

With Lagrange multipliers, where $f_j(q_1 \dots q_n) = 0$ are equations of constraint:

$$\frac{\partial \mathcal{L}}{\partial q_i} \longrightarrow \frac{\partial \mathcal{L}}{\partial q_i} + \sum_j \lambda_j(t) \frac{\partial f_j}{\partial q_i}$$

Transformed radial equation: $u''(\phi) = -u(\phi) - \frac{\mu}{\ell^2 u^2(\phi)} F$

Keplerian orbits: $F = -\gamma/r^2$ $r(\phi) = \frac{r_o}{1 + \epsilon \cos \phi}$ $r_o = \frac{\ell^2}{\gamma \mu}$ $E = \frac{\gamma^2 \mu}{2\ell^2} (\epsilon^2 - 1)$