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Traditional Roots, New Beginnings: Transitions in Undergraduate Research in Mathematics at ETSU

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1. Background and Transitions

I have run an REU program more or less continuously since the Summer of 1991. Starting at Michigan Tech (1991-1999), the program moved with me to Tennessee (2000–2004; 2006–2008) when I became department chair at ETSU. Various incarnations of the program have been run under monikers such as *Discrete Random Structures*, *Probabilistic Methods in Graph Theory, Combinatorics, and Number Theory*, *Discrete Probability and Associated Limit Theorems*, etc., indicating quite clearly that this was a probability REU with inspiration being derived from the AMS 05 (Combinatorics) and 11 (Number Theory) classifications. This phase of the program has been well described in the First PURM Conference article "The Michigan Tech REU Program in Probability," Proceedings of the Conference on Summer Undergraduate Mathematics Programs, Joe Gallian, ed., American Mathematical Society, pp. 93–104, 2000.

During the Summers of 2002 and 2003, however, I began to notice that I personally had some talent in the area of discrete mathematics. I could ask the right questions and guide students in undergraduate research in these non-stochastic areas of inquiry. My latest REU site award is accordingly titled *Probability and Discrete Mathematics*, and at least half of the projects do not involve probability in any form.

Transition Point No. 1: *The program is no longer rooted as firmly in Stochastics as before.*

In the Fall of 2004, we extensively revised our curriculum at ETSU in response to a 120-hour curriculum conversion. A new track in Quantitative Biology was added to the existing tracks in Mathematics, Statistics and Mathematics Education. However, *completion of a three hour undergraduate research requirement became mandatory* regardless of the chosen track, i.e., undergraduate research is now a core requirement. Work at the level of an REU student was not expected, but in many cases, students were indeed able to perform at that level. In many other cases, the work was original but perhaps not as deep as that produced by my summer REU students.

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Transition Point No. 2: *I have begun to believe that undergraduate research is appropriate for all math majors, regardless of their ability, and that it should not be confined only to select students such as those in honors programs.*

After working on undergraduate research with students at all levels, I have come to realize that a C student feels as immense a sense of ownership of his/her achievements as does a straight A peer, and is, debatably, inspired to a larger extent. This realization has had a ripple effect. I am in the process of redesigning my REU webpage to send the message to applicants that a very important criterion for selection will be the "potential value added to the student's future endeavors."

Transition Point No. 3: *I expect that my short list will no longer contain the very best applicants in the traditional sense, but rather the students most likely to experience a long-term impact from the summer experience.*

Elsewhere in this volume I have written of my involvement with talent expansion in quantitative biology, and the NSF-STEP program in which a team of mathematicians and biologists will lead 60 students in a research intensive curriculum in quantitative and computational biology. I have already worked with REU and ETSU students on research in areas such as shotgun sequencing and random palindromic sites in genomes, and my deep involvement with the STEP program (I am PI) will undoubtedly create for me a parallel undergraduate research director persona.

Transition Point No. 4: *I am now as interested in stochastic models in genetics and molecular biology as I am in stochastics and discrete mathematics.*

In 2003, we launched at ETSU a new MS program in "Pre-Collegiate Mathematics" with tracks for in-service K–8 and 7–12 teachers. The first cohort of teachers from Southwest Virginia has now graduated with an MS in K-8 Pre-Collegiate mathematics. Their curriculum consisted of 5 math classes, 3 education classes, a three hour research thesis on classroom practice and innovation, and, last but not least, a *Research Experiences for Teachers* requirement, inspired by the following heart-felt quote from Al Cuoco.

"There are very few absolutes in education, but theres one thing of which I am absolutely certain: The best teachers are those who have a research-like experience in mathematics. Working for an extended period of time on a hard problem that has no apparent approach or solution has profound effects on how one perceives the nature of the enterprise. Teachers who have done this type of research are much less likely to think of mathematics as an established body of facts than are teachers who have simply taken a set of courses. They are more likely to stay engaged in teaching after they start teaching. And they are much more likely to organize their classes around large investigations rather than low-level exercises. An ideal teacher preparation program combines the kind of orchestrated assimilation of the main results in mathematics with the much messier unstructured explorations that come from working with a mentor and grappling with a research project."

The teachers have worked in teams on very creditable research projects (described elsewhere in this article), often with input from their own young students. Additionally, their work has inspired that of several REU students. I plan on applying for RET supplements to my Summer of 2007 and 2008 awards; if granted, these will be my fourth and fifth RET supplements in eighteen years.

Transition Point No. 5: *I have come to believe that research is appropriate for students and teachers of mathematics at all levels.*

I will now offer snapshots of what has transpired over the last three years. Readers will notice, I hope, the emergence of each of the above-mentioned five sets of transitions.

2. Activities

2.1. The Summer of 2004. This was a glorious summer for undergraduate math research at ETSU. Debbie Knisley ran her first MAA-SUMMA REU with four students working on chemical and biological graph theory. The students were Ary Clemons (ETSU), Francesca Duncan (Tuskegee College), Mariam Konate (Virginia Tech) and Jeremy Smith (UT-Knoxville).

The Math and Biology departments ran their first NSF-UBM summer program with participation mainly from ETSU (7 students) and Rice (1 student). The students were Mike Phillips (ETSU, advisors Jeff Knisley and Steve Karsai), Leanna Horton (ETSU, advisors Godbole and Karl Joplin), Particia Carey and Georganna Rosel (ETSU, advisors Hugh Miller, Joplin and Jeff Knisley), Emily Mullersman and Jennifer Whittington (Rice and ETSU, advisors Lev Yampolsky and Jeff Knisley) and Brad Wild and Daniel Lamb (ETSU, advisors Debbie Knisley and Celia McIntosh). Techniques used extensively were Poisson approximation, neural network modeling, game theory, and graph theory. Leanna Horton's work on palindromes in genomes was conducted jointly with REU students James Gardner (ETSU) and Anne Shiu (Chicago).

Overall, I consider the Summer of 2004 to be one of the more successful years of my REU program. Students were Gardner and Shiu, mentioned earlier and now grad students at Emory and UC Berkeley respectively, Nathaniel Watson (Washington University, now at Cal Berkeley), Carl Yerger (Harvey Mudd, now at Cambridge/Georgia Tech), Annalies Vuong (UC Santa Barbara, now at Dartmouth. Annalies won honorable mention for the Alice Schafer Prize the same year that former student Melody Chan won it, fresh from a very productive summer at Duluth), Nadia Heninger (UC Berkeley, now at Princeton), and Ian Wyckoff (Arizona State, now at Georgetown Law School). Among the highlighs of the summer were the resolution of the Stacking Conjecture in graph pebbling by Wyckoff and Vuong, a series of six papers written by Watson and Yerger (some in collaboration with Vuong and Gardner), the collaborative work between REU and UBM students mentioned earlier, a resolution of a conjecture on vertex neighbor integrity (made by former Duluthite Marci Gambrell) by Heninger, Vuong and Shiu, and a hearty attack on a very hard and still unsolved combinatorics problem by the same three students. Several papers have emerged out of these efforts.

Last but not least three K-8 teachers worked with me in teams on RET style research as part of their MS requirements. Donette Carter, Mary Jo Neal, and Crystal Hall did number theoretic work on coincidences in multinomial coefficients, coming up with the $p \sim \{p+1, p+2 \dots q-1\}$ conjecture (p, q are adjacent primes); Sherry Mullins, Beverly Owens and Laura Robinson studied the graph theoretic properties of "standardized planarizations of K_n"; and Robin Herndon, Rita McFaddin, Crystal Anderson, James Ray and Janeane Young worked on "625 generalizations of Karpekar's process." Each of these problems was further worked on by my 2006 REU students (see below).

2.2. The 2004-05 Academic Year. MATH 4010 (the required undergraduate research class) was offered for the first time with me as instructor. This is a

so-called writing intensive (WI) and oral intensive (OI) class operated under the scrutiny of ETSU WI and OI committees. Students worked in teams and were required to give four oral presentations as they turned in 5, 10, 15 and 20 page reports – each a refinement of the previous. This first year was rather successful:

- Patty Carey, Josh Fair and Jason Jones began work on "A Generalization of the Erdős-Ko-Rado theorem". They did creditable work but only Patty chose to continue beyond the Fall semester. Her paper is now under review by *Journal of Combinatorial Designs* and was presented (by me) at the Fields Institute Workshop on Covering Arrays, Ottawa, June 2006.
- Tom Intardonato and Garth Fine's work "Birthday Problem with Dependence" was presented by me at a Workshop on deBruijn Cycles held at the Banff International Research Station in December 2004. A paper might emerge with possible co-authors being experts in compound Poisson approximation.
- Mark Dula and Cristian Madar continued the work of 2003 REU students Alexa Mater and Deanna Turk, which I presented at the IMA in October 2003. The work extends random genome reconstruction to include the case of Markov nucleotide generation. A joint paper with four students is in preparation.

2.3. The Summer of 2005.

- I had not applied for an REU renewal in the Fall of 2004. Consequently, I had a year off from the REU;
- I directed the RET research of two teams of teachers. The projects were "Generalizations of the $M\&m$ problem," and "Variations of the $3X + 1$ problem";
- Debbie Knisley ran her second MAA-SUMMA REU program, with four students working with her on chemical and biological graph theory. The students were Tywanna Anderson (ETSU), Huda Hussein (ETSU) Veranda Moffet (Mississippi Valley) and Glenda Span (Mississippi Valley);
- The NSF-UBM collaboration between Math and Biology offered its second summer REU program. I did not take part in activities but my student Patty Carey was a participant. Some participants won a $100 award at the Undergraduate Poster Session at the Joint Math Meetings. There were some repeat students from the summer of 2004; new recruits were Holly Hicks and Shannon McConnell who worked with the protein folding group of McIntosh and Debbie Knisley; Dmitri Yampolsky who joined Karsai and Jeff Knisley's complexity group, and Erin Ashton and Jennifer Cooke who were part of the microarray group led by Joplin, Yampolsky, Edith Seier, and Jeff Knisley

2.4. The 2005-06 Academic Year. During this academic year students worked independently on undergraduate research projects with Edith Seier. Patty Carey worked independently with me on "Standard Planarization of K_n and $K_{n,n}$, which has proven to be an extremely hard problem that has stymied even my best summer REU students.

In the formal offering of MATH 4010, the following projects were completed:

- Brad Wild and Stephanie Goins worked on several novel aspects of the longest monotone sequence problem. I hope soon to see their small case studies appear on the *Integer Sequences* website;
- Tracy Holt and Jeff Worley's project "Domination cover pebbling of certain families of graphs" was presented at the 2006 Southeastern Combinatorics conference and we hope to submit a paper to the *Proceedings of the 2007 Conference*;
- Brett Kindle and Torey Burton worked on "The appearance of the first 123 pattern," which I presented at the Permutation Patterns conference in Reykjavik, June 2006.

2.5. The Summer of 2006. Much of the work from this summer is still in progress. Highlights included results on universal cycles for multisets (Josh Zahl, Caltech, and Toby Johnson, Yale), random tree compression (Zahl), random compression of random graphs (Joe Marincel, Washington University), random domino tilings (Katie Bennedetto, William and Mary), planarizations of Sierpiński type graphs (Benedetto and John Symms, University of Utah), first occurrences of 123 patterns (Symms), planarizations of complete graphs and circulants (Benedetto, Johnson, Helen Hauser, Ohio University, and Michael Wijaya, University of Rochester), pebbling thresholds for diameter two graphs (Ariel Levavi, CMU, and Hauser), coincidences in multinomial coefficients (Igor Konfisakher, Washington University and Wijaya), chordal permutation graphs (Levavi and Marincel), and a complete resolution of the Kaprekar problem by Zahl.

2.6. The 2006-07 Academic Year. Students in MATH 4010 this year have worked with me in the formally "taught" Fall class, but others have worked with collegues Jeff Knisley, Michel Helfgott, and George Poole on a one-on-one basis. My students are getting ready for their final presentations as I write this. The projects are as follows:

- Bijan Khairollahi is working on "Equitable division of pizza," and I am confident that when completed it will combine nicely with work of RET teacher Floyd Brown to make a publishable paper (the original version was rejected by *Math Magazine*);
- Beth Haun and Courtney Sanders' work on "Star of David Graphs" will be publishable in an "entry level" combinatorics journal, as will
- Glenn Quarles, Eric George, Joe Barwick, and Brooks Dearstone's paper "U-cycles of DNA and non-bijective functions;
- David Simpson and Nicole Holder's work on "3-good permutations" will likely lead to a submission to Neil Sloane's website of integer sequences.

In addition to these projects, Elizabeth Harris is making good progress on "Square induced Sierpiński graphs" as part of a Research Discovery Federal Work Study position, while Ryen Lapham's work on "2-balanced permutations" will be submitted to the integer sequence website as well.

3. Looking ahead

I am fairly certain that my NSF-REU days will end after the Summer of 2008. I simply do not have the time to spend 6 or more hours a day "in the trenches." Eighteen years of such work has been the defining part of my career, and I will miss my Mudders and Arizona State honors students and Harvard Math Concentrators,

but I do look forward to other challenges. My UGR direction will continue through the academic year – with ETSU Math majors and ETSU-STEP students, and over the summers with ETSU-STEP students (one project per year). Over the summers too, I hope to play a secondary role in Debbie Knisley's NSF-MAA-NSA-SUMMA REU program for underrepresented minorities, in which she guides students to use graph invariants in the pursuit of certain kinds of secondary RNA structures (I have not been successful in attracting many underrepresented students to my REU program).

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