

Math 4230 Assignment 10, due Wednesday, May 4th.

(1) Compute the integral $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$ by using the residue theorem.

(2) Compute the integral

$$\int_0^{2\pi} \frac{d\theta}{9 \sin^2(\theta) + 25 \cos^2(\theta)}$$

using the residue theorem and the substitution $z = e^{i\theta}$.

(3) Compute the integral

$$\int_{-\infty}^{\infty} \frac{x^2 + 1}{x^4 + 1} dx$$

using the residue theorem.

(4) For the function $w = e^z$, describe the image in the w -plane of the following regions in the z -plane:

(a) The strip $0 < \operatorname{Im}(z) < \pi$.

(b) The rectangle $1 < \operatorname{Re}(z) < 2$, $0 < \operatorname{Im}(z) < 2\pi$.

(5) Describe and sketch the image of the circle $|z - 3| = 1$ under the following transformations:

(a) $w = 1/z$.

(b) $w = \frac{z-3}{z-2}$.

(6) Find a Möbius transformation that maps the lower half-plane into the circle $|w - 1| < 1$.