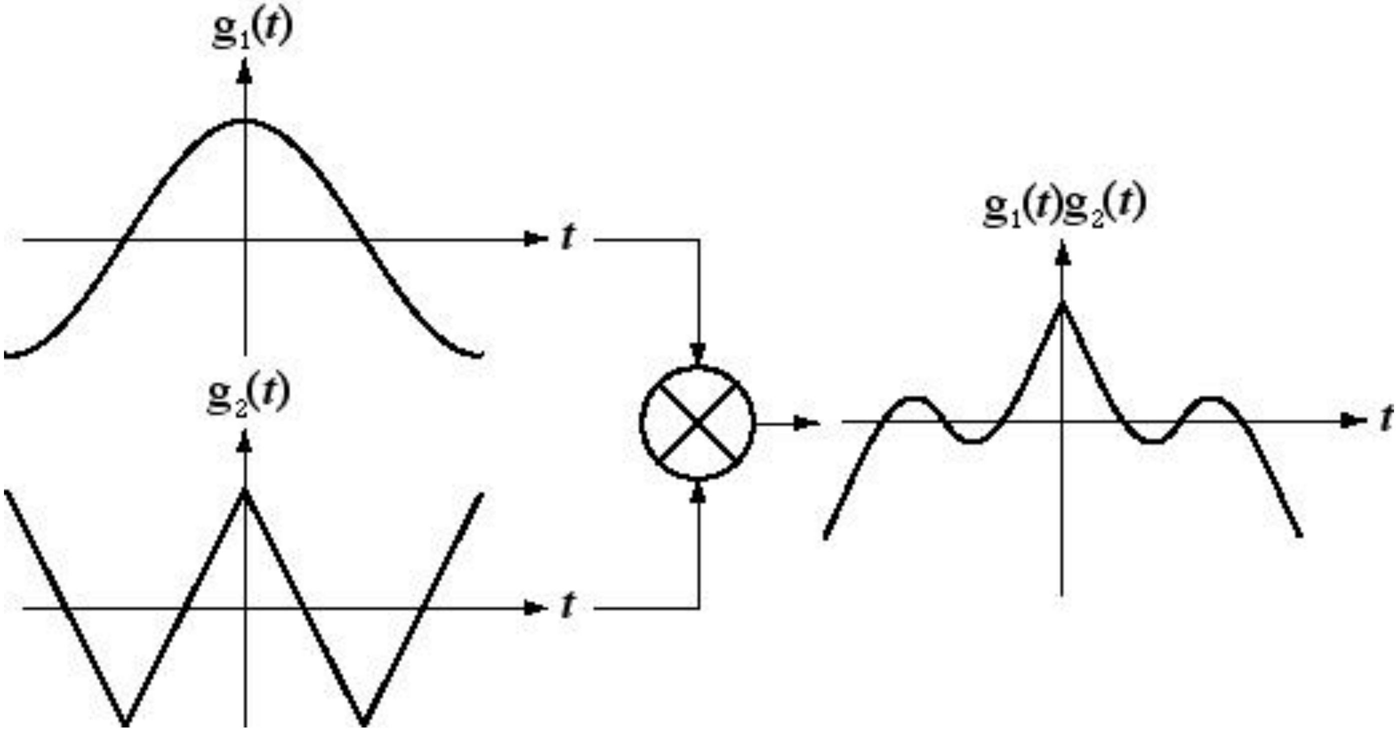
$$g(t) = g_e(t) + g_o(t)$$

Where

$$g_e(t) = \frac{g(t) + g(-t)}{2}$$

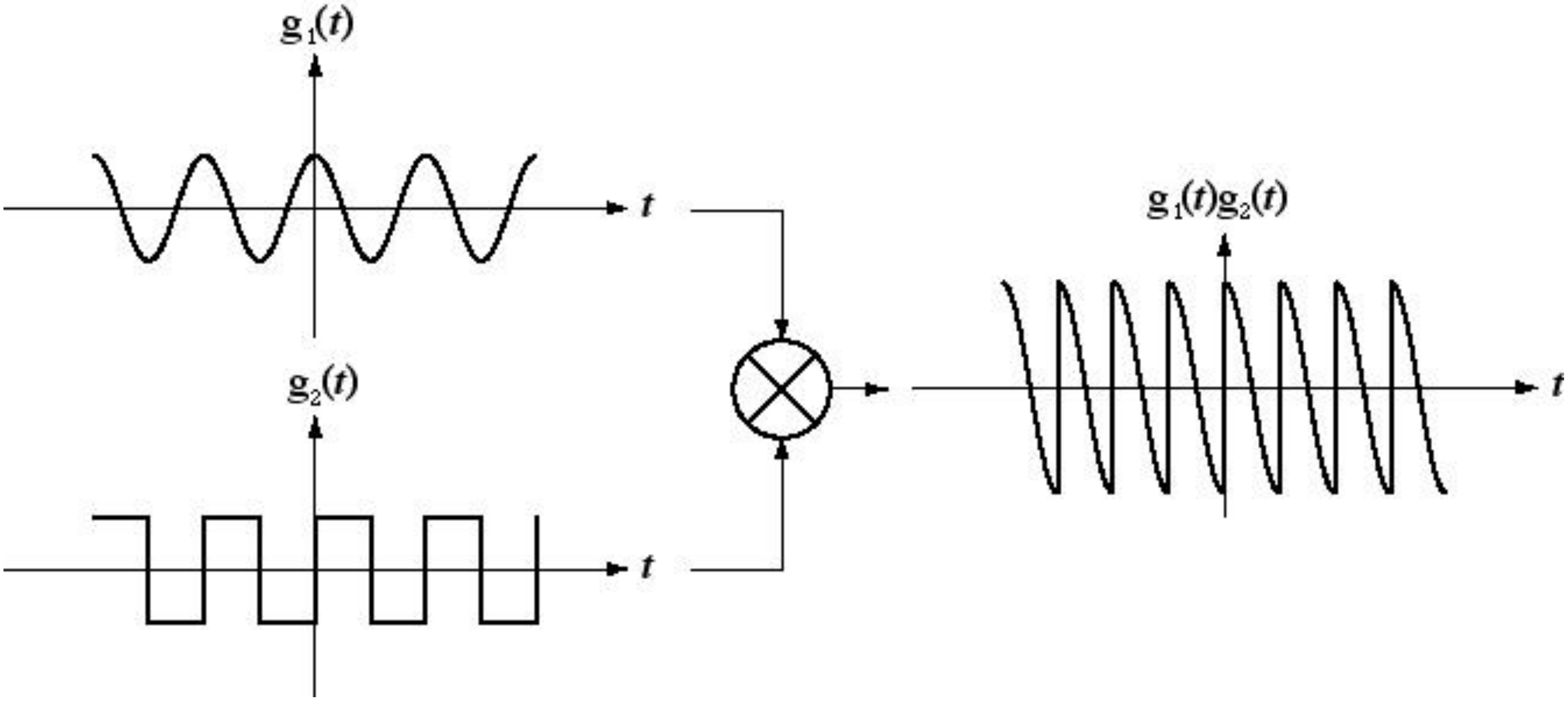
$$g_o(t) = \frac{g(t) - g(-t)}{2}$$

Products of Even and Odd CT Functions



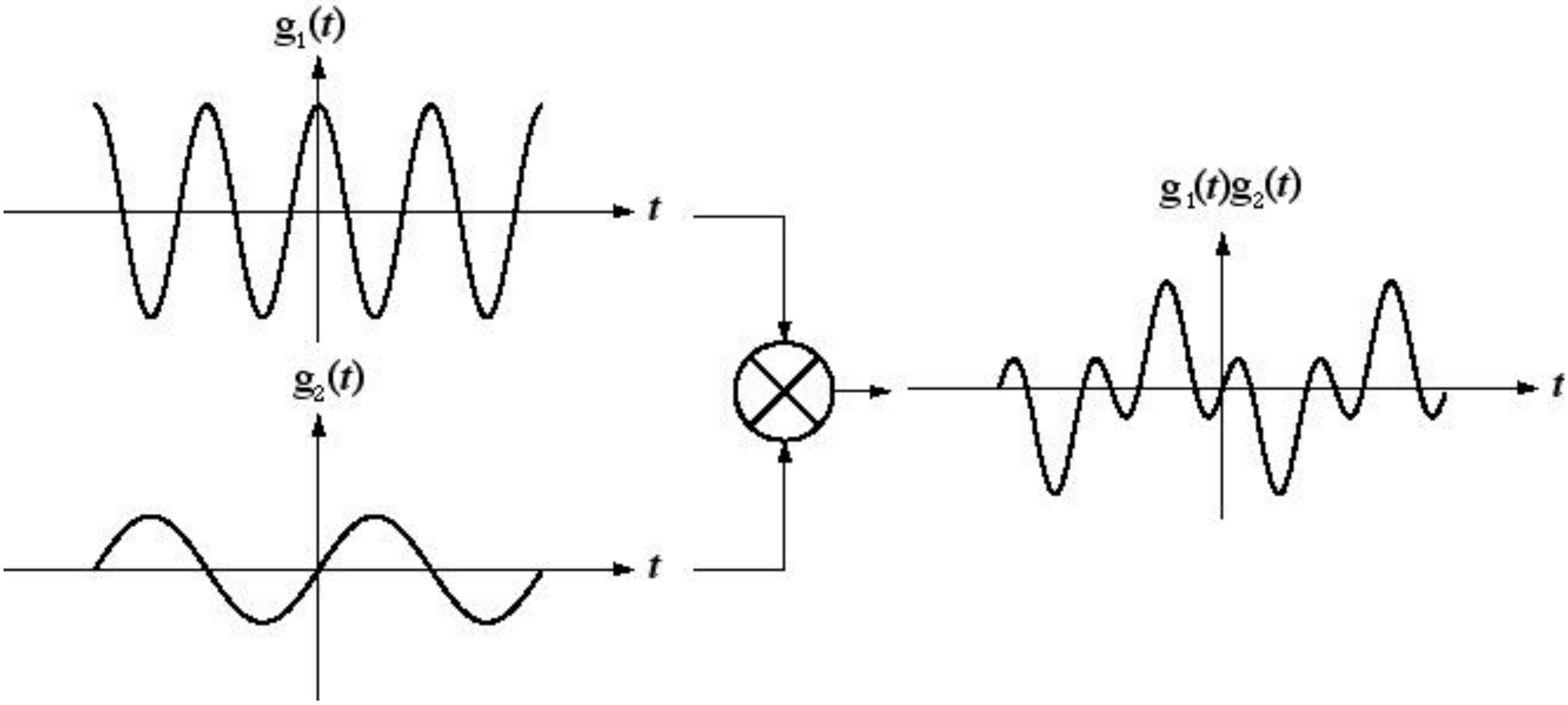
Even x Even = Even

Products of Even and Odd CT Functions



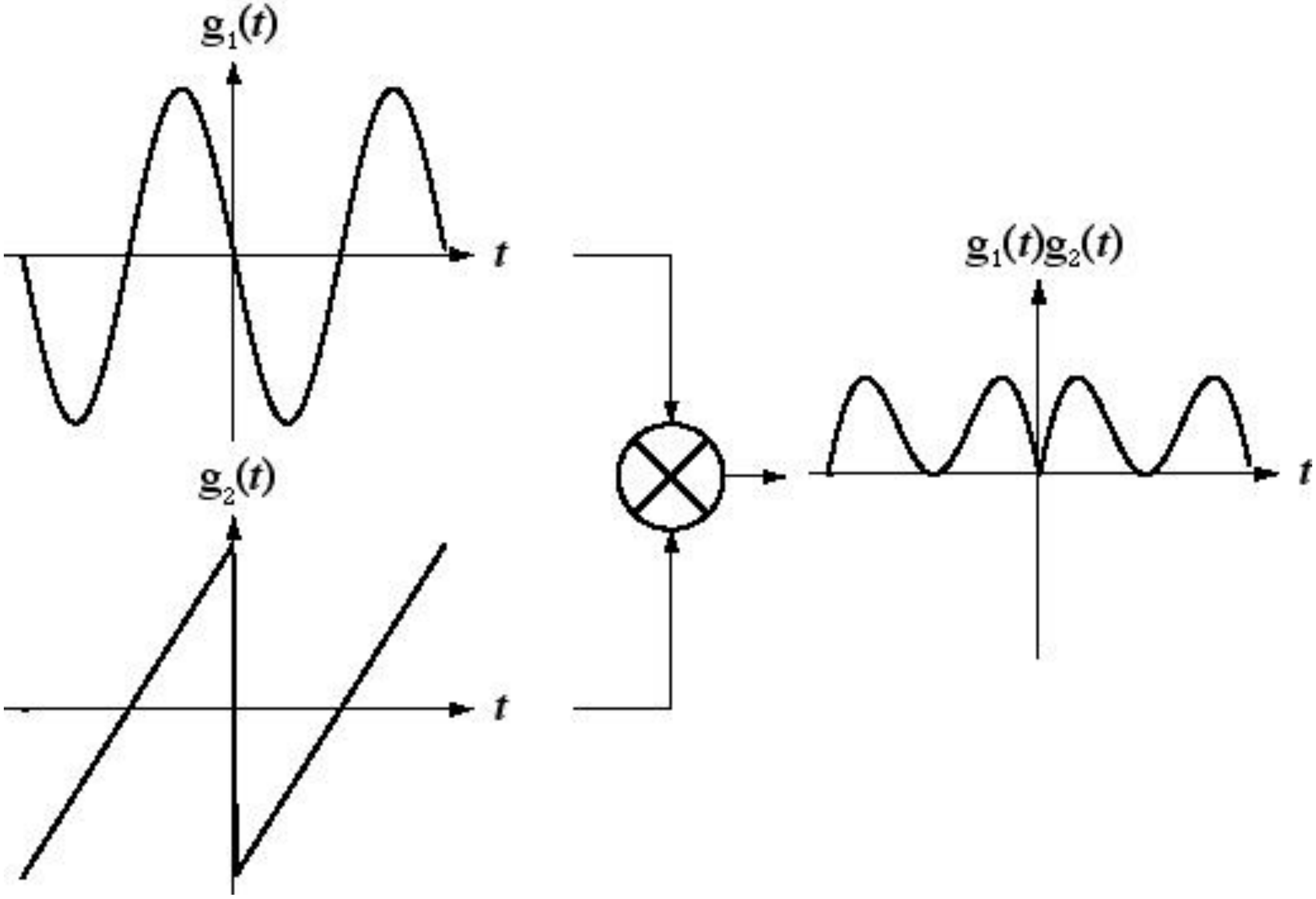
Even x Odd = Odd

Products of Even and Odd CT Functions



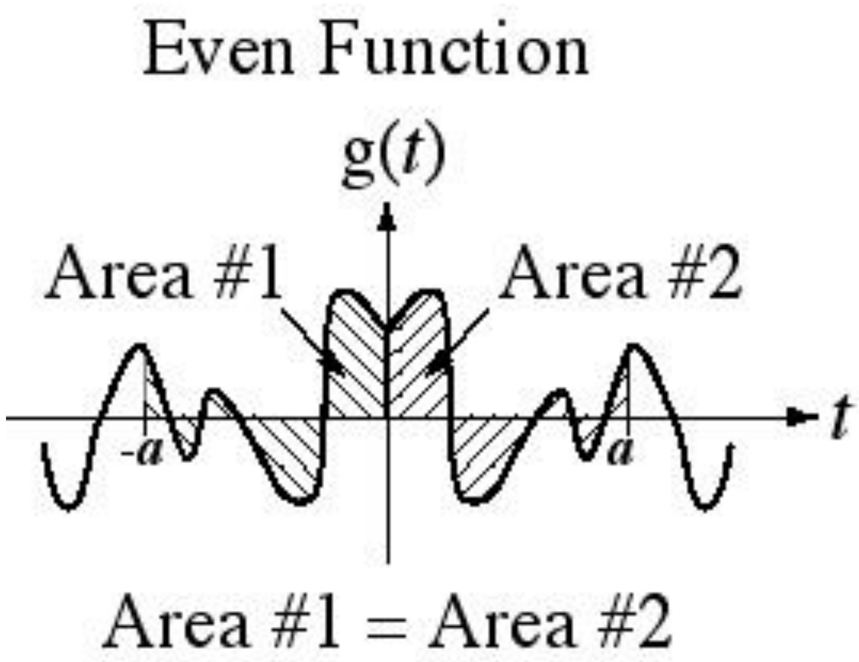
Even x Odd = Odd

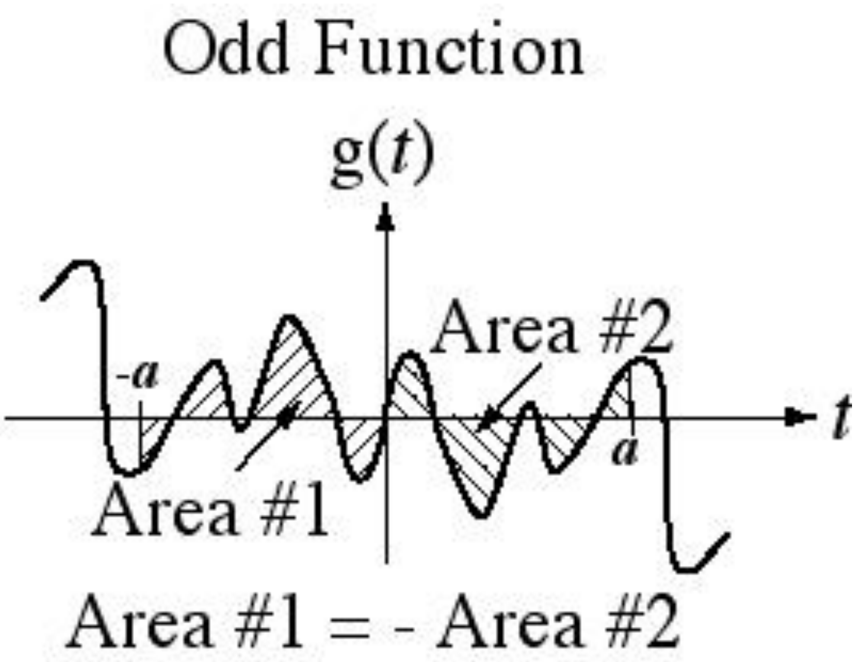
Products of Even and Odd CT Functions



Odd x Odd = Even

Integrals of Even and Odd CT Functions





$$\int_{-a}^{a} g(t)dt = 2\int_{0}^{a} g(t)dt$$

$$\int_{-a}^{a} g(t)dt = 0$$

Continuous Time Periodic Functions

Function is periodic with period T, if

$$g(t) = g(t + nT)$$

What is the effect on periodic function of time shifting by nT?

Examples of Periodic Signals

$$g(t) = 3\sin(400\pi t)$$

$$g(t) = 2 + t^2$$

$$g(t) = \sin(12\pi t) + \sin(6\pi t)$$

$$g(t) = \sin(\pi t) + \sin(6\pi t)$$

Discrete Time Signals

Continuous Time

$$x(t) = A\sin(2\pi f_0 t)$$

Discrete Time

$$x[nT_s] = A\sin[2\pi f_o nT_s]$$

$$= A\sin[2\pi f_o T_s n]$$

$$= A\sin[\frac{2\pi f_o}{f_s} n]$$

$$x[nT_s] = A\sin[\frac{2\pi f_o}{f_s} n]$$

$$x[n] = A\sin[2\pi \frac{f_o}{f_s} n]$$

$$x[n] = A\sin[2\pi \frac{T_s}{T_o} n]$$

$$x[n] = A\sin[2\pi K n]$$

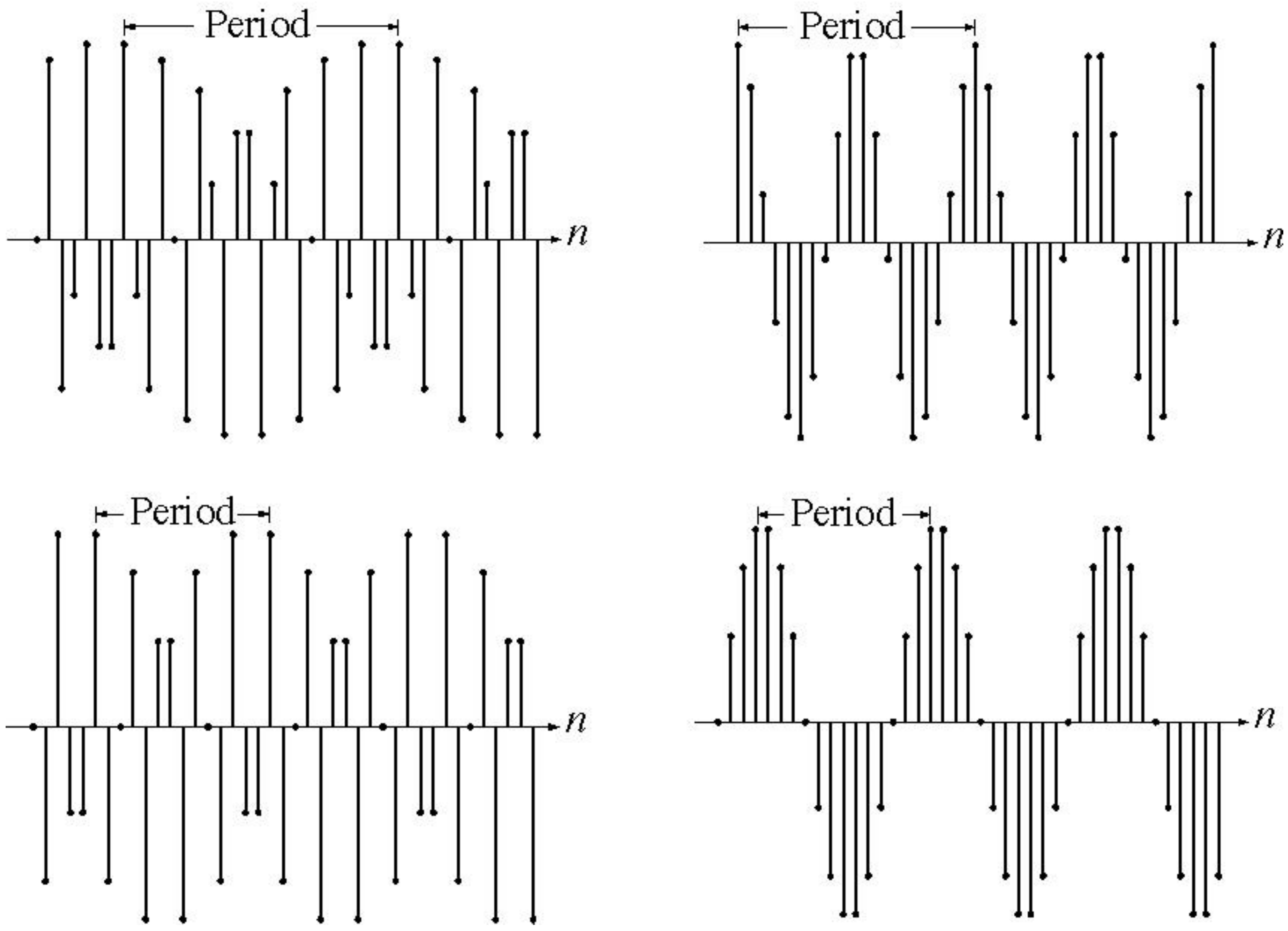
To be periodic, "Kn" has to be an integer for some "n"
=> K has to be a ratio of integers

$$x[n] = A\sin[2\pi \frac{p}{q} n]$$

Period = q

Discrete-Time Sinusoids

Periodic Sinusoids



How Many CT periodic Cycles are Present in One DT Periodic Cycle

$$x[n] = A\sin[2\pi \frac{p}{q} n]$$

Period = q

$$x[n] = A\sin[2\pi p \frac{n}{q}]$$

$$x[n] = A\sin[2\pi p]$$

When n = q
one DT period

=> There are p cycles of CT periodic sinusoidal function per one cycle of DT periodic sinusoidal function

Examples

$$g_1[n] = \cos[\frac{2\pi}{5} n]$$

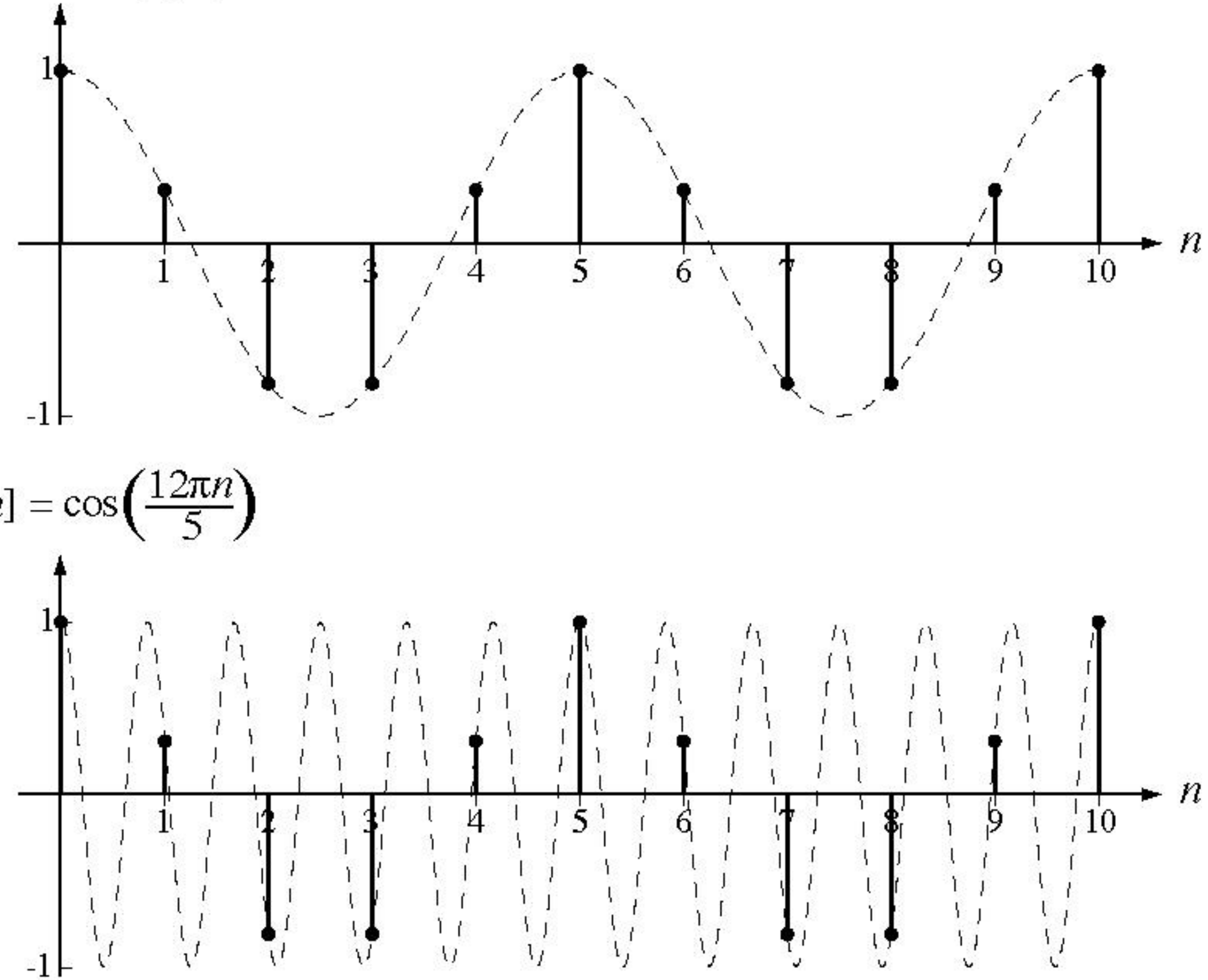
Period = 5

$$g_2[n] = \cos[\frac{12\pi}{5} n]$$

Period = 5

$$g_1[n] = \cos\left(\frac{2\pi n}{5}\right)$$

$$g_2[n] = \cos\left(\frac{12\pi n}{5}\right)$$



Two Discrete Sinusoids could be similar?

Two different-looking DT sinusoids,

$$g_1[n] = A\cos(2\pi K_1 n + \theta) \quad \text{and} \quad g_2[n] = A\cos(2\pi K_2 n + \theta)$$

may actually be the same. If

$$K_2 = K_1 + m, \quad \text{where } m \text{ is an integer}$$

then (because n is discrete time and therefore an integer),

$$A\cos(2\pi K_1 n + \theta) = A\cos(2\pi K_2 n + \theta)$$

(Example on next slide)

Examples

$$g_1[n] = \cos[\frac{2\pi}{5}n]$$

Period = 5

$$g_2[n] = \cos[\frac{12\pi}{5}n]$$

Period = 5

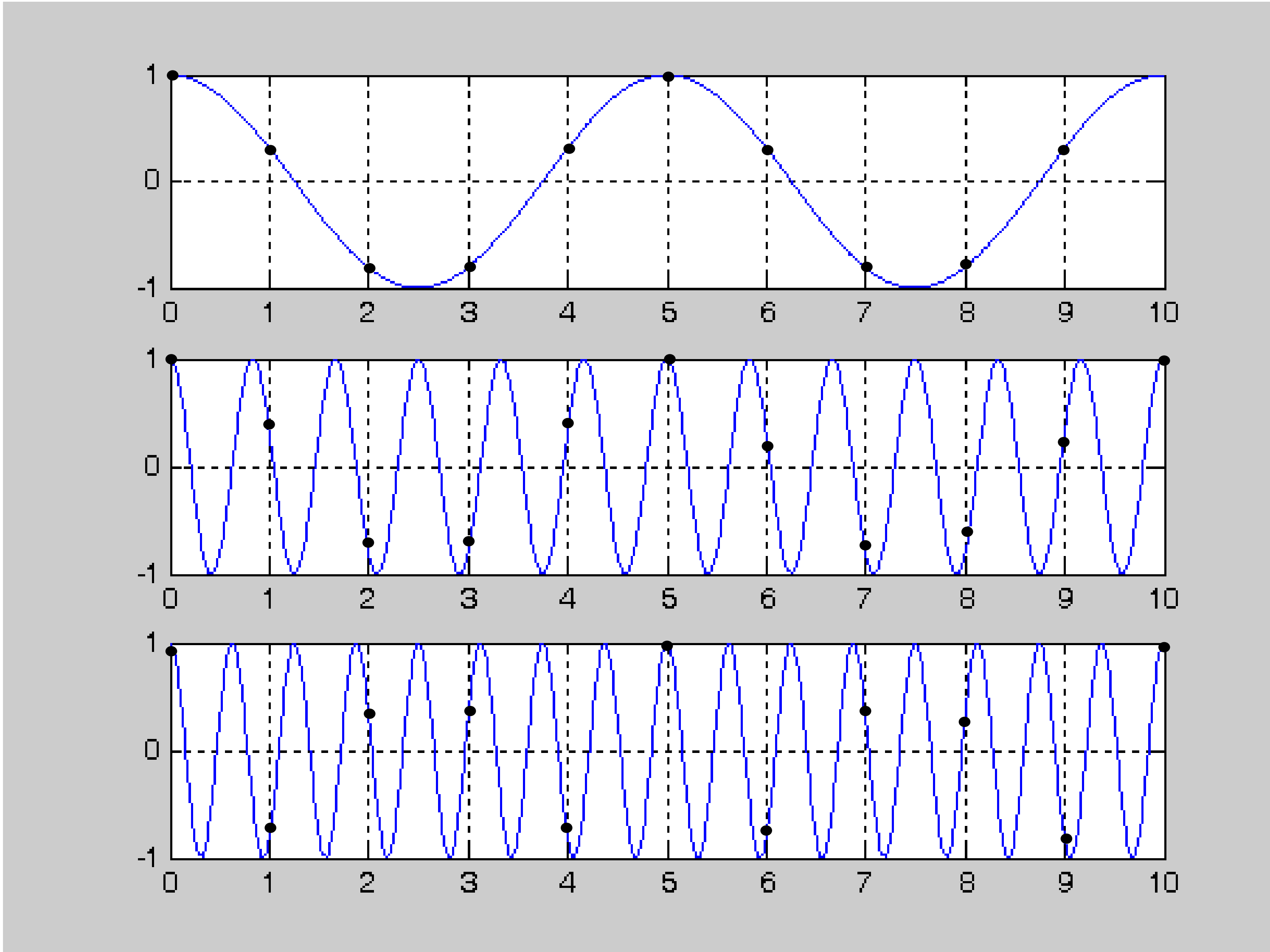
$$g_3[n] = \cos[\frac{16\pi}{5}n]$$

Period = 5

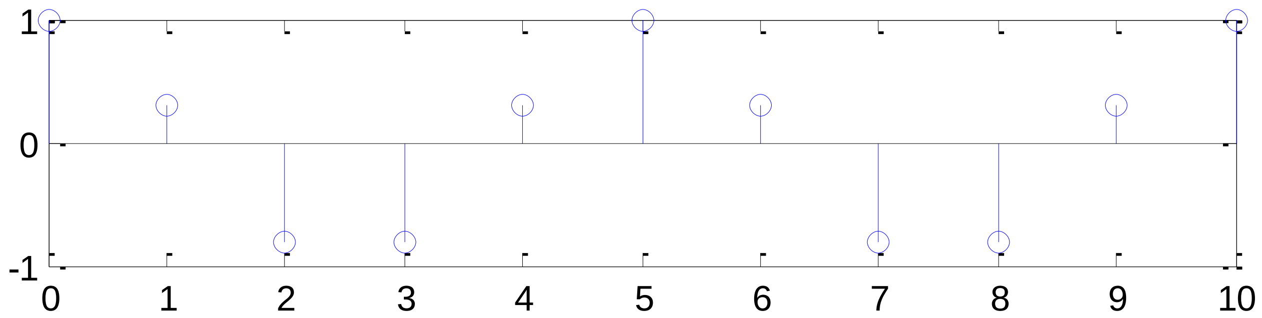
$$\cos[\frac{2\pi}{5}n]$$

$$\cos[\frac{12\pi}{5}n]$$

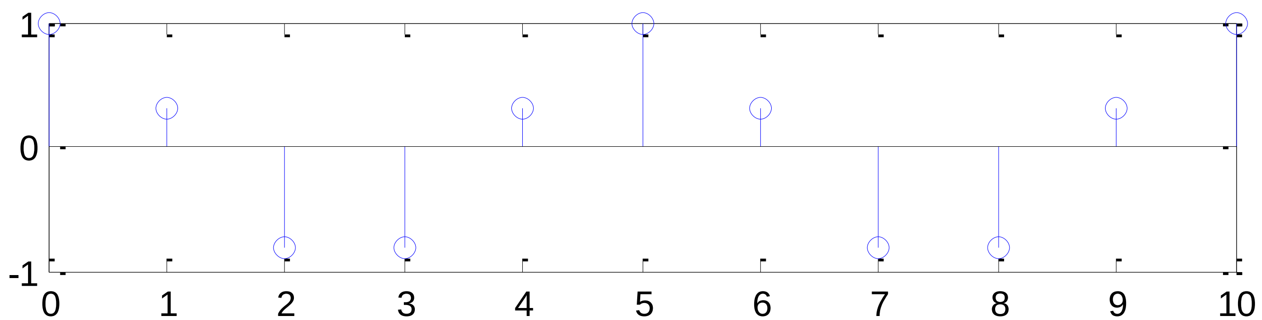
$$\cos[\frac{16\pi}{5}n]$$



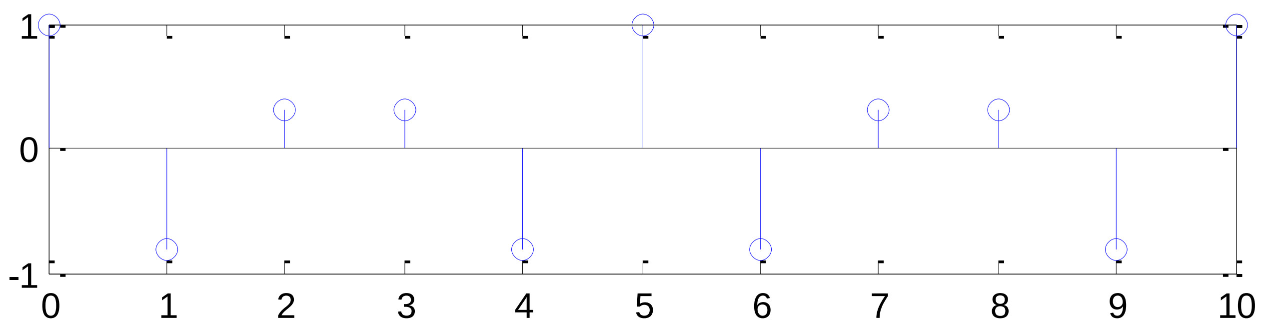
$$\cos[\frac{2\pi}{5}n]$$



$$\cos[\frac{12\pi}{5}n]$$



$$\cos[\frac{16\pi}{5}n]$$



Other Discrete Functions

Unit Impulse Function

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Please note:

$$\delta[n] = \delta[an]$$

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n] = x[0]$$

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0]$$

Discrete Sampling

Unit Step Function

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Unit Ramp Function

$$ramp[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Rectangular Function

$$rect_{N_w}[n] = \begin{cases} 1 & |n| \le N_w \\ 0 & |n| > N_w \end{cases}$$

Please note that

$$rect_{N_w}[n] = u[n + N_w] - u[n - N_w - 1]$$

Transformations on Discrete Time Functions

Amplitude Shifting

$$g[n] \rightarrow A + g[n]$$

Amplitude Scaling

$$g[n] \rightarrow A g[n]$$

Time Shifting

$$g[n] \rightarrow g[n-a]$$

Same as continuous time

Time Scaling

$$g[n] \rightarrow g[\frac{n}{a}]$$

Tricky! Isn't it?

Example of Time Shifting

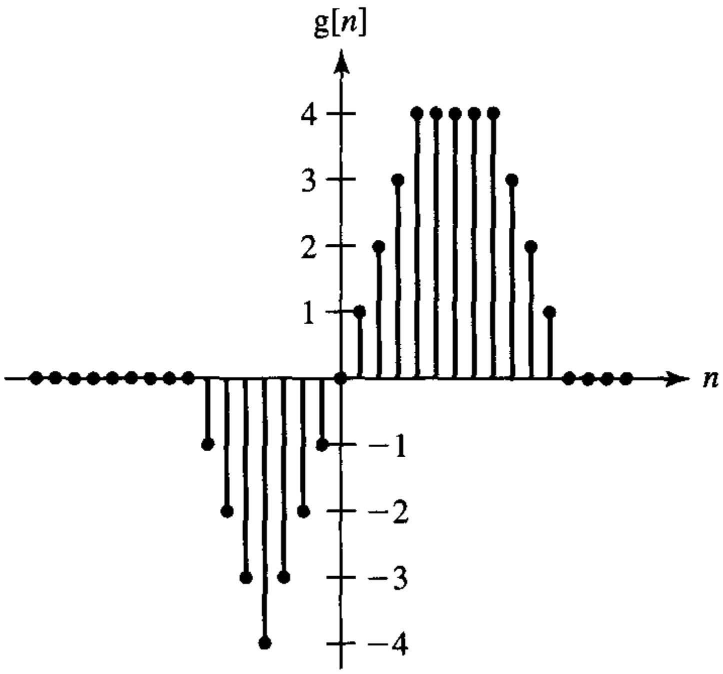


Figure 2.68
Graphical definition of a DT function g[n], where g[n] $= 0$ and $|n| > 15$.

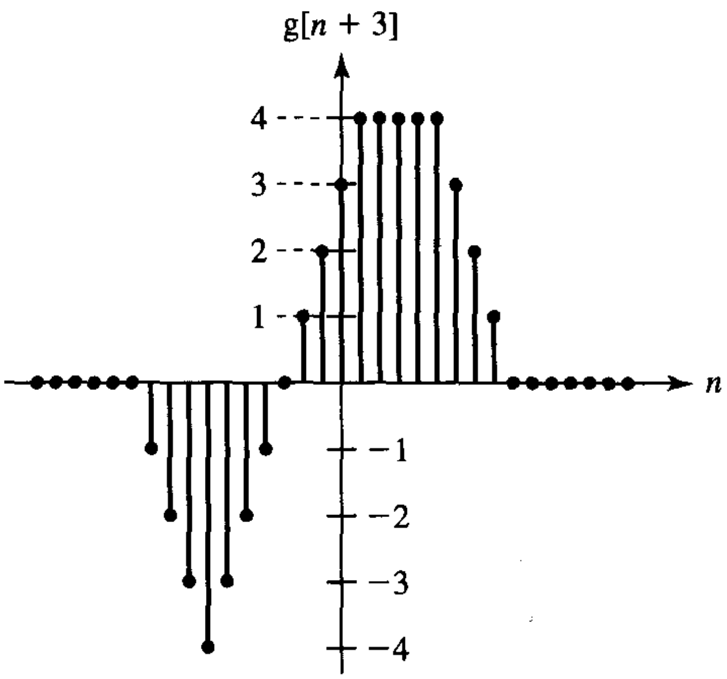


Figure 2.69
Graph of g[$n + 3$] illustrating the time-shifting functional transformation.

Example of Time Compression

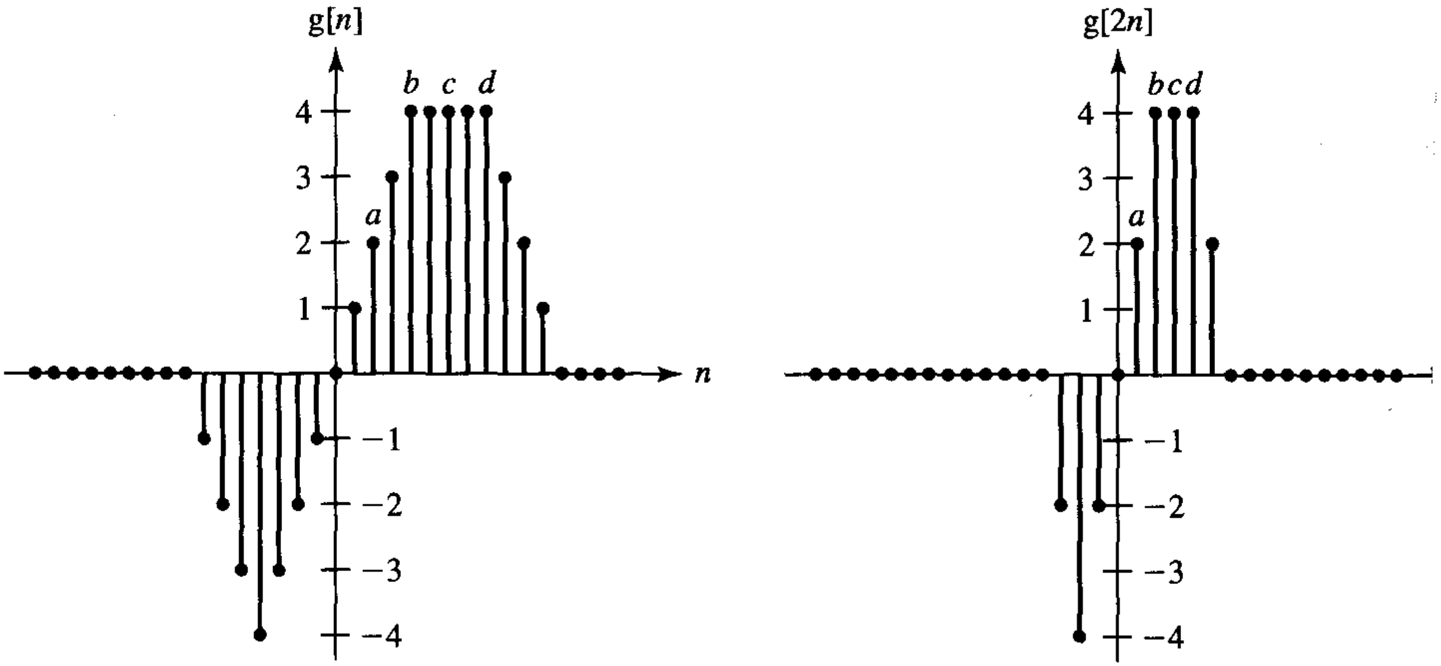


Figure 2.70
Time compression for a DT function.

Discrete Time Even and Odd Functions

Function is Even if $g[n] = g[-n]$

Function is Odd if $g[n] = -g[-n]$

If function is neither even nor odd, then

$$g[n] = g_e[n] + g_o[n]$$

Where

$$g_e[n] = \frac{g[n] + g[-n]}{2}$$

$$g_o[n] = \frac{g[n] - g[-n]}{2}$$

Differencing and Accumulation

$$\Delta g[n] = g[n+1] - g[n]$$

$$\sum_{n=-\infty}^{\infty} g[n]$$

Energy of a Signal

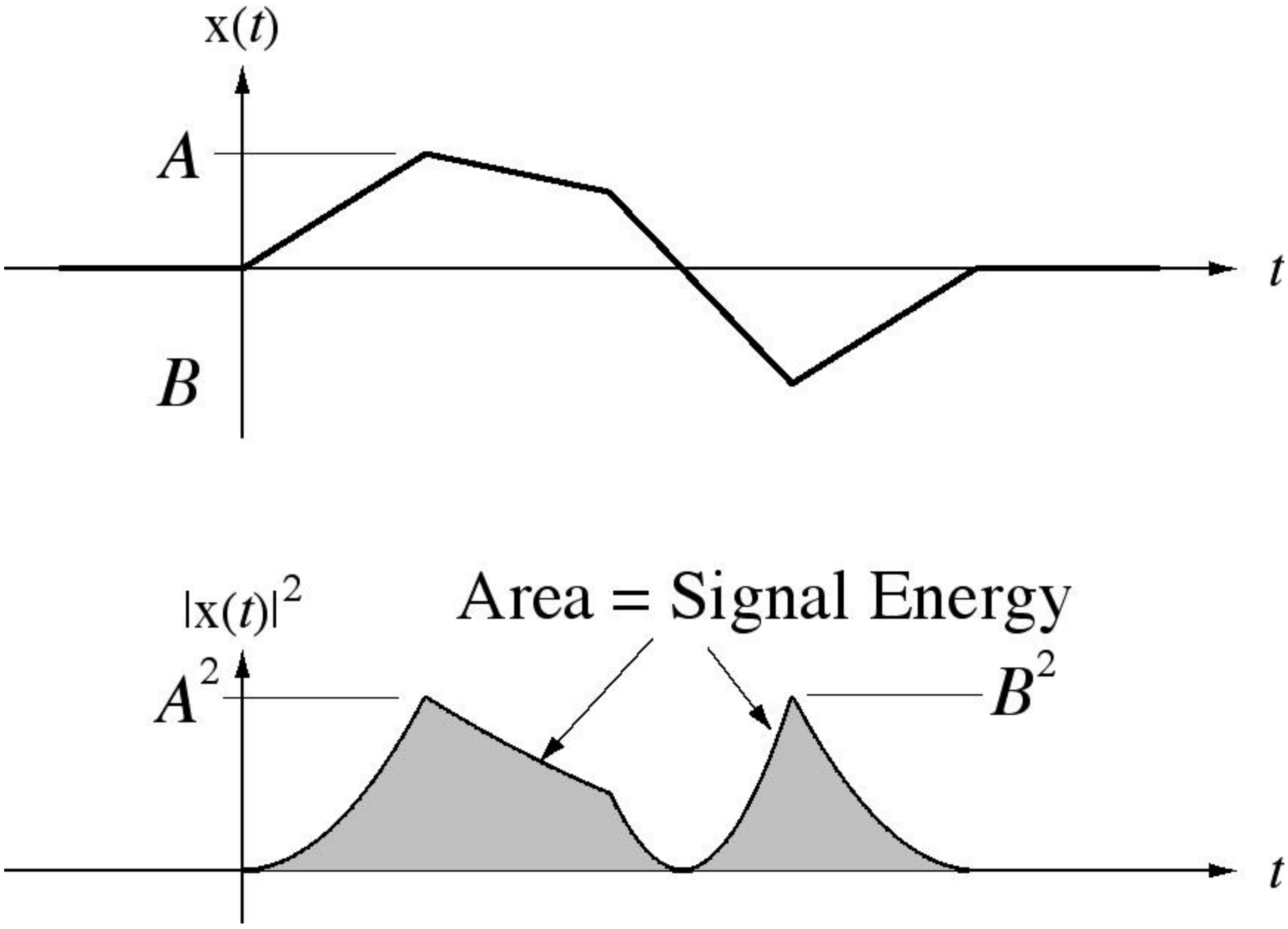
For Continuous Time Signals

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

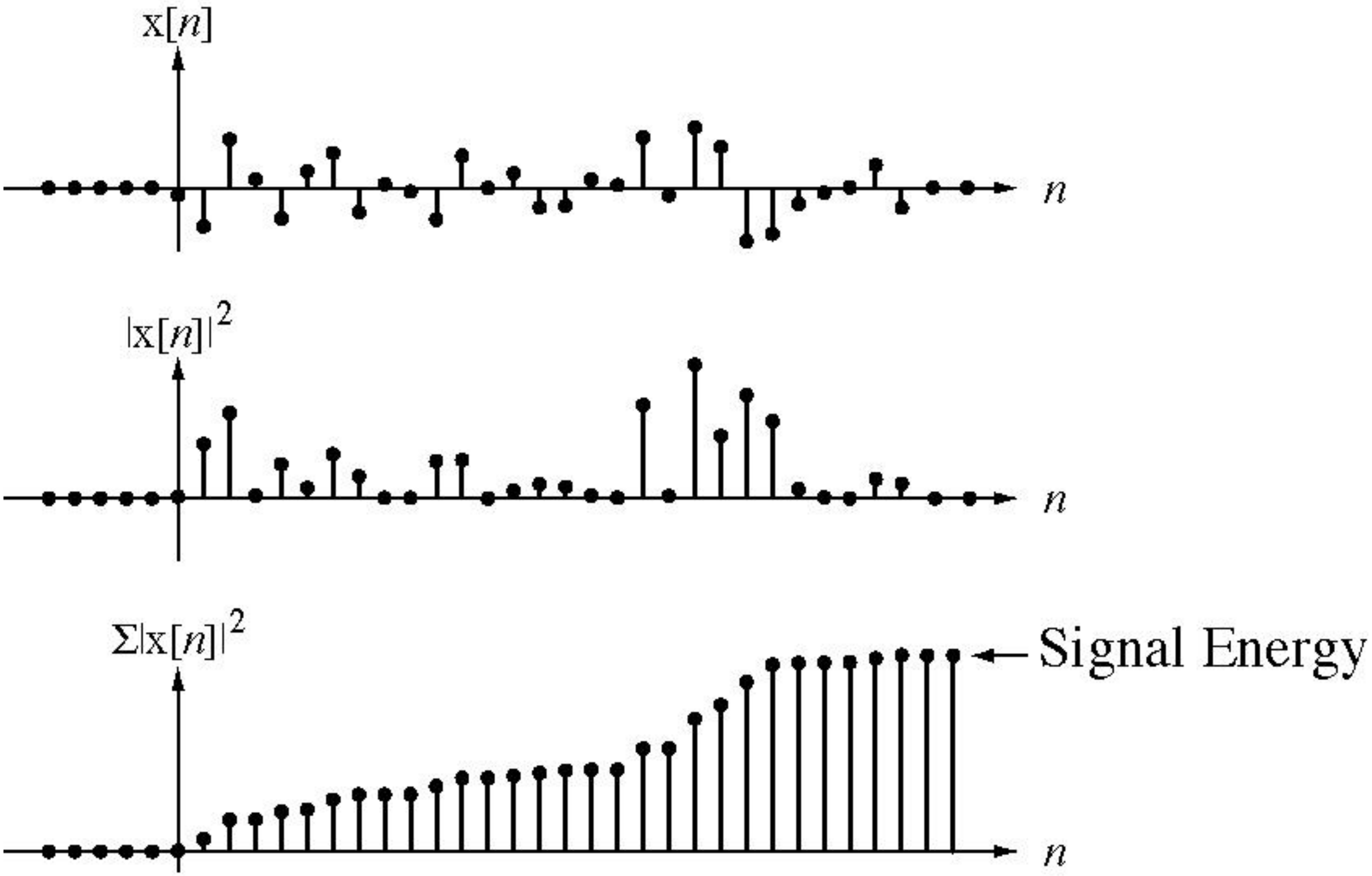
For Discrete Time Signals

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Visual Example of Energy of a Signal – CT Signal



Visual Example of Energy of a Signal – DT Signal



Power of a Signal

Some signals have infinite signal energy. In that case It is more convenient to deal with average signal power.

For Continuous Time Signals

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

For Discrete Time Signals

$$P_x = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N-1} |x[n]|^2$$

Power of a Periodic Signal

For a periodic CT signal, x(t), the average signal power is

$$P_x = \frac{1}{T}\int_T \left|\mathrm{x}(t)\right|^2 dt$$

where T is any period of the signal.

For a periodic DT signal, x[n], the average signal power is

$$P_x = \frac{1}{N}\sum_{n=\langle N\rangle} \left|\mathrm{x}[n]\right|^2$$

where N is any period of the signal.

Energy and Power Signals

*A signal with finite signal energy is called an **energy signal**.*

*A signal with infinite signal energy and finite average signal energy is called a **power signal.***