Math 4230 Assignment 4, due Wednesday, February 16th.

(1) Chapter 3.2, #6: Establish the trigonometric identities $\sin^2(z) + \cos^2(z) = 1$ and $\sin(z_1 + z_2) = \sin(z_1)\cos(z_2) + \sin(z_2)\cos(z_1)$ for the complex sine and cosine.

(2) Find all complex numbers $z$ such that $e^{iz} = 4$.

(3) Ch 3.2 #22: Prove that for any $m$ distinct complex numbers $\lambda_1, \lambda_2, \ldots, \lambda_m$ ($\lambda_i \neq \lambda_j$ for $i \neq j$), the functions $e^{\lambda_1 z}, e^{\lambda_2 z}, \ldots, e^{\lambda_m z}$ are *linearly independent* over $\mathbb{C}$. In other words, show that if $c_1 e^{\lambda_1 z} + c_2 e^{\lambda_2 z} + \cdots + c_m e^{\lambda_m z} = 0$ for all $z$, the $c_1 = c_2 = \cdots = c_m = 0$. [HINT: Proceed by induction on $m$. In the inductive step, divide by one of the exponentials and then take the derivative.]

(4) Solve the following equations for all possible values of $z$:
(a) $e^z = 4i$
(b) $Log(z^3 - 1) = \frac{i\pi}{2}$
(c) $e^{3z} + 27 = 0$.

(5) Chapter 3.3, #12: Find a branch of the function $\log(z^2 + 1)$ that is analytic at $z = 0$ and is equal to $2\pi i$ at $z = 0$.

(6) Chapter 3.4, #4: Find a function $\phi(z)$ which is harmonic in the upper half plane ($Im(z) > 0$) and which is equal to 0 on the real axis for $x < -1$ and $x > 2$, and equal to $\pi$ for $-1 < x < 2$.

(7) Find all the values of the following quantities:
(a) $i^{2i}$
(b) $(2i + 1)^2$
(c) $(-1)^{3/5}$.

(8) Chapter 3.5, #4: Is 1 raised to any complex power always equal to 1?