Math 4230 Assignment 9, due Friday, April 22nd.

(1) A theorem of Weierstrass says that if we have functions f_j which are analytic in a domain, and if $|f_j| < M_j$ and $\sum M_j$ converges then $\sum f_j$ converges and we can compute a series expansion by adding together the series for the f_j. As an example of this, find the series expansion of $\sum_{n=1}^{n=\infty} \frac{z^n}{1-z^n}$ around $z_0 = 0$ by expanding each term $\frac{z^n}{1-z^n}$ in a series and adding them together.

(2) Find and classify the isolated singularities of the following functions:
   (a) $\frac{z^2+1}{z^3(z-1)}$.

   (b) $e^z z^4$.

   (c) $\sin(1 - 1/z)$.

(3) Construct a function which has a zero of order 3 at $z = 0$ and an essential singularity at $z = 1$.

(4) Classify the behavior at ∞ for the following functions:
   (a) $\frac{2z+1}{z-1}$.

   (b) $\frac{\cos(z)}{z}$.

(5) Construct a function which has poles of order 1 at both $z = \infty$ and $z = 0$.