Manjul Bhargava: Budding Superstar

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"We are watching him very closely.
He is going to be a superstar.
He's amazingly mature mathematically.
He is changing the subject in a fundamental way."
Peter Sarnak, Princeton mathematician, speaking of Manjul Bhargava

Manjul Bhargava, the son of a chemist father and mathematician mother, was born in Hamilton, Ontario, Canada and grew up in Long Island, New York. At an early age Manjul's mother introduced him to the two disciplines that would become his passions– mathematics and classical Indian music. By the time he was in grade school he began thinking about the distribution of primes and was skilled at playing a pair of Indian drums called the tabla. At the age of 12 he took several months off from school to study with the renowned tabla teacher Pandit Prem Prakash Sharma. In high school Manjul was the winner of the First Annual New York State Science Talent Search. As an undergraduate at Harvard he won the $2500 Hoopes Prize for his outstanding senior thesis and the $1000 Morgan Prize given jointly by three professional mathematical societies for outstanding research by an undergraduate student. These prizes were for work he began at the Duluth REU and expanded on in his senior thesis. Manjul also was a three-time recipient of an award at Harvard for excellence in teaching and he was selected to lead the commencement of the 1600 Harvard graduates of the class of 1996.

After graduating from Harvard *summa cum laude*, Bhargava went to Princeton to work under Andrew Wiles, who is famous for solving the centuries-old puzzle known as Fermat's Last Theorem. In his Ph.D. thesis, Bhargava investigated a "composition law" first formulated by the legendary mathematician Carl Gauss in 1801 for combining two quadratic equations (equations with a form such as $x^2 + 3xy + 6y^2 = 0$) in a way that was very different from normal addition and revealed a lot of information about number systems. Bhargava tackled an aspect of the problem where no significant progress had been made in over 200 years. He made a major breakthrough in that area by discovering 13 more composition laws and developing a coherent mathematical framework to explain them. He also applied his theory of composition to solve a number of other long-standing problems concerning the distribution of certain types of algebraic objects known as number rings, number fields, and class groups. What made Bhargava's work especially remarkable is that he was able to explain all his revolutionary ideas using

only elementary mathematics. In commenting on Bhargava's results Wiles said "He did it in a way that Gauss himself could have understood and appreciated." Wiles considers Bhargava's dissertation one of the strongest he's seen in 24 years at Princeton.

One of Bhargava's remarkable accomplishments has to do with the so-called "Fifteen Theorem," which was originally proved by Princeton mathematicians John Conway and William Schneeberger in 1993. A classical result of Lagrange states that every positive integer can be written in the form $a^2 + b^2 + c^2 + d^2$ (for example, $15 = 3^2 + 2^2 + 1^2 + 1^2$). Conway and Schneeberger greatly generalized Lagrange's theorem by allowing any positive-valued expression in which all the terms were second degree (like the one given earlier). They proved the amazing fact that if all the integers from 1 to 15 could be written in the prescribed form then every positive integer could be written in that form. However, Conway and Scheeberger were not happy with their proof and never published it. When Bhargava heard about this he dropped everything he was doing and began thinking about the problem in a geometric context. In this way he was able to find a significantly simpler proof that generalized the result in several beautiful ways. For example, he proved that to check whether a quadratic form represents all primes, one need only check the primes up to 73.

Bhargava draws much of his mathematical inspiration from his love of music. He has studied with Zakir Hussain, a renowned maestro of the tabla, and was instrumental in convincing Hussain to spend the semester at the Princeton in the fall of 2005. His course titled "Introduction to the Music of India," designed by Hussain and Bhargava, was one of the most popular in the music department that semester. In Manjul's point of view, music and math are both forms of art, only in different languages.

Although barely beyond 30 years old, Bhargava has already received many honors. In 2000, he received the first Clay Mathematics Institute Long-Term Prize Fellowship given to mathematicians under 30 years of age who have already contributed profound ideas to the discipline. The Clay Fellowship provides for employment under ideal research conditions with no teaching duties or committee work. Even before he obtained his Ph.D. degree in 2001 Bhargava began receiving offers of tenured Associate Professorships from top Ivy league schools. By 2003 these offers were increased to full professorships. In mid 2003 he accepted Princeton's offer. He is among the youngest full professors in the history of Princeton. Part of Princeton's attraction for Manjul is its wooded setting and the fact that Princeton was willing to let him take time off to visit India to study with tabla masters. Manjul performs on the tabla extensively in the New York and Boston areas.

Bhargava's other honors include being the only mathematician on Popular Science magazine's first list of "Brilliant 10," an annual celebration of ten scientists who are shaking up their fields. In 2003 he received the Hasse Prize from the Mathematical Association of America for mathematical exposition. The paper for which he won the Hasse Prize – "The factorial function and generalizations" published in the American Mathematical Monthly (November 2000) – explains his new generalization of the factorial function and its connections with some classical problems in number theory, ring theory, and combi-

natorics. In 2005 Manjul was featured on National Public Radio's Morning Edition as part of a series exploring the intersection of art and science. You can hear him playing the tabla and the interview at http://www.npr.org/templates/story/story.php?storyId=4111253.

In 2004 he received a Packard Foundation Fellowship, which enables scientists to pursue innovative lines of scientific inquiry and to inspire and train the next generation of young scientists and engineers and the Blumenthal Award for the Advancement of Research in Pure Mathematics at the annual meeting of the American Mathematical Society in 2005. The latter is presented once every four years for the most substantial Ph.D. thesis produced anywhere in the world in the four year interval between awards. Bhargava's most two recent awards are the Clay Research Award, which recognizes extraordinary achievement in mathematics and the first $10,000 SASTRA Ramanujan Prize, presented to young scholars for their outstanding contributions in the field of mathematics.

Princeton colleague Peter Sarnak has said "For a guy so young I can't remember anybody so decorated at his age. He certainly started out with a bang and has not let it get to his head, which is unusual. Of course he couldn't do what he does if he wasn't brilliant. It's his exceptional talent that's so striking."

Unlike many prominent mathematicians, Manjul has great interest in teaching mathematics courses for students who are not intending to major in mathematics. Princeton has encouraged him to design his own course titled "An Introduction to Mathematical Thinking." Manjul says "I am really excited about taking advantage of the teaching and research opportunities that Princeton and the Clay Mathematics Institute have provided me." He wants to spend more time over the next few years on efforts to popularize mathematics among the general population. "I hope that one day people in the general public will be able to appreciate the beauty and universality of mathematics, just as they already do with various other creative arts and applied sciences."