Seventy-Five Years of the Putnam Mathematical Competition*

Joseph A. Gallian

Abstract. We review the 75-year history of the world's foremost college-level mathematics competition. Evidence is provided to support the assertion that exceptional performance in premier secondary-school-level and college-level mathematics competitions correlates well with an exceptional research career in mathematics.

1. INTRODUCTION. The annual William Lowell Putnam Mathematical Competition for undergraduate mathematics students in the United States and Canada sponsored by the Mathematical Association of America (MAA) began in 1938 with 163 individuals and 42 teams. One of the goals of the competition is to identify the best and most promising students in mathematics in the two countries. Prizes in the first few years were \$500, \$300, and \$200 for the top three teams and \$50 each for the top five ranking individuals, who were designated as Putnam Fellows. By the year 1997, the prizes for the top five teams had risen to \$25,000, \$20,000, \$15,000, \$10,000, and \$5,000, while team members of the top five teams received \$1,000, \$800, \$600, \$400, and \$200, respectively. Each Fellow received \$2,500, those ranking in the next ten (because of ties this is approximate) received \$1,000, and those in the following 10 received \$250. Moreover, each year, one Putnam Fellow received the William Lowell Putnam Fellowship for graduate study at Harvard. The number of participants exceeded 1,000 for the first time in 1961 when 1,094 individuals and 165 teams took part. In the 75th competition in 2014, there were 4,320 participants and 431 teams. Through 2014, 140,314 people have taken the exam.

In the first 22 competitions, the number of problems varied from 11 to 14, but beginning with the 23rd competition in 1962, the exams have had 12 problems worth ten points apiece. Institutions entering teams must designate the three team members before the competition is held. The team score is the sum of the ranks of the three team members. Points in the range 3–7 are rarely, if ever, given. Although the first two problems of each session are intended to be approachable by large numbers of participants, the most recent year when the median score was above 3 was 1998. Moreover, a median score of 0 is not uncommon.

The fact that the team members are designated in advance and the method of summing the ranks for team scoring cause some peculiar results on occasion. In 1959, for instance, Harvard had four Putnam Fellows but finished fourth in the team competition, and in 1966, 1970, 2005, and 2006, MIT had three Putnam Fellows but did not win the competition.

2. TEAM PERFORMANCE. By a wide margin, Harvard has the best record in the Putnam competition. Through 2014, Harvard has won the team competition 29 times, while its closest rival for team titles, Caltech, has won ten times. MIT is in

*This is an updated and supplemented version of [1] published in 2004. See [2] for more on the Putnam competition.

http://dx.doi.org/10.4169/amer.math.monthly.124.1.54
MSC: Primary 01A05, Secondary 00A05

third place with eight titles, with four of these coming since 2003. [Editor's note: MIT won its ninth team title in 2015.] Tied for fourth place with four team titles each are Washington University and the University of Toronto. All four of Toronto's team titles occurred in the first six years of the competition. Harvard is tied with MIT and Princeton for the most second place finishes (11), and Harvard also has the most third place finishes (13).

Curiously, the Harvard team did not place in the top five in the first six competitions, but it has placed in the top five in 59 of the 75 competitions held through 2014. Caltech's glory years were the six years 1971–1976 when they won the team competition five times. Excluding Harvard, only once has the same institution won three years in a row. That was Caltech in 1971–1973. Between 1976 and 1986, Washington University won the team title four times and placed second four times. During that period, Wash U had only two Putnam Fellows. Between 1990 and 2000, Duke became Harvard's top rival by winning three times and finishing second to Harvard twice. After finishing in the top five 24 times and in second place nine times prior to 2006, Princeton won its first and only team title in 2006 despite having 21 Putnam Fellows.

The only state universities in the U. S. to win the team competition are Michigan State (three times) and the Universities of California at Davis (once) and at Berkeley (once). The highest place ever achieved by a liberal arts college was second by Oberlin College in 1972 beating out third place Harvard. That same year, Swarthmore finished fourth, ahead of MIT. Harvard's longest winning streak was eight years (1985–1992), and its longest stretch without winning was 15 years (1967–1981). The only tie for first place occurred in 1984 between the University of California at Davis and Washington University. Amazingly, in 1986, 1987, and 1990, every member of Harvard's team was a Putnam Fellow.

A complete list of the top five schools and top five individuals each year can be found at http://en.wikipedia.org/wiki/Putnam_competition.

3. INDIVIDUAL ACCOLADES. As for producing Putnam Fellows, Harvard is again the overwhelming winner with 104 versus MIT's second place 65. On the other hand, between 2001 and 2014, MIT outdid Harvard in Putnam Fellows 34 to17. Harvard has had four Putnam Fellows in the same competition on four occasions. MIT had an unprecedented five Putnam Fellows in 2014 (because of a three-way tie for fourth place, there were six Putnam Fellows in 2014) and four Putnam Fellows in 2013. Oddly, Harvard did not record its first Putnam Fellow until the sixth competition. Since then, the longest period in which Harvard did not have a Putnam Fellow is three years, and that happened only once. With the exception of 2004, Harvard has had a Putnam Fellow every year since 1990. Because of tie scores, in 15 competitions, there have been six Putnam Fellows, while in 1959 a four-way tie for fifth place resulted in eight. Fourteen of the 16 competitions in which there were more than five Putnam Fellows have occurred since 1970. Through 2014, there have been 283 individuals who have been Putnam Fellows, for a total of 393, counting multiplicity. Only eight people—Don Coppersmith, Arthur Rubin, Bjorn Poonen, Ravi Vakil, Gabriel Carroll, Reid Barton, Daniel Kane, and Brian Lawrence—have been Putnam Fellows four times. Twenty-one people have been three-time winners: Andrew Gleason, Edward Kaplan, Donald J. Newman, James Herreshoff, Samuel Klein, Randall Dougherty, Eric Carlson, David Ash, Noam Elkies, David Moews, David Grabiner, Kiran Kedlaya, Lenny Ng, J. P. Grossman, Ciprian Manolescu, Aaron Pixton, Arnav Tripathy, Yufei Zhao, Xiaosheng Mu, Evan O'Dorney, and Zipei Nie. Zhao missed being a four-time Fellow by one point in 2007. In Ash's fourth attempt at the Putnam in 1984,

he finished tied for sixth, just two points short of being a Putnam Fellow for a fourth time. It should be noted that some of the three-time winners only took the exam three times. Barton is the only person ever to win four gold medals in four attempts in the International Mathematical Olympiad for high school students. Barton also won two gold medals in the International Olympiad in Informatics. O'Dorney won the U. S. National Spelling Bee.

Of the 283 individuals who have been Putnam Fellows, 75 (about 27%) were Fellows more than once. It appears that there have never been two members of the same immediate family who have been Putnam Fellows. The first certain occurrence of a woman finishing in the Honorable Mention or higher categories was in 1948. In the announcement in this MONTHLY [3] she is listed as "M. Djorup (Miss), Ursinus College." Because many participants use the initials of their first and middle names (e.g., R. P. Feynman), it is possible that Djorup is not the first woman to achieve Honorable Mention or better status. The first woman Putnam Fellow was Ioana Dumitriu from New York University in 1996; the second was Melanie Wood from Duke in 2002; the third was Ana Caraiani from Princeton in 2003 and 2004. Since the ages of participants are not noted, there is no way to know who the youngest and oldest people to win the competition were. Most likely, the youngest is Arthur Rubin, who was a winner in 1970 at age 14. John Tillinghast, David Ash, Noam Elkies, and Lenny Ng were Putnam Fellows at 16.[1] The likely oldest winner is Samuel Klein, who was born in 1934 and won the competitions in 1953, 1959, and 1960.

Unlike the early years of the Putnam competition, in the past 35 years or so, many of those who have done exceptionally well in the Putnam competition have participated as high school students in problem solving summer training camps in the United States and elsewhere in preparation for the annual International Mathematical Olympiad (IMO). The IMO competition started in 1959. Now, about 100 countries take part, with each country allowed to have up to six competitors participate. Unlike the Putnam, which limits each participant to at most four contests, there is no limit on the number of times one person can participate in the IMO. The problems require no calculus or higher mathematics. About 8% of the participants receive gold medals (roughly 40), 16% silver, and 24% bronze. The competition takes place over two days with each three-problem session lasting 4.5 hours. In recent years, many of the international students who represented their countries in the IMO have come to the United States for their undergraduate degrees. As a consequence, the winners of Putnam competitions now come from many countries. The 2006 Putnam competition illustrates this well. All five 2006 Putnam winners were IMO gold medal recipients, and 12 of the top 26 scorers in competition represented countries other than the United States or Canada in the IMO. In 2007, five of the six Putnam Fellows were IMO gold medalists, and nine of the top 24 in the Putnam competition represented countries other than the United States or Canada in the IMO. Between 2010 and 2014, all but one Putnam Fellow was an IMO gold medalist.

Over the 75 competitions between 1938 and 2014, there have been only four perfect scores—one in 1987, two in 1988, and one in 2010. Although the top five scorers are always listed alphabetically, it is known that the 1987 perfect score was achieved by David Moews. What is amazing about this score is that the 1987 exam was a difficult one. The median score was one point, and 26 points put one in the top 200 (out of 2,170 participants). In 1987, the second-highest score was 108, the third-highest score in 1988 was 119, and the second-highest in 2010 was 118. The winners of the 1987 and 1988 competitions rank among the strongest groups of Putnam Fellows ever.

[1] [1] had Elkies as the youngest winner known then.

Among them are Bjorn Poonen and Ravi Vakil, both four-time Putnam Fellows; David Moews and David Grabiner, both three-time Putnam Fellows; and Mike Reid, a two-time Putnam Fellow. In contrast to the 1988 scores, of the 1,260 contestants in the 1963 competition, the highest score was 62.

Two changes were made in 1992 regarding the recognition of individuals. In previous competitions, the announcements of winners alphabetically identified the top ten as the five highest ranking participants and the next five highest. The next group of 30–35 highest ranking people was designated "Honorable Mention." In 1992, the announcement of the results put the top 25 into four categories: the five highest ranking individuals, the next five highest, then the next five highest, and the next ten highest. Beginning in 1997, the top 25 (approximately) finishers were put into three categories: the five highest ranking individuals, the next ten highest, then the next ten highest. The number in the Honorable Mention group remained at about 30–35. In an effort to encourage more women to take the contest, the other change made in 1992 was the addition of an "Elizabeth Lowell Putnam Award" given from time to time to a female participant with a high score. Through 2014, there have been 11 individual winners. Of these, Ioana Dumitriu and Alison Miller won it three times, and Ana Caraiani and Melanie Wood won it twice. The 2004 competition was a high-water year for women. In addition to Caraiani and Miller, two other women finished in the top 15, four more received Honorable Mention, and another 11 finished in the top 200. Two of Princeton's three-member team, which was second to MIT, were women.

For most of the years between the late 1940s and the early 1990s, Harvard far outpaced all others schools in the number of individuals receiving Honorable Mention status or higher. In 1991, Harvard had 11, and MIT had just one in that group. By 1993, MIT narrowed the margin to 8–6 in favor of Harvard. The first time that MIT surpassed Harvard was 1998 with the totals 11–9. In recognition of the significantly increasing number of participants, between 2002 and 2014, the number of those designated Honorable Mention has gradually increased from approximately 45 to 60. Since 1998, MIT has widened its edge over Harvard in the number of individuals receiving Honorable Mention status or higher with the widest margin of 34–6 occurring in 2012. In part, the recent disparity between MIT and Harvard is due to the fact that MIT has about four times as many math majors as Harvard. Among the ten institutions in the 2012 competition that had the greatest number of people to achieve Honorable Mention or higher status, the number from MIT exceeded the total of the other nine. Of the top 201 finishers in 2013, 57 were from MIT. During the years 2012–2014, MIT had 12 Putnam Fellows, Harvard had four, and all other schools combined had zero. Despite MIT's deep pool of talent, between 1998 and 2014, Harvard won the team competition eight times to MIT's five times. In 2012, MIT had 12 who placed in the top 25 to Harvard's three, but only one MIT team member was in the top 25, whereas Harvard had two team members in the top five. That put Harvard in first place and MIT in second.

4. A PUTNAM WHO'S WHO. Over the years, many distinguished mathematicians and scientists have participated in the Putnam. Among them are Fields Medalists John Milnor, David Mumford, Daniel Quillen, Paul Cohen, John G. Thompson, and Manjul Bhargava. Milnor, Mumford, and Quillen were Putnam Fellows; Cohen was in the second five; Thompson received Honorable Mention; Bhargava was in the top 25. Physics Nobel Laureates who have received Honorable Mention or better are Richard Feynman, a Putnam Fellow in 1939; Kenneth G. Wilson, a two-time Putnam Fellow; Steven Weinberg; and Murray Gell-Mann. The Nobel Prize winner in Economics John Nash (of "A Beautiful Mind" fame), to his great disappointment, finished in the

second five of 147 individuals in 1947. Thompson won the $1,000,000 Abel Prize in 2008, Milnor won it in 2011, and Nash won it in 2015. Eric Lander, one of the principal leaders in the Human Genome Project, finished in the second five in 1976. Both Mumford and Lander are MacArthur Fellows. Craig Gentry, 1993 Putnam Fellow, is a MacArthur Fellow. Distinguished computer scientist Donald Knuth received Honorable Mention in 1959. American Mathematical Society presidents who did well in the Putnam are Irving Kaplansky (Putnam Fellow, 1938), Andrew Gleason (Putnam Fellow, 1940, 1941, 1942), Felix Browder (Putnam Fellow, 1946), David Vogan (Putnam Fellow, 1972), and AMS (American Mathematical Society) and MAA President Ron Graham (Honorable Mention, 1958). Putnam Fellows in the National Academy of Sciences include Elwyn Berlekamp, Felix Browder, Eugenio Calabi, Andrew Gleason, Melvin Hochster, Roger Howe, Irving Kaplansky, George W. Mackey, John W. Milnor, David Mumford, Daniel G. Quillen, Lawrence A. Shepp, Peter W. Shor, Richard G. Swan, David Vogan, and Kenneth G. Wilson. Cohen, Browder, Thompson, and Mumford are National Medal of Science laureates. Many others who have done well in the Putnam have won the prestigious research awards given by the AMS. The 1956 Harvard team had both a future Nobel Prize winner (Wilson) and a future Fields Medalist (Mumford). Both were Putnam Fellows that year, and Harvard's team finished first.

One might wonder how the winners of the AMS/MAA/SIAM (Society of Applied and Industrial Mathematicians) Morgan Prize for outstanding research by an undergraduate student have done in the Putnam Competition. Of the 20 recipients through 2015, Wood, Barton, Kane, Manolescu, Pixton, and Larson have been Putnam Fellows. Three-time Putnam Fellows Kedlaya, Ng, and Zhao received Honorable Mention for the Morgan Prize.

5. AN IMO WHO'S WHO. The previous section makes a strong case that exceptional performance on the Putnam often correlates well with exceptional distinction in research. Because the Putnam competition is restricted to students attending U. S. and Canadian institutions, it is worth examining the connection between performance on the International Mathematical Olympiad (IMO) and research. Indeed, many top performers in the IMO have had distinguished research careers. Six of the past 12 Fields Medal winners were IMO medal winners with five of the six winning gold medals and four achieving perfect scores. Among the notable IMO performers are: Maryam Mirzakhani two gold, a perfect score, 2014 Fields Medal; Artur Avila one gold, 2014 Fields Medal; Terence Tao one bronze (age 11), one silver (age 12), one gold (age 13), 2006 Fields Medal, $3,000,000 2014 Breakthrough Prize; László Lovász three golds, one silver, two perfect scores, winner of Kyoto, Wolf, von Neumann, Gödel, Knuth Prizes; Grigori Perelman one gold, perfect score, 2006 Fields Medal (declined), 2010 $1,000,000 Millenium Prize (declined); Timothy Gowers IMO gold, perfect score, 1998 Fields Medal; Richard Borcherds IMO one gold, one silver, 1998 Fields Medal; Jean-Christophe Yoccoz IMO one gold, perfect score, one silver, 1994 Fields Medal; Stanislaw Smirnov IMO two gold, two perfect scores, 2010 Fields Medal; Vladimir Drinfeld IMO one gold, perfect score, 1990 Fields Medal; and Jacob Lurie IMO one gold, perfect score, one silver, Morgan Prize, $3,000,000 2014 Breakthrough Prize, MacArthur Fellow.

Of course, it is people in their teens or early 20s who focus on mathematics competitions, whereas the large majority of world-class mathematicians blossom in their late 20s or beyond. Still, these premier competitions have served to identify many of the very best and most promising students in mathematics.

REFERENCES

1. J. A. Gallian, The first sixty-six years of the Putnam competition, *Amer. Math. Monthly* **111** (2004) 691–699, http://dx.doi.org/10.2307/4145042.
2. L. F. Klosinski, The Putnam competition: Origin, lore, structure, in *A Century of Advancing Mathematics*, Ed. by S. F. Kennedy et al., Mathematical Association of America, Washington DC, 2015. 387–392, http://www.jstor.org/stable/10.4169/j.ctt15r3zq0.
3. G. W. Mackey, The William Lowell Putnam Mathematical Competition, *Amer. Math. Monthly* **55** (1948) 630–632. http://dx.doi.org/10.2307/2305618.

JOE GALLIAN received a B.A. degree from Slippery Rock State University in 1966 and Ph.D. from Notre Dame in 1971. He has been at the University of Minnesota Duluth since 1972. He is a past president of the MAA and an inaugural Fellow of the AMS. Between 1977 and 2015, 222 students participated in his summer undergraduate research program. Among them are 38 Putnam Fellows (74 counting multiplicity) and 29 IMO gold medalists (46 counting multiplicity).

Department of Mathematics and Statistics, University of Minnesota Duluth, Duluth, MN 55812
jgallian@d.umn.edu

A Novel Way to Prove the Irrationality of $\sqrt{2}$

We present an interesting and simple way to prove the irrationality of $\sqrt{2}$.

Theorem. *$\sqrt{2}$ cannot be represented as a ratio of two integers.*

Proof. Let $f(x) = x^{-1} - \lfloor x^{-1} \rfloor$. For any positive rational number $\frac{a}{b}$, we can express $f(\frac{a}{b})$ as $\frac{b \bmod a}{a}$.

If we assume that $\frac{a}{b}$ is reduced to lowest terms, i.e., $\gcd(a, b) = 1$, the fraction $\frac{b \bmod a}{a}$ is also reduced to lowest terms. Since the numerator of the second fraction is always less than the numerator of the first and they both are reduced to lowest terms, we can conclude that these fractions are different: $f\left(\frac{a}{b}\right) \neq \frac{a}{b}$.

To calculate the value of $f(\sqrt{2} - 1)$ we use the fact that $\frac{1}{\sqrt{2}-1} = \sqrt{2} + 1$:

$$f(\sqrt{2} - 1) = \sqrt{2} + 1 - \left\lfloor \sqrt{2} + 1 \right\rfloor = \sqrt{2} - 1.$$

This calculation implies that there is no rational number $\frac{a}{b} = \sqrt{2} - 1$. ■

Underlying this proof is the fact that the continued fraction representation of $\sqrt{2}$ is periodic and has a period length of one. More generally, this argument can be used to prove that any number of the form $\sqrt{n^2 + 4}$ is irrational for $n > 0$.

—Submitted by Saveliy Ustuzhanin

http://dx.doi.org/10.4169/amer.math.monthly.124.1.59
MSC: Primary 11J72