Math 4230 Assignment 6, due Wednesday, March 2.

(1) Where is the function $f(z) = \dfrac{z}{z^2+36}$ analytic? Use your answer to compute $\int_\Gamma f(z)dz$ if Γ is the circle $|z| = 3$ traversed positively starting at $z = 3$.

(2) Where is the function $f(z) = Log(3 - z)$ analytic? Use your answer to compute $\int_\Gamma f(z)dz$ if Γ is the circle of radius 2 centered at -2 traversed positively starting at $z = 0$.

(3) Suppose $f(z)$ is analytic in the unit disk D ($D = \{z \in \mathbb{C}$ such that $|z| < 1\}$) and its derivative is continuous there. Suppose we also have a bound on the derivative in D: $|f'(z)| \leq M$. Show that for any two points z_1 and z_2 in D, $|f(z_2) - f(z_1)| \leq 2M$.

(4) Find a homotopy (a continuous deformation) $z(s,t)$ of the contour Γ_0 to the contour Γ_1 if Γ_0 is the unit circle $|z| = 1$ traversed positively starting from 1 and Γ_1 is the cardioid $c(t) = e^{i2\pi t} - e^{i4\pi t}/2$, $t \in [0,1]$.

(5) Evaluate the $\displaystyle\int_\Gamma \frac{z+1}{z^2+36}dz$ if Γ is the circle $|z| = 12$ traversed positively starting at $z = 12$.