## Repeated Measures ANOVA

One Factor, Correlated Measures:
Same reasoning of Correlated Measures t-test
More Power (and more efficient)
Pulls out relatively small differences among treatments
Relative to Big differences among subjects
Removes Differences among subjects from error term

## Subjects vs. Treatments

## Levels of independent variable, $X$

| Subjects | $\boldsymbol{X}_{1}$ | $\boldsymbol{X}_{2}$ | $\boldsymbol{X}_{3}$ |
| :--- | :---: | :---: | :---: |
| $S_{1}$ | 57 | 60 | 64 |
| $S_{2}$ | 71 | 72 | 74 |
| $S_{3}$ | 75 | 76 | 78 |
| $S_{4}$ | $\frac{93}{74}$ | $\underline{92}$ | $\underline{96}$ |
| $\bar{X}$ | 75 | 78 |  |

$\uparrow$
Large
Differences
(Error)
$\downarrow$
$\leftarrow$ Small Differences (Effect) $\rightarrow$

## Partitioning The Variance

$\mathrm{SS}_{\text {Total }}=\mathrm{SS}_{\text {Subjects }}+\mathrm{SS}_{\text {Treatment }}+\mathrm{SS}_{\text {Error }}$
$\mathrm{SS}_{\text {Error }}$ Is the Variability which a single subject would have If you repeatedly measured him without changing treatment

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## The Structure of the ANOVA

Partitioning the Total Sum of Squared Deviations From the Grand Mean

Spontaneous Variability of Subject Change is not the same for each Subject

## D.V.: Test Score

Subjects

If you test your subjects repeatedly:
Counter Balance for (e.g.) practice effects/fatigue

## POC: Piece of Cake

|  | 1st choice | Control | Notes |
| :---: | :---: | :---: | :---: |
| S1 | 57 | 60 | 64 |
| S2 | 71 | 72 | 74 |
| S3 | 75 | 76 | 78 |
| S4 | 93 | 92 | 96 |

## Step 1: Find The Total SS

|  | Raw | Cell | Square |
| :---: | :---: | :---: | :---: |
|  | Data | Deviations | Deviations |
| S1T1 | 57 | -18.7 | 348.4 |
| S2T1 | 71 | -4.7 | 21.8 |
| S3T1 | 75 | -0.7 | 0.4 |
| S4T1 | 93 | 17.3 | 300.4 |
| S1T2 | 60 | -15.7 | 245.4 |
| S2T2 | 72 | -3.7 | 13.4 |
| S3T2 | 76 | 0.3 | 0.1 |
| S4T2 | 92 | 16.3 | 266.8 |
| S1T3 | 64 | -11.7 | 136.1 |
| S2T3 | 74 | -1.7 | 2.8 |
| S3T3 | 78 | 2.3 | 5.4 |
| S4T3 | 96 | 20.3 | 413.4 |
| Grand Mean= | 75.67 | SS-Total= | 1754.7 |

## Step 2: Compute Between Subjects SS

| Subject |  |  |  |
| :--- | ---: | ---: | ---: |
| Means | Deviation | Sq Dev | Sq Dev*3 |
| 60.3 | -15.3 | 235.1 | 705.33 |
| 72.3 | -3.3 | 11.1 | 33.33 |
| 76.3 | 0.7 | 0.4 | 1.33 |
| 93.7 | 18.0 | 324 | 972 |
|  |  |  |  |
|  |  | SS-Sub= | $\mathbf{1 7 1 2}$ |
|  |  | MS-Sub= | $\mathbf{5 7 0 . 7}$ |

## Step 3: Compute Treatment SS

| Treatment Means | 74 | 75 | 78 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Deviation | -1.7 | -0.7 | 2.3 |  |  |  |
| Sq Dev | 2.8 | 0.4 | 5.4 |  |  |  |
| Sq dev*4 | 11.1 | 1.8 | 21.8 | SS-Treat= | $\mathbf{3 4 . 7}$ | "/2 |
|  |  |  |  | MS-Treat= | $\mathbf{1 7 . 3}$ |  |

## Step 4: Compute SS Error

| SS-Total "- | SS-Sub- | SS-Treat | "= | SS-Error |
| ---: | ---: | ---: | ---: | ---: |
| 1754.7 |  | 1712 | 34.7 |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  | MS-Error= |

This is the Same as the Interaction Term in a 2-Way ANOVA
$\mathrm{IV}_{1}$ : Treatment
$\mathrm{IV}_{2}$ : Subject
Interaction: Subject x Treatment

## Step 5: Determine Degrees of Freedom

In general:

$$
\begin{aligned}
d f_{\text {tot }} & =N_{\text {tot }}-1 \\
d f_{\text {subjects }} & =N_{s}-1 \\
d f_{\text {treat }} & =N_{t}-1 \\
d f_{\text {error }} & =\left(N_{s}-1\right)\left(N_{t}-1\right)
\end{aligned}
$$

For the strategy study:

$$
\begin{aligned}
d f_{\text {tot }} & =12-1=11 \\
d f_{\text {subjects }} & =4-1=3 \\
d f_{\text {strategies }} & =3-1=2 \\
d f_{\text {error }} & =(3)(2)=6
\end{aligned}
$$

Just like Interaction df

## Step 6: Calculate MS \& F

## F = MS-Treat I MS-Error <br> " 17.3 | $1.333=13.003$

TABLE 12.4 ANOVA summary table for the exam strategy study

| Source | SS | $d f$ | $M S$ | $F$ |
| :--- | ---: | :---: | :---: | :---: |
| Subjects | 1712.000 | 3 |  |  |
| Strategies | 34.667 | 2 | 17.333 | 13.00 |
| Error | 8.000 | $\frac{6}{11}$ | 1.333 |  |
| Total | 1754.667 |  |  |  |
|  | $F_{.05}(2,6)$ | $=5.14$ | $F_{01}(2,6)=10.92$ |  |

## If ANOVA is Significant

Use Tukey Test to compare treatments

$\mathrm{N}_{\mathrm{t}}$ is Number of Subjects in your
Experiment

## Caution

-Caryover effects
Counter-balance
Vs. Trend Analysis
-Populations Normally Distributed

