Chapter 6 Nonlinear Equations Practice Problems

Use Excel and VBA to solve the following problems. Document your solutions using the Expert Problem Solving steps outlined in Table 1.2.

1. Find the real and imaginary roots of the following equations using Bairstow’s method:
   
   (a) \( x^4 - x^3 + 2x^2 - 3x + 2 = 0 \)
   
   (b) \( x^3 + 2x - 1 = 0 \)
   
   (c) \( x^6 + 3x^5 - 41x^4 - 87x^3 + 400x^2 + 444x - 720 = 0 \)
   
   (d) \( x^5 - 2x^4 + 3x^3 - x^2 + 2x - 1 = 0 \)

   **Answers:** (a) \(-4458 \pm 0.9142i, 0.5708 \pm 0.3969\)
   
   2. Find real roots of the following equations using a variety of methods from this chapter:

   (a) \( \sin x + x = 1 \)
   
   (b) \( \sin x + x^2 = 1 \)
   
   (c) \( \sin x + x^3 = 1 \)
   
   (d) \( \sin x - x = 1 \)
   
   (e) \( \sin^2 x + x = 1 \)
   
   (f) \( \cos x + \sin x = 1 \)
   
   (g) \( \sin^2 x + e^{-x} = 10 \)
   
   (h) \( x^2 + 2\ln x - \cos x = 5 \)
   
   (i) \( \tan x + x = 3 \)

   **Answers:** (a) 0.511 (b) 0.637 (c) 0.706 (d) \(-1.93\) (e) 0.642 (f) 1.57 (g) \(-2.24\) (h) 1.86 (i) 1.09

3. Find real roots of the following simultaneous equations using a variety of methods from this chapter.

   \( 3x^2 + y^2 - 2z - 4 = 0 \)

   (a) \( xy + z - 2 = 0 \)
   
   \( x + y - 2z + 3 = 0 \)

   (b) \( x^2 + y \sin x = 2 \)
   
   \( y^2 + x \sin y = 4 \)

   \( x^2 + y^2 + z^2 = 1 \)

   (c) \( xe^{-x} + 3y + \sin z = 3 \)
   
   \( x^2 + 2ye^{-x} + z = 2 \)

   **Answers:** (a) \( x = 1.68, y = 0.157, z = 2.26 \) (b) \( x = 0.827, y = 1.79 \) (c) \( x = 0.124, y = 0.767, z = 0.629 \)

4. Solve the following present worth equation for the rate of return, \( i \):

   \[
   2000 \frac{(1+i)^8 - 1}{i(1+i)^8} + 500(1+i)^8 = 10000
   \]
5. Consider the Van der Waal’s equation of state for a gas:

\[
P + \frac{a}{V^2} (V - b) = RT
\]

where
- \( P \) = pressure, atm
- \( V \) = specific molar volume, L/mol
- \( R \) = gas constant, 0.08206 L atm/mol K
- \( T \) = temperature, K
- \( a, b \) = constants

Calculate the specific molar volume for air at 1 atm pressure and 290 K, and for hydrogen at 5 atm pressure and 350 K.

Air: \( a = 1.33 \frac{atm \cdot L^2}{mol \cdot K} \) \( b = 0.0366 \frac{L}{mol} \)

\( V_{\text{Air}} = \frac{RT}{P} - \frac{a}{RT^2} \)

H₂: \( a = 0.245 \frac{atm \cdot L^2}{mol \cdot K} \) \( b = 0.0266 \frac{L}{mol} \)

6. Consider the Redlich-Kwong equation of state:

\[
P = \frac{RT}{V - b} - \frac{a}{T^{0.5}V(V - b)}
\]

where
- \( P \) = pressure, atm
- \( V \) = specific molar volume, L/mol
- \( R \) = gas constant, 0.08206 L atm/mol K
- \( T \) = temperature, K

\[a = 0.4278 \frac{R^2T^2}{P_c^2} \quad b = 0.0867 \frac{RT_c}{P_c}\]

Estimate the specific molar volume of CO₂ at a pressure of 1 atm and temperature of 283 K, where the critical values for CO₂: \( P_c = 78 \text{ atm}, T_c = 304 \text{ K} \).

7. Solve Example 6.11 by Steffensen’s method.

8. Add the calculation for the bubble point temperature at the end of distillation to the system of nonlinear equations in Example 6.17.

9. Solve the following system of equations.
   a. Use successive substitution (iteration). Show the values for each iteration.
   b. Use the Solver.

\[
3x_1 - \cos(x_2x_3) - \frac{1}{2} = 0
\]
\[
x_1^2 - 625x_2^2 = 0
\]
\[
e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0
\]

10. Consider the following function. Use Newton’s method to find the root to the following equation. Show the intermediate values. (Do not use the Solver).

\[4 \cos x = e^x\]

11. Solve the following system of nonlinear equations. Show your work.
12. Find the root of the following nonlinear equation using Newton’s method. Show your work.

\[ e^{-x} = 3x \]

\[ f(x) = 3x - e^{-x} \]

\[ df/dx = 3 + e^{-x} \]

13. Find the roots of the following system of equations using the Solver.

\[ x_1 \exp(x_1 + x_2) - 2 = 0 \]
\[ x_1x_2 - 0.1 \exp(-x_2) = 0 \]

14. Solve the following system of equations for \( x \) and \( y \). Show your work.

\[ (2x)^{2/3} + y^{2/3} = (9)^{1/3} \]
\[ x^2/4 + y^2 = 1 \]

15. Find all roots for \( x \) and \( y \) of the following simultaneous equations. Show your work.

\[ y = \cosh(x) \]
\[ x^2 + y^2 = 2 \]

16. Solve the following system of equations.

\[ x_1^2 + 2x_2^2 - x_2 - 2x_3 = 0 \]
\[ x_1^2 - 8x_2^2 + 10x_3 = 0 \]
\[ \frac{x_1^2}{7x_2x_3} - 1 = 0 \]

12. The following system of equations represents mole balances around a steady-state reactor. Use Excel’s Solver to find A, B, and C.

\[ 0.15(1.25 - A) + 3(1.2B^{1.5} - 0.3A^2) = 0 \]
\[ 3\left( \frac{3.7C}{1.5 + C^{0.8}} - 1.2BC \right) = 0.15B \]
\[ 3\left( 0.8B - 3.7C^2 \right) = 0.15C \]

13. Refer to the following system of equations:

\[ x^3 - 10x + y - x + 3 = 0 \]
\[ y^3 + 10y - 2x - 2z - 5 = 0 \]
\[ x + y - 10z + 2 \sin(z) + 5 = 0 \]

a. Create a VBA user function for each equation.

b. Use Excel’s Solver and your VBA functions to solve the system of equations \( x \), \( y \), and \( z \).
14. The following equations are used to calculate the pipe diameter in ft for a maximum volumetric flow rate of a fluid expressed in gpm, with a maximum allowable pressure drop per unit length of pipe expressed as psi/ft.

\[
P = 144L \left( \frac{\Delta P}{\Delta L} \right)
\]

\[
q = 0.002228Q
\]

\[
V = \frac{4q}{\pi d^2}
\]

\[
Re = \frac{dVp}{\mu}
\]

\[
f = 0.0014 + \frac{0.125}{Re^{0.32}}
\]

where

- \( P \) = pressure drop in pipe, psf (pounds per square foot)
- \( L \) = length of pipe, ft
- \( \Delta P / \Delta L \) = allowable pressure drop per length of pipe, psi/ft
- \( q \) = volumetric flow rate, gpm
- \( Q \) = volumetric flow rate, ft³/min
- \( V \) = maximum allowable linear velocity through the pipe, ft/s
- \( Re \) = Reynolds number, dimensionless
- \( \rho \) = fluid density, lb/ft³
- \( \mu \) = fluid viscosity, lb/ft s
- \( f \) = Darcy friction factor, dimensionless

Determine the diameter of a pipe that has to deliver 1000 gpm of water at 70°F with a maximum pressure drop of 0.003 psi/ft.

15. The Underwood equation relates the fraction of feed to a multicomponent distillation column that joins the liquid stream in the column, \( q \), to the relative volatilities, \( \alpha_i \), and mole fractions of the different components in the feed, \( x_{Fi} \), using the parameter \( \phi \), as follows:

\[
1 - q = \sum_i \frac{\alpha_i x_{Fi}}{\alpha_i - \phi}
\]

The value of the parameter, \( \phi \), is between the values of the relative volatilities of the light and heavy key components. The key components are the ones that the design engineer wants to separate. Calculate the value of \( \phi \) for a multicomponent feed (\( q=0.5 \)) to a distillation column having the following data:

<table>
<thead>
<tr>
<th>Component</th>
<th>Mole fraction in feed</th>
<th>Relative volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethane</td>
<td>0.2</td>
<td>7.2</td>
</tr>
<tr>
<td>Propane</td>
<td>0.3</td>
<td>5.8</td>
</tr>
<tr>
<td>Butane</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Pentane</td>
<td>0.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

16. Solve the following equation for \( \theta \) given the values of \( z \) and \( K \) in the table.
\[ \sum_{i=0}^{6} \frac{z_i (1 - K_i)}{1 + \theta (K_i - 1)} = 0 \]

<table>
<thead>
<tr>
<th>i</th>
<th>z_i</th>
<th>K_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.01</td>
<td>16.2</td>
</tr>
<tr>
<td>1</td>
<td>0.13</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
<td>0.08</td>
<td>2.6</td>
</tr>
<tr>
<td>3</td>
<td>0.27</td>
<td>1.98</td>
</tr>
<tr>
<td>4</td>
<td>0.06</td>
<td>0.91</td>
</tr>
<tr>
<td>5</td>
<td>0.13</td>
<td>0.72</td>
</tr>
<tr>
<td>6</td>
<td>0.32</td>
<td>0.28</td>
</tr>
</tbody>
</table>

**Answer:**

17. For an exothermic reversible reaction, \(2A + B \leftrightarrow C + D\), the relation between the equilibrium constant, \(K\), and the fractional conversion at equilibrium, \(X\), is given by the following equation:

\[
K = \frac{X^2}{4C_{A0}(1 - X)^2 \left( \theta - \frac{X}{2} \right)}
\]

where
\[
\theta = \frac{C_{B0}}{C_{A0}}
\]

The equilibrium constant is a function of temperature, \(T\), according to the following equation

\[
K = 20000 \exp \left[ -\frac{30(T - 300)}{T} \right]
\]

where \(T\) = equilibrium temperature, K

The temperature at equilibrium, \(T\), is related to the fractional conversion, \(X\), and of \(A\) by the equation assuming adiabatic conditions:

\[
X = 5 \times 10^{-6} (T - T_0)^2 + 5 \times 10^{-3} (T - T_0)
\]

where \(T_0\) is the inlet temperature, K

Determine the equilibrium temperature and the fractional conversion of \(A\) when the initial concentration of \(A\) is 3.2 mol/L, \(\theta = 0.8\), and \(T_0 = 300\) K.

18. Find the root to \(f(x)=0\) in the range \(0 < x < 5\) for \(A = 7.5, B = 0.855\) with a relative convergence error < \(10^{-5}\), using the following methods. Use a VBA user-defined function of the following equation.

\[
f(x) = x^2 - A \sin \left( \frac{x}{B} \right)
\]

a. Graphical method. Follow the graphing guidelines to prepare your plot.

b. Bisection in an Excel worksheet.

c. Newton’s method in an Excel worksheet.
19. Find the roots for x, y, and z in the following system of equations. Use any method.

\begin{align*}
xyz - 1 & = 0 \\
x^2 + y^2 + z^2 - 4 & = 0 \\
x^2 + 2y^2 - 3 & = 0
\end{align*}

\textit{Answer:} x = 2.13

20. Write a macro for solving a nonlinear equation using the method of bisection. Test your macro using one of the practice problems for this chapter.

21. Write a user-defined function to find the root of a formula for an algebraic equation in a worksheet by Newton’s method (similar to the function ROOT in Figure 6.15).