TOPIC 4. POPULATION PREDICTIONS

Population predictions are based on the initial number of animals in the population, the reproductive potential of the animals, and on the mortality at different times during the reproductive process and in the classes within the population.

Reproductive potentials represent the ultimate number of young that can be produced. This is highest when expressed as a conception rate (CORT). In utero mortality results in fewer births than conceptions; the birth rate (BIRT) is an expression of realized natality. Mortality of neonates is often high in free-ranging populations, so the weaning rate (WERT) is an even better ecological expression of recruitment of individuals into a population. Then, the number of females that survive to reproduce is the best estimate of the reproductive potential of a population, since it is at that level of individual survival that productivity once again enters into population dynamics.

The compiling of N into sex and age classes is the first consideration when preparing to make population predictions. Knowing the reproductive rates of those present to reproduce is the beginning of the potential population predictions. Mortalities are then considered, representing depressants of productivity, resulting in realized populations that are less than the potential ones.

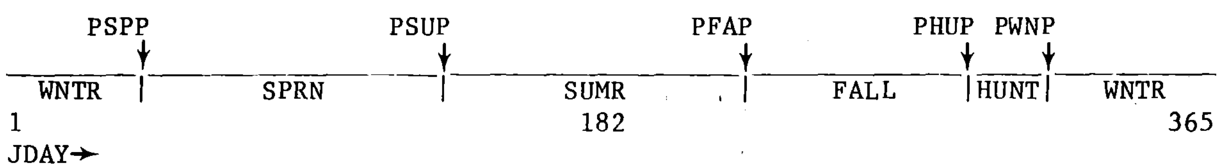
Two methods of population predictions are described in UNITS 4.1 and 4.2. Arithmetic functions are described in UNIT 4.1, using additions to and subtractions from the total population by each class. This approach requires considerable arithmetic. Exponential predictions are described in UNIT 4.2, using a short equation to make the predictions for each age or weight class, or after weighted mean natality and mortality rates have been determined for the population.

UNIT 4.1. ARITHMETIC SUMMATIONS

The basic idea behind arithmetic summations of recruitment of individuals to and removal from a population is very simple. Individuals are added to the population at birth and subtracted from the population at death. It is necessary to move survivors from one age class to the next at the completion of each annual cycle. When weight classes are used, the numbers in each weight class need to be adjusted too, of course, as survivors increase in weight. If range conditions deteriorate and weights do not increase as expected, then the distibution of numbers in weight classes will not be in proportion to age changes.

The summations must begin at some point in the annual cycle. The pre-hunt population is a good starting point as there are many estimates of fall, prehunt populations available in the literature.

The annual cycle was divided into functional rather than arbitrary periods of time at the beginning of this CHAPTER. The hunting season (HUNT) is one on those functional periods, a time of specific mortality. The winter season (WNTR) is another time of specific kinds of mortality. The time from the end of winter to parturition (SPNG) is another logical period in the annual cycle as animals move from winter range to summer range, dispersing for the reproductive season. The time period from parturition through weaning of the young (SUMR) is another logical period as the females are responsible for themselves and their young, and the survival of the young determines the number recruited into the population. The fall season (FALL), occurring after the young are weaned or at least not dependent on the dam for milk, and a time of weight recovery for the dam, is the fifth period in the annual cycle. It is at the beginning of this period that the young from the previous summer move to the 1/2 year old class, and all other age classes move up one year.



Some mortality factors are present throughout the year and need to be considered during each time period. Road kills, for example, may occur at any time, but rates vary in relation to the presence of roads and traffic, and to the movement patterns of the animals. They are usually higher during the period from the end of winter to parturition when animals are dispersing, for example.

A simplified diagram of population decrements and increments in three age classes for each sex is illustrated on the next page. The initial number in the class is subject to mortality, and the number of mortalities in the class (NBMO) is subtracted from NUAC. The number surviving in the class (NSAC) produces a number of young per female (NYPF), which combine to become the recruitment into the first (0-1) age class in the next prehunt population. Note that the number of females (NFER) is added to the 0-1 AGCL for females, and the number of males is added to the 0-1 male AGCL. The calculations for males involve only mortality, of course.

CLASS	ACYI	
ACa-.	0-1	→ NUAC - NBMO = NSAC
ACb-.	1-2	→ NUAC - NBMO = NSAC
ACc-.	2-3+	→ NUAC - NBMO = NSAC

	ACYI				
ACaa.	0-1	→ NUAC - NBMO = NSAC	→ NYPF		NFER
ACbb.	1-2	→ NUAC - NBMO = NSAC	→ NYPF	→ MFSR	
ACcc.	2-3+	→ NUAC - NBMO = NSAC	→ NYPF		NMAR

The definitions are:

ACYI = age class by year intervals,
NUAC = number in the age class,
NBMO = number of mortalities,
NSAC = number surviving in each age class,
NYPF = number of young per female,
MFSR = male to female sex ratio,
NFER = number of females recruited into the population, and
NMAR = number of males recruited into the population.

Note how the arrows show a beginning N followed by mortality which leads to the number of survivors. These survivors reproduce, and the young are recruited into the population. When the young of the year are recruited into the pre-hunt population, all older members move up one year to reflect the annual cycle from one pre-hunt population to the next.

The oldest age class accumulates all the survivors over 2 years old in the above example. Wild ruminant populations may be divided into several age classes, and smaller numbers are usually observed in the older age classes up to the last one, especially in populations that are exposed to hunting.

The flow of information illustrated should be clearly understood before going on to further divisions of the population into more age classes or of the annual cycle into smaller time periods. When further divisions are made, the format remains the same but the number of repetitious steps necessary for the completion of an annual cycle increases.

AGE-RELATED SUMMATIONS

Arithmetic summations for age-related population data provide a fairly simple approach to population predictions, and are based on the division of the population into age classes. The format for completing a sequence of summations is shown below.

CLASS	ACYI	NUAC	NBMO	NSAC
ACa-.	0-1	50	- 10	= 40
ACb-.	1-2	38	- 3	= 35
ACc-.	2-3+	12	- 1	= 11

CLASS	ACYI	NUAC	NBMO	NSAC	NYPF	NYPC	MFSR	NMAR	NFER
ACaa.	0-1	44	- 12	= 32	x 0.06	= 1.92	1.1:1.0	1.00	0.92
ACbb.	1-2	34	- 3	= 31	x 0.70	= 21.7	1.1:1.0	11.3	10.4
ACcc.	2-3+	22	- 1	= 21	x 1.30	= 27.3	1.1:1.0	14.2	13.1

The numbers illustrate the sequence and results of the calculations. To repeat another cycle, move the NSAC from the first year to the next age class as indicated by the dashed lines, as the number surviving in each age class (NSAC) becomes the number in the next higher age class; one year's outputs become the next year's inputs.

The use of such a format makes it possible to keep track of changes in the causes of mortality. The total number of mortalities (NBMO) in each class is determined by adding together the mortalities from different causes such as hunting, poaching, winter effects, predation, road kills, etc. over the period being considered. Suppose 100 animals were subject to the following mortalities:

```
NUAC = 100
                 - 16 (hunting)
                 -  2 (crippling loss)
                 -  4 (poaching)
                 -  3 (predation)
                 -  6 (winter losses)
                 -  8 (road kills)
                 -  1 (other causes)
                 ----
                   40 = NBMO
NSAC =  60
```

Some of the mortality factors above are compensatory. If hunting mortality is low, winter mortality may be high. If hunting mortality is high, winter mortality may be low. The amount of compensation depends on hunting regulations and on other factors, such as weather and range conditions. The possibilities of compensations should not be overstated as an argument of ecological certainty.

Reproductive rates--NYPF in this example--are derived from observed values, from age or weight data, or from indices to these rates (See TOPIC 1,2, and 3). The use of arithmetic summations forces one to estimate biological factors such as reproductive rates, and that is good mental excercise.

The number of mortalities may be expressed as a rate or ratio in relation to the initial number in the age class. In the example above, the 40 mortalities are expressed as 40/100 = 0.40 = MORT. If mortality rates rather than numbers are used, then the format on the next page is used. The use of reproductive rates and mortality rates rather than the number of mortalities is much more customary since absolute numbers of both the population and mortalities are seldom known. Rates allow one to keep all of the evaluations together on a relative basis.

CLASS	ACYI	MORT	x	NUAC	=	NBMO		NSAC
ACa-.	0-1	0,20						
			x	50	=	10		
					-	10	=	40
ACb-.	1-2	0.08						
			x	38	=	3.0		
					-	3.0	=	35
ACc-.	2-3+	0.08						
			x	12	=	1.0		
					-	1.0	=	11

CLASS	ACYI	MORT	x	NUAC	=	NBMO		NSAC	x	NYPF	=	NYPC	MFSR	NMAR	NFER
ACaa.	0-1	0.27													
			x	44	=	11.9									
					-	11.9	=	32.1	x	0.06	=	1.93	1.1:1.0	1.00	0.93
ACbb.	1-2	0.09													
			x	34	=	3.1									
					-	3.1	=	30.9	x	0.70	=	21.6	1.1:1.0	11.2	10.4
ACcc.	2-3+	0.05													
			x	22	=	1.1									
					-	1.1	=	20.9	x	1.3	=	27.2	1.1:1.0	14.1	13.1

The definitions are:

ACYI = age class by year intervals,
MORT = mortality rate,
NUAC = number in the age class,
NBMO = number of mortalities,
NSAC = number surviving in each age class,
NYPF = number of young per female,
MFSR = male to female sex ratio,
NMAR = number of males recruited into the population, and
NFER = number of females recruited into the population.

If survival rates have been determined, the format on the next page is used.

CLASS	ACYI	SURT	x	NUAC	=	NSAC
ACa-.	0-1	0.80	x	50	=	40.0
ACb-.	1-2	0.92	x	38	=	35.0
ACc-.	2-3+	0.92	x	12	=	11.0

CLASS	ACYI	SURT	x	NUAC	=	NSAC	x	NYPF	=	NYPC	MFSR	NMAR	NFER
ACaa.	0-1	0.73	x	44	=	32.1	x	0.06	=	1.93	1.1:1.0	1.00	0.93
ACbb.	1-2	0.91	x	34	=	30.9	x	0.70	=	21.6	1.1:1.0	11.2	10.4
ACcc.	2-3+	0.95	x	22	=	20.9	x	1.30	=	27.2	1.1:1.0	14.1	13.1

The definitions are:

ACYI = age class by year intervals,
SURT = suvival rate,
NUAC = number in the age class,
NSAC = number surviving in each age class,
NYPF = number of young per female,
MFSR = male to female sex ratio,
NMAR = number of males recruited into the population, and
NFER = number of females recruited into the population.

WEIGHT-RELATED ARITHMETIC SUMMATIONS

Weight-related summations are completed in the same way as the age-related ones, except that the population is divided into weight classes rather than age classes. There are few data on the relationships between natality, mortality and weight, but derivation of such rates may prove to be biologically useful. The format below shows the sequence of calculations, and WORKSHEETS that follow provide space for completed analyses based on weight. Knowing the use of weight-related inputs may alert field biologists to the need for more attention to weight and its relationship to natality and mortality.

CODEN VO-NU BEPA ENPA ANIM KEY WORDS----------------- AUTHORS---------- YEAR

rata

CODEN VO-NU BEPA ENPA ANIM KEY WORDS----------------- AUTHORS---------- YEAR

anam

CODEN VO-NU BEPA ENPA ANIM KEY WORDS----------------- AUTHORS---------- YEAR

bibi

CODEN VO-NU BEPA ENPA ANIM KEY WORDS----------------- AUTHORS---------- YEAR

ovca

CODEN VO-NU BEPA ENPA ANIM KEY WORDS----------------- AUTHORS---------- YEAR

ovda

CODEN VO-NU BEPA ENPA ANIM KEY WORDS----------------- AUTHORS---------- YEAR

obmo

CODEN VO-NU BEPA ENPA ANIM KEY WORDS----------------- AUTHORS---------- YEAR

oram

CODEN VO-NU BEPA ENPA ANIM KEY WORDS----------------- AUTHORS---------- YEAR

JFUSA 50--3 206 207 biga yield tables for biga hrds kelker,gh 1952

The formats illustrated are duplicated as WORKSHEETS at the end of this UNIT. The use of such WORKSHEETS is more for illustration of an accounting procedure than for large scale predictions of populations, since such WORKSHEETS require the tabulation of data as inputs, followed by outputs that are again tabulated as inputs for the next cycle. The use of such organized WORKSHEETS helps one to become acquainted with the recruitment of animals from one age class to the next. When several large and real populations are being evaluated, electronic computing equipment should be programmed to do all of the summations, of course, functioning as electronic "worksheets."

REFERENCES, UNIT 4.1

ARITHMETIC SUMMATIONS

SERIALS

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS------------------	AUTHORS----------	YEAR
JWMAA	11--2	177	183	od--	computing rate of increase	kelker,g	1947
NAWTA	34---	372	387	od--	ceel, optimum yield, popul	gross,je	1969
OETAT	130--	1	72	od--	computer simul deer, calif	anderson,fm; con/	1974

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS------------------	AUTHORS----------	YEAR
				odvi			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS------------------	AUTHORS----------	YEAR
				odhe			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS------------------	AUTHORS----------	YEAR
				ceel			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS------------------	AUTHORS----------	YEAR
				alal			

CHAPTER 19, WORKSHEET 4.1a

Arithmetic predictions based on number of mortalities

The sets of blanks below and on the next page may be used to determine arithemetic predictions based on number of mortalities for a population with nine age classes.

SPECIES: ______________ TIME PERIOD: ______________

LOCATION: ______________ REFERENCE: ______________

CLASS	ACYI	NUAC		NBMO		NSAC
ACa-.	0-1	____	-	____	=	____
ACb-.	1-2	____	-	____	=	____
ACc-.	2-3	____	-	____	=	____
ACd-.	3-4	____	-	____	=	____
ACe-.	4-5	____	-	____	=	____
ACf-.	5-6	____	-	____	=	____
ACg-.	6-7	____	-	____	=	____
ACh-.	7-8	____	-	____	=	____
ACi-.	8-9+	____	-	____	=	____

CLASS	ACYI	NUAC		NBMO		NSAC		NYPF		NYPC	MFSR	NMAR	NFER
ACaa.	0-1	____	-	____	=	____	x	____	=	____	____	____	____
ACbb.	1-2	____	-	____	=	____	x	____	=	____	____	____	____
ACcc.	2-3+	____	-	____	=	____	x	____	=	____	____	____	____
ACdd.	3-4	____	-	____	=	____	x	____	=	____	____	____	____
ACee.	4-5	____	-	____	=	____	x	____	=	____	____	____	____
ACff.	5-6	____	-	____	=	____	x	____	=	____	____	____	____
ACgg.	6-7	____	-	____	=	____	x	____	=	____	____	____	____
AChh.	7-8	____	-	____	=	____	x	____	=	____	____	____	____
ACii.	8-9+	____	-	____	=	____	x	____	=	____	____	____	____

CHAPTER 19, WORKSHEET 4.1b

Arithmetic predictions based on mortality rate

The sets of blanks below and on the next page may be used to determine arithemetic predictions based on mortality rate for a population with nine age classes.

SPECIES: ____________________ TIME PERIOD: ____________________

LOCATION: ____________________ REFERENCE: ____________________

CLASS	ACYI	MTRT	x	NUAC	=	NBMO		NSAC
ACa-.	0-1	____	x	____	=	____		
					-	____	=	____
ACb-.	1-2	____	x	____	=	____		
					-	____	=	____
ACc-.	2-3	____	x	____	=	____		
					-	____	=	____
ACd-.	3-4	____	x	____	=	____		
					-	____	=	____
ACe-.	4-5	____	x	____	=	____		
					-	____	=	____
ACf-.	5-6	____	x	____	=	____		
					-	____	=	____
ACg-.	6-7	____	x	____	=	____		
					-	____	=	____
ACh-.	7-8	____	x	____	=	____		
					-	____	=	____
ACi-.	8-9+	____	x	____	=	____		
					-	____	=	____

CLASS	ACYI	MTRT	x	NUAC	=	NBMO	NSAC	x	NYPF	=	NYPC	MFSR	NMAR	NFER
ACaa.	0-1													
ACbb.	1-2													
ACcc.	2-3													
ACdd.	3-4													
ACee.	4-5													
ACff.	5-6													
ACgg.	6-7													
AChh.	7-8													
ACii.	8-9+													

CHAPTER 19, WORKSHEET 4.1c

Arithmetic predictions based on survival rate

The sets of blanks below and on the next page may be used to determine arithemetic predictions based on survival rate for a population with nine age classes.

SPECIES: ____________________ TIME PERIOD: ____________________

LOCATION: ____________________ REFERENCE: ____________________

CLASS	ACYI	SURT	x	NUAC	=	NSAC
ACa-.	0-1	___	x	___	=	___
ACb-.	1-2	___	x	___	=	___
ACc-.	2-3	___	x	___	=	___
ACd-.	3-4	___	x	___	=	___
ACe-.	4-5	___	x	___	=	___
ACf-.	5-6	___	x	___	=	___
ACg-.	6-7	___	x	___	=	___
ACh-.	7-8	___	x	___	=	___
ACi-.	8-9+	___	x	___	=	___

CLASS	ACYI	SURT	x	NUAC	=	NSAC	x	NYPF	=	NYPC	MFSR	NMAR	NFER
ACaa.	0-1	____	–	____	=	____	x	____	=	____	____	____	____
ACbb.	1-2	____	–	____	=	____	x	____	=	____	____	____	____
ACcc.	2-3	____	–	____	=	____	x	____	=	____	____	____	____
ACdd.	3-4	____	–	____	=	____	x	____	=	____	____	____	____
ACee.	4-5	____	–	____	=	____	x	____	=	____	____	____	____
ACff.	5-6	____	–	____	=	____	x	____	=	____	____	____	____
ACgg.	6-7	____	–	____	=	____	x	____	=	____	____	____	____
AChh.	7-8	____	–	____	=	____	x	____	=	____	____	____	____
ACii.	8-9+	____	–	____	=	____	x	____	=	____	____	____	____

UNIT 4.2: EXPONENTIAL PREDICTIONS

It is generally known that population growth has an "exponential" form. This generality is often visualized as having some kind of an increasing effect through several reproductive cycles. An interesting feature of this generality is that, when natality and mortality rates remain constant, population changes are perfectly exponential; the fit is perfect (R^2 = 1.00).

The formula for an exponential relationship between a dependent variable Y and an independent variable X is:

$$Y = a\ e^{bX}$$

In the population context, X = the time period ahead one wants to predict the number, Y, present. The intercept, a, is the initial number present and the slope of the line, b, (+ or -), expresses population changes. The use of this formula is best illustrated with an example.

Suppose a population consists of one male and one female, and the female has one young. If there is no mortality, the population is three after one reproductive cycle. Substituting these numbers into the formula:

$$Y = 2\ e^{(0.40547)\ (1)} = 3$$

where the value of b, 0.40547, is in the table discussed below.

Predictions may be made for as many years as the reproductive and mortality rates apply, i.e. have not changed. If X = 5 in the sample above, then Y = 15.19. There are variations in these rates due to fluctuations in environmental conditions, the effects of different population levels, changes in range conditions, etc. so one must use caution when predicting several years ahead.

Particular combinations of natality and mortality rates in populations with a 1:1 sex ratio result in specific b values. Thus a table of b values for instant reference and quick prediction of future populations may be derived. Such a table, much abbreviated, looks like this:

ABBREVIATED TABLE OF b VALUES TO BE USED WHEN THE SEX RATIO IS 1:1

Mortality	Reproductive rates				
rates	0.0	0.5	1.0	1.5	2.0
0.0	0	+0.22374	+0.40547	+0.55962	+0.69315
0.4	-0.51083	-0.28769	-0.10536	+0.04879	+0.18232
0.8	-1.60944	-1.38630	-1.20397	-1.04982	-0.91629

The use of this table is simple. Determine the reproductive rate and locate that column. Determine the mortality rate and locate that row. Move down the reproductive rate column until the mortality rate row is intercepted. Read the b value, and substitute it in the formula. Then ask how many years ahead you wish to predict N (YAPN) and the number of animals present in the initial population (NAIP = a), and substitute them in the formula. The answer, Y, or predicted N (PRDN) may be quickly obtained on any calculator with an exponential function key.

The formula, rewritten to include the four-letter symbols given above, is:

$$\text{PRDN} = \text{NAIP } e^{(b)(\text{YAPN})}$$

The following illustration verifies the use of the formula. Suppose 10 animals--5 males and 5 females--are being considered, and that the females have one young each. The males (5) plus the females (5) plus their young (5) results in 15 animals if there is no mortality. NAIP is 10, b = 0.40547, and YAPN is 1.0. The equation and the solution are:

$$\text{PRDN} = (10)\ e^{(0.40547)(1)} = 15$$

The following illustration uses more animals and predicts for more years. Suppose the reproductive rate is 1.0 and the mortality rate is 0.40. The b value on the table is -0.10536. Suppose the prediction is for three years ahead (YAPN = 3), and the number of animals in the initial population (NAIP) is 100. The equation and the solution are:

$$\text{PRDN} = 100\ e^{(-0.10536)(3)} = 73$$

It must be emphasized that the abbreviated table of b values on Page 51 is derived for a 1:1 sex ratio. If the sex ratio is 1:1, if the stated number of animals in the initial population (NAIP = 100) is correct, and if the reproductive and mortality rates are correct for the 3-year period, the tabulated b value applies and the predicted number, 73, must be correct. If the reproductive and mortality rates change, a new b value must be used. The factors affecting the correctness of population predictions are biological rather than mathematical.

Sex ratios, reproductive rates, mortality rates, and the initial number in a population, all biological characteristics of a population, are difficult and expensive to determine, yet they are absolutely essential components in this relationship. Suppose that a reasonable estimate of the population or even an index to the population size, is known rather than a definite value for a, or NAIP. PRDN in relation to NAIP will be proportional, no matter what value of NAIP was used. To illustrate, suppose NAIP is 44 rather than 100 in the previous example. The new PRDN is:

$$\text{PRDN} = (44)\ e^{(-0.10536)(3)} = 32$$

The ratios 73/100 and 32/44 both equal 0.73. The predicted population is 73% of the original in each case after 3 years with these natality and mortality rates.

The table of b values has a feature that may be used to determine the desired b value for any combination of natality and mortality rates. The algebraic sum of the reproductive rate b (bRPR) at zero mortality and the mortality rate b (bMTR) at zero reproductive rate is the b for the two combined. Thus:

$$PRDN = NAIP\ e^{(bMTR + bRPR)(YAPN)}$$

This may be illustrated by adding the bRPR for a reproductive rate of 1.0 (+0.40547) and the bMTR for a mortality rate of 0.40 (-0.51083), resulting in a b value for the population (bPOP) of [0.40547 + (-0.51083) = -0.10536].

The algebraic addition of bMTR + bRPR suggests that the population b (bPOP) is easily determined from the two parameters. The bMTR and bRPR values on the lists below may be used for direct inputs when the sex ratio is 1:1.

TABLE OF b VALUES FOR ALGEBRAIC ADDITION OF MORTALITY AND REPRODUCTIVE RATES

MTRT	bMTR	RPRT	bRPR
0.00	0.00000	0.00	0.00000
0.04	-0.04082	0.10	0.04879
0.08	-0.08338	0.20	0.09531
0.12	-0.12783	0.30	0.13976
0.16	-0.17435	0.40	0.18232
0.20	-0.22314	0.50	0.22374
0.24	-0.27444	0.60	0.26236
0.28	-0.32850	0.70	0.30010
0.32	-0.38566	0.80	0.33647
0.36	-0.44629	0.90	0.37156
0.40	-0.51083	1.00	0.40547
0.44	-0.57982	1.10	0.43825
0.48	-0.65393	1.20	0.47000
0.52	-0.73397	1.30	0.50078
0.56	-0.82098	1.40	0.53063
0.60	-0.91629	1.50	0.55962
0.64	-1.02165	1.60	0.58779
0.68	-1.13943	1.70	0.61519
0.72	-1.27297	1.80	0.64185
0.76	-1.42712	1.90	0.66782
0.80	-1.60944	2.00	0.69315
0.84	-1.83258	2.10	0.71784
0.88	-2.12026	2.20	0.74194
0.92	-2.52573	2.30	0.76547
0.96	-3.21888	2.40	0.78846
1.00	infinite	2.50	0.81093

The use of the table of bMTR and bRPR values for determining bPOP will require interpolation when mortality and reproductive rates do not coincide exactly with those listed. A linear interpretation is illustrated by the following examples.

Suppose the mortality rate is 0.33. When MTRT = 0.32, bMTR = -0.38566, and when MTRT = 0.36, bMTR = -.44629. The difference between these two values is 0.06063 for the 0.04 change in MTRT. The actual mortality rate of 0.33 is one-fourth of the distance from 0.32 to 0.36. Thus 0.06063/4 = 0.01516, so the interpolated bMRT for 0.33 = (-0.38566) + (-0.01516) = -0.40082. Be sure to use the negative sign properly.

Suppose the reproductive rate is 1.47. When RPRT = 1.40, bRPR = 0.53063, and when RPRT = 1.50, bRPR = 0.55962. The difference in b values is 0.02899. Dividing by 10, multiplying by 7, and adding to the bRPR for RPRT = 1.40, the interpolated bRPR = [(0.02899)/(10)] x 7 = 0.02029; 0.53063 + 0.02029 = 0.55092. Thus:

$$\text{bPOP} = (-0.40082) + (0.55092) = 0.15010$$

An example using the above interpolations follows. If a population consists of 10 males and 10 females, then the number expected after one annual cycle with the mortality and reproductive rates given is:

$$\begin{array}{lrl} 10 - 3.30 = & 6.70 & \text{males} \\ 10 - 3.30 = & 6.70 & \text{females; } 6.70 \times 1.47 = 9.85 \text{ young} \\ & + \underline{9.85} & \text{young} \\ & 23.25 & \text{total} \end{array}$$

Using the exponential equation:

$$\text{PRDN} = 20\ e^{(0.15010)\ (1)} = 23.24 \text{ total.}$$

Remember, PRDN may be calculated for as many years ahead as the mortality and natality rates apply.

If reasonable estimates of the population are available before and after a specified number of years and either the reproductive or mortality rate (but not both) is known, then the equation may be rearranged to determine the unknown rate. Suppose the reproductive rate for a population is known and the mortality rate is unknown. The rearranged formula is:

$$[(\ln\ (\text{PRDN/NAIP})]/\text{YAPN} = \text{bPOP}$$

Since bPOP = bMTR + bRPR, the equation to solve for bMTR is:

$$\text{bMTR} = \text{bRPR} - \text{bPOP}$$

Solving for unknowns can be further extended to evaluate the contributions of different causes of mortality to the overall mortality rate by subtracting known contributions from the total mortality. Suppose that total mortality was estimated to be 0.4 and known mortality due to hunting was known to be 0.15. Then mortality due to other causes has to be 0.40 less 0.15, which is 0.25. Careful use of some definite mathematical relationships can increase our understanding of some difficult-to-obtain biological relationships.

The disadvantage of this table is its dependence on a 1:1 sex ratio. This problem is complicated by the fact that sex ratios of the adult population are often different from the sex ratio at birth. The exponential prediction does make it easy to predict potential populations within the limitations described, however, and that may be useful when reasonable approximations over long time spans are needed.

REFERENCES, UNIT 4.2

EXPONENTIAL PREDICTIONS

SERIALS

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
NAWTA	34---	372	387	od--	ceel, optim yield, populat	gross,je	1969
OETAT	130--	1	72	od--	computer simul deer, calif	anderson,fm; con/	1974

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				odvi			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				odhe			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				ceel			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
BPURD	3----	27	30	alal	rata, ecological modeling	harbo,s; haber,g/	1978

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
ECMOD	1---1	303	315	rata	comp simul, bg car dynamic	walters,cj; hilb/	1975

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				anam			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				bibi			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				ovca			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				ovda			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				obmo			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
				oram			

CODEN	VO-NU	BEPA	ENPA	ANIM	KEY WORDS-----------------	AUTHORS----------	YEAR
BISNA	19--6	524	528		pop model, test pop, tools	silliman,rp	1969
ECOLA	56--4	855	867	many	rates, prey and predat pop	tanner,jt	1975

CHAPTER 19, WORKSHEET 4.2a

Exponential population predictions

The basic procedures for predicting exponential changes in populations having 1:1 sex ratios have been described in UNIT 4.2. The formula on Page 53 is convenient to use along with the TABLE OF b VALUES FOR ALGEBRAIC ADDITION OF MORTALITY AND REPRODUCTIVE RATES, also on Page 53. As a review, write the formula in the space below.

Write the symbol definitions below:

PRDN =

NAIP =

bMTR =

bRPR =

bPOP =

YAPN =

Demonstrate the use of the formula by selecting 100 as NAIP, 0.36 as MTRT, 1.40 as RPRT, and 1 as YAPN. From the b table, bMTR = ________, and bRPR = ________. The equation is:

PRDN = = ________

Repeat the calculation using the calculated PRDN as the new NAIP. The equation is:

PRDN = = ________

Repeat the calculation using the second calculated PRDN as the second new NAIP. The equation is:

PRDN = = ________

Now do a 3-year prediction in one calculation. Begin with the original NAIP (100), and substitute 3 for YAPN. The equation is:

PRDN = = ________

The exercise above provides practice in making the calculation, and it verifies the similarity of the answers when three one-year cycles and one three-year cycle are used.

Now you are ready to set up your own problems. Complete data tabulations and calculation of PRDN in the space below.

NAIP = ________

MTRT = ________

bMTR = ________

RPRT = ________ PRDN = = ________

bRPR = ________

bPOP = ________

YAPN = ________

* * * * * *

NAIP = ________

MTRT = ________

bMTR = ________

RPRT = ________ PRDN = = ________

bRPR = ________

bPOP = ________

YAPN = ________

* * * * * *

NAIP = ________

MTRT = ________

bMTR = ________

RPRT = ________ PRDN = = ________

bRPR = ________

bPOP = ________

YAPN = ________