Math 4230 extra credit problems. Due March 25th. Important: you must cite any sources you use.

(1) Show that Newton's method will converge to the closest root of a quadratic polynomial $p(z) = z^2 + b^2$. What happens if the initial z_0 is on the line equidistant from the roots?

(2) Computational: write a program that, given a polynomial $p(z)$, colors a grid of points according to which root of $p(z)$ that Newton's method converges to (if it doesn't converge within a certain number of iterations, indicate that by a different color). You should hand in your code, plus an example of its output.

(3) Computational: for a quadratic polynomial $p(z) = az^2 + bz + c$, the Julia sets can be computed by backwards iteration: using a variety of input points (either randomly generated or from a grid), for a point z_i compute a value z_{i+1} such that $p(z_{i+1}) = z_i$. You should hand in your code, plus an example of its output.

(4) Show that a for a polynomial $p(z) = z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0$ (where $a_i \in \mathbb{C}$), if $|z| > 1$ and $|z| \geq 2\sum_{i=0}^{n-1} |a_i|$ then $|z|^n/2 \leq |p(z)| \leq 3|z|^n/2$.

(5) Suppose $f(z)$ is analytic in the entire complex plane, and it is doubly-periodic in the sense that $f(z) = f(z+1)$ for all z and $f(z) = f(z+w)$ for all z where w is a complex number with non-zero imaginary part. Show that $f(z)$ must be a constant.

(6) For what rational functions $R(z) = \frac{P(z)}{Q(z)}$ (P and Q are polynomials) is the modulus of $R(z)$ equal to 1 for all unit length z (i.e. for $|z| = 1$).