Chapter 9

Electromagnetic Waves

The four Maxwell equations in vacuum consist of two dynamical equations (i.e. at least one time derivative)

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (9.1) \]
\[ \nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \quad (9.2) \]

and two constraint equations (i.e. contain no time derivatives)

\[ \nabla \cdot \mathbf{E} = 0 \quad (9.3) \]
\[ \nabla \cdot \mathbf{B} = 0. \quad (9.4) \]

The two dynamical equations are coupled first order differential equations for \( \mathbf{E} \) and \( \mathbf{B} \), but one can decouple them by first expanding the curl of curls and using the constraints,

\[ \nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E} \quad (9.5) \]
\[ \nabla \times (\nabla \times \mathbf{B}) = \nabla (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = -\nabla^2 \mathbf{B} \quad (9.6) \]

and then by re-expressing the Laplacian of the two vector fields:

\[ \nabla^2 \mathbf{E} = \nabla \times \frac{\partial \mathbf{B}}{\partial t} \]
\[ = \frac{\partial}{\partial t} (\nabla \times \mathbf{B}) \]
\[ = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (9.7) \]
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and

\[ \nabla^2 B = -\frac{1}{c^2} \nabla \times \frac{\partial E}{\partial t} = -\frac{1}{c^2} \frac{\partial}{\partial t} (\nabla \times E) = \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2}. \]  
(9.8)

The decoupled equations are the well-known wave equations

\[ \nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{\varepsilon_0 \mu_0}{c^2} \frac{\partial^2 E}{\partial t^2} \]  
(9.9)

\[ \nabla^2 B = \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2} = \frac{\varepsilon_0 \mu_0}{c^2} \frac{\partial^2 B}{\partial t^2} \]  
(9.10)

that are often written using the box operator (a generalization of the Laplace operator to space-time)

\[ \Box \equiv -\frac{\partial^2}{c^2 \partial t^2} + \nabla^2 = -\frac{\partial^2}{c^2 \partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \]  
(9.11)

9.1 Wave Equation

Let start with a wave equation for a scalar function \( f(z,t) \) in one space and one time dimension,

\[ \Box f = -\frac{\partial^2 f}{v^2 \partial t^2} + \frac{\partial^2 f}{\partial z^2} = 0 \]  
(9.12)

with speed of propagation \( v \). In electrodynamics \( v = c \) and for a wave traveling on a string

\[ v = \sqrt{\frac{T}{\mu}} \]  
(9.13)

where \( T \) is tension and \( \mu \) is mass per unit length.

The most general solution of (9.12) is given by

\[ f(z,t) \equiv a(r) + b(l) \]  
(9.14)

\[ r(z,t) \equiv z - vt \]  
(9.15)

\[ l(z,t) \equiv z + vt. \]  
(9.16)

\[ \frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 f}{c^2 \partial t^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \]  
(9.11)
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where \(a(r)\) and \(b(l)\) are arbitrary (but at least twice differentiable) functions. These solutions can be verified by direct substitution

\[
\frac{-\partial^2 f}{c^2 \partial t^2} + \frac{\partial^2 f}{\partial z^2} = -\frac{d^2 a}{c^2 dr^2} \left( \frac{\partial r}{\partial t} \right)^2 + \frac{d^2 a}{dr^2} \left( \frac{\partial r}{\partial z} \right)^2 - \frac{d^2 b}{c^2 dl^2} \left( \frac{\partial l}{\partial t} \right)^2 + \frac{d^2 b}{dl^2} \left( \frac{\partial l}{\partial z} \right)^2
\]

\[
= -\frac{d^2 a}{c^2 dr^2} c^2 + \frac{d^2 a}{dr^2} - \frac{d^2 b}{c^2 dl^2} c^2 + \frac{d^2 b}{dl^2} = 0 \quad (9.17)
\]

Note that the wave equation (9.12) is linear since the sum of any two solutions is a solution as well.

Perhaps the most familiar solutions of the wave equation are sines and cosines. For example a right-moving cosine wave is

\[
f(x, t) = A \cos(k(x - ct) + \delta). \quad (9.18)
\]

The three constants that parametrize (mathematically speaking) and describe (physically speaking) the wave solutions are the

- amplitude \(A\)
- phase constant \(\delta\) and
- wave number \(k\).

Instead of wave number one often uses other observable quantities such as

- wavelength

\[
\lambda = \frac{2\pi}{k} \quad (9.19)
\]

- period

\[
T = \frac{2\pi}{kv} \quad (9.20)
\]

- frequency

\[
\nu = \frac{1}{T} = \frac{v}{\lambda} = \frac{kv}{2\pi} \quad (9.21)
\]

- angular frequency

\[
\omega = 2\pi \nu = kv. \quad (9.22)
\]

The newly defined quantities can be used to rewrite the cosine wave solutions in a different form such as

\[
f(z, t) = A \cos(kz - \omega t + \delta). \quad (9.23)
\]
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It is often convenient to define a complex wave function as
\[ \tilde{f}(z, t) \equiv \tilde{A}e^{i(kz-\omega t)} \]  
(9.24)

where
\[ \tilde{A} = Ae^{i\delta}. \]  
(9.25)

Then using the Euler’s formula
\[ e^{i\theta} = \cos \theta + i \sin \theta. \]  
(9.26)

one can show that
\[ \text{Re} \left( \tilde{f}(z, t) \right) = \text{Re} \left( \tilde{A}e^{i(kz-\omega t)} \right) \]
\[ = \text{Re} \left( Ae^{i(kz-\omega t+\delta)} \right) \]
\[ = \text{Re} \left( A \cos (kz - \omega t + \delta) + iA \sin (kz - \omega t + \delta) \right) \]
\[ = A \cos (kz - \omega t + \delta) = f(z, t). \]  
(9.27)

Of course the only reason to use the complex wave functions is to simplify the solutions of the wave equations by using exponents instead of sines and cosines. Due to linearity of the wave equation if the complex and imaginary parts of the complex wave function solve the wave equation separately the complex wave function will solve it as well. Converse is also true.

The constants which parametrize the (sinusoidal) solutions of wave equation must be determined from boundary (and initial) conditions. In problems where the speed of propagation in different regions is different, e.g.

\[ -\frac{\partial^2 f}{v_1^2 \partial t^2} + \frac{\partial^2 f}{\partial z^2} = 0 \quad \text{for } z < 0 \]  
(9.28)
\[ -\frac{\partial^2 f}{v_2^2 \partial t^2} + \frac{\partial^2 f}{\partial z^2} = 0 \quad \text{for } z > 0 \]  
(9.29)

the constants are determined by matching the function \( f \) and its first derivative \( \frac{\partial f}{\partial z} \) at the boundary,

\[ \lim_{z \to 0^+} f(z, t) = \lim_{z \to 0^-} f(z, t) \]  
(9.30)
\[ \lim_{z \to 0^+} \frac{\partial f(z, t)}{\partial z} = \lim_{z \to 0^-} \frac{\partial f(z, t)}{\partial z}. \]  
(9.31)

For example, consider an incident right-moving wave
\[ \tilde{f}_I = \tilde{A}_I e^{i(k_1 z-\omega t)} \]  
(9.32)
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reflected left-moving wave
\[ \tilde{f}_R = \tilde{A}_R e^{i(-k_1 z - \omega t)} \] (9.33)

and transmitted right-moving wave
\[ \tilde{f}_T = \tilde{A}_T e^{i(k_2 z - \omega t)}. \] (9.34)

and the net wave function is given by
\[ \tilde{f}(z, t) = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & \text{for } z < 0 \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & \text{for } z > 0 \end{cases} \] (9.35)

Note that the frequency of oscillations must be the same in all regions in order for the string to remain continuous, but according to (9.22) the wave number (and wavelength) depends on the speed of propagation
\[ \frac{k_2}{k_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \] (9.36)

By matching the boundary conditions (9.30) and (9.31) in two regions at \( z = 0 \) we get
\[ \tilde{A}_I + \tilde{A}_R = \tilde{A}_T \] (9.37)
\[ k_1 (\tilde{A}_I - \tilde{A}_R) = k_2 \tilde{A}_T \] (9.38)

or
\[ \tilde{A}_R = \left( \frac{k_1 - k_2}{k_1 + k_2} \right) \tilde{A}_I \] (9.39)
\[ \tilde{A}_T = \left( \frac{2k_1}{k_1 + k_2} \right) \tilde{A}_I. \] (9.40)

and in terms of propagating speeds
\[ \tilde{A}_R = \left( \frac{v_2 - v_1}{v_2 + v_1} \right) \tilde{A}_I \] (9.41)
\[ \tilde{A}_T = \left( \frac{2v_2}{v_2 + v_1} \right) \tilde{A}_I. \] (9.42)

or
\[ A_Re^{i\delta_R} = \left( \frac{v_2 - v_1}{v_2 + v_1} \right) A_I e^{i\delta_I} \] (9.43)
\[ A_T e^{i\delta_T} = \left( \frac{2v_2}{v_2 + v_1} \right) A_I e^{i\delta_I}. \] (9.44)
If the second strings is lighter $\mu_2 < \mu_1$ then from (9.13) the propagating speeds $v_2 > v_1$ and the phase constants are

$$\delta_R = \delta_I = \delta_T$$  \hspace{1cm} (9.45)

and thus

$$A_R = \left( \frac{v_2 - v_1}{v_2 + v_1} \right) A_I$$  \hspace{1cm} (9.46)

$$A_T = \left( \frac{2v_2}{v_2 + v_1} \right) A_I.$$  \hspace{1cm} (9.47)

If the first strings is lighter $\mu_1 < \mu_2$ then from (9.13) the propagating speeds $v_1 > v_2$ and the phase constants are

$$\delta_R + \pi = \delta_I = \delta_T$$  \hspace{1cm} (9.48)

and thus

$$A_R = \left( \frac{v_1 - v_2}{v_2 + v_1} \right) A_I$$  \hspace{1cm} (9.49)

$$A_T = \left( \frac{2v_2}{v_2 + v_1} \right) A_I.$$  \hspace{1cm} (9.50)

### 9.2 Plane Waves

So far we were concern with oscillations of a scalar function $f(z,t)$ in $1+1D$. We now switch to two vector functions $E$ and $B$ in $3+1D$ described by wave equations

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$  \hspace{1cm} (9.51)

$$\nabla^2 B = \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2}$$  \hspace{1cm} (9.52)

The situation is not much more complicated since all six degrees of freedom are completely $E_x, E_y, E_z, B_x, B_y, B_z$ are completely decoupled from each other. Perhaps the only important difference that in addition to solving the wave equations we should impose additional constraints

$$\nabla \cdot E = 0$$  \hspace{1cm} (9.53)

$$\nabla \cdot B = 0.$$  \hspace{1cm} (9.54)
For example, if we only consider plane waves so that the solutions do not depend on two of the coordinates

\[
\begin{align*}
\mathbf{E}(x, y, z, t) &= \mathbf{E}(z, t) \\
\mathbf{B}(x, y, z, t) &= \mathbf{B}(z, t).
\end{align*}
\]  

(9.55)

(9.56)

then in the complex wave function representation

\[
\begin{align*}
\tilde{\mathbf{E}}(z, t) &= \tilde{\mathbf{E}}_0 e^{i(kz - \omega t)} \\
\tilde{\mathbf{B}}(z, t) &= \tilde{\mathbf{B}}_0 e^{i(kz - \omega t)}
\end{align*}
\]  

(9.57)

(9.58)

(where \(\tilde{\mathbf{E}}_0\) and \(\tilde{\mathbf{B}}_0\) play the role of complex amplitudes \(\tilde{A}\) from previous section) and

\[
\begin{align*}
\mathbf{E} &= Re(\tilde{\mathbf{E}}) \\
\mathbf{B} &= Re(\tilde{\mathbf{B}}).
\end{align*}
\]  

(9.59)

(9.60)

But in order to satisfy (9.53) and (9.54) we must set

\[
\begin{align*}
Re \left( \frac{\partial}{\partial z} \tilde{\mathbf{E}}_0 e^{i(kz - \omega t)} \right) &= 0 \\
Re \left( \frac{\partial}{\partial z} \tilde{\mathbf{B}}_0 e^{i(kz - \omega t)} \right) &= 0
\end{align*}
\]  

(9.61)

(9.62)

which is satisfied only if

\[
\begin{align*}
Re (\tilde{\mathbf{E}}_0)_z &= 0 \\
Re (\tilde{\mathbf{B}}_0)_z &= 0
\end{align*}
\]  

(9.63)

(9.64)

and without loss of generality we set

\[
\left( \tilde{\mathbf{E}}_0 \right)_z = \left( \tilde{\mathbf{B}}_0 \right)_z = 0.
\]  

(9.65)

Such plane wave solutions are called transverse since both electric and magnetic fields must be perpendicular to the direction of propagation. Moreover, the Faraday’s law

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}
\]  

(9.66)

implies that

\[
\begin{align*}
k \left( \tilde{\mathbf{E}}_0 \right)_y &= -\omega \left( \tilde{\mathbf{B}}_0 \right)_x \\
k \left( \tilde{\mathbf{E}}_0 \right)_x &= -\omega \left( \tilde{\mathbf{B}}_0 \right)_y
\end{align*}
\]  

(9.67)

(9.68)
or
\[ \tilde{B}_0 = \frac{k}{\omega} \left( \hat{z} \times \tilde{E}_0 \right). \] (9.69)

Thus the electric and magnetic fields and the direction of propagation are all mutually orthogonal and the magnitudes of their real amplitudes are related by
\[ B_0 = \frac{k}{\omega} E_0 = \frac{1}{c} E_0. \] (9.70)

For example, if
\[ \begin{align*}
E_0 &= E_0 \hat{x} \\
B_0 &= B_0 \hat{y}
\end{align*} \] (9.71)
then the complex wave solutions is
\[ \begin{align*}
\tilde{E}(z, t) &= \tilde{E}_0 e^{i(kz - \omega t)} \hat{x} \\
\tilde{B}(z, t) &= \tilde{B}_0 e^{i(kz - \omega t)} \hat{y}
\end{align*} \] (9.73)
and the real wave solution is
\[ \begin{align*}
E(z, t) &= E_0 \cos (kz - \omega t + \delta) \hat{x} \\
B(z, t) &= \frac{1}{c} E_0 \cos (kz - \omega t + \delta) \hat{y}
\end{align*} \] (9.75)

Such solution is known as a monochromatic plane wave. More generally the wave can be propagating an an arbitrary direction often described by wave vector \( \mathbf{k} \) so that the plane wave solution can be written as
\[ \begin{align*}
\tilde{E}(r, t) &= \tilde{E}_0 e^{i(k \cdot r - \omega t)} \hat{n} \\
\tilde{B}(r, t) &= \tilde{B}_0 e^{i(k \cdot r - \omega t)} \left( \mathbf{k} \times \hat{n} \right) = \frac{1}{c} \left( \mathbf{k} \times \tilde{E} \right)
\end{align*} \] (9.77)
where \( \hat{n} \) is the polarization vector (defined as a unit vector pointing in the direction the electric field) which must be perpendicular to the wave vector
\[ \hat{n} \cdot \mathbf{k} = 0. \] (9.79)

The energy density in monochromatic electromagnetic waves is given by
\[ u = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} \mu_0 \epsilon_0 E^2 \right) = \epsilon_0 E^2 \] (9.80)
with equal electric and magnetic contributions. For a plane wave propagating in \( z \) direction
\[ u = \epsilon_0 E^2 = \epsilon_0 E_0^2 \cos^2 (kz - \omega t + \delta) \] (9.81)
and the Poynting vector is
\[ S = \frac{1}{\mu_0} (E \times B) = c \epsilon_0 E_0^2 \cos (kz - \omega t + \delta) \hat{z} = cu \hat{z}. \] (9.82)

and the momentum density is
\[ g = \frac{1}{c^2} S = \frac{1}{c} u \hat{z}. \] (9.83)

For very large frequencies of oscillations (as for typical waves of light) one can only measure time-averaged quantities

\[ \langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2 \] (9.84)
\[ \langle S \rangle = \frac{1}{2} c \epsilon_0 E_0^2 \hat{z} \] (9.85)
\[ \langle g \rangle = \frac{1}{2c} \epsilon_0 E_0^2 \hat{z}. \] (9.86)

For example the time average power per unit area is called \textbf{radiation intensity}
\[ I \equiv \langle S \rangle \] (9.87)

and the momentum transfer per unit time per unit area is called the \textbf{radiation pressure}
\[ P = \frac{1}{A} \frac{\Delta p}{\Delta t} \] (9.88)

For the monochromatic plane wave traveling in the direction orthogonal to the area of interest the radiation intensity is
\[ I = \frac{1}{2} c \epsilon_0 E_0^2 \] (9.89)

and the radiation pressure for absorbing surface
\[ P = \frac{1}{A} \frac{\langle g \rangle A c \Delta t}{\Delta t} = \langle g \rangle c = \frac{1}{2\epsilon_0 E_0^2}. \] (9.90)

and for reflecting surface
\[ P = \frac{1}{A} \frac{2\langle g \rangle A c \Delta t}{\Delta t} = 2\langle g \rangle c = \epsilon_0 E_0^2. \] (9.91)
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9.3 Waves in matter

Inside matter the electric permittivities $\epsilon$ and magnetic permeabilities $\mu$ might differ from those in vacuum ($\epsilon_0$ and $\mu_0$) and thus the speed of propagation of electromagnetic waves

$$v = \sqrt{\frac{1}{\epsilon \mu}} \quad (9.92)$$

might be different in different media. It is then convenient to define the so-called index of refraction

$$n \equiv \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}} \quad (9.93)$$

so that

$$v = \frac{c}{n} \quad (9.94)$$

However, for most materials,

$$\mu \approx \mu_0 \quad (9.95)$$

and

$$n \approx \sqrt{\frac{\epsilon}{\epsilon_0}} \equiv \sqrt{\epsilon_r} \quad (9.96)$$

where $\epsilon_r$ is called the dielectric constant.

If the media are linear than the macroscopic Maxwell equations once again given by two dynamical equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (9.97)$$
$$\nabla \times \mathbf{B} = \frac{1}{v^2} \frac{\partial \mathbf{E}}{\partial t} \quad (9.98)$$

and two constraint equations

$$\nabla \cdot \mathbf{E} = 0 \quad (9.99)$$
$$\nabla \cdot \mathbf{B} = 0. \quad (9.100)$$

Then all of the results of the previous section apply to such media with a trivial substitution

$$\epsilon_0 \rightarrow \epsilon$$
$$\mu_0 \rightarrow \mu$$
$$c \rightarrow v. \quad (9.101)$$
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What is less trivial is when the electromagnetic wave passes from one media to another. It is possible to show that at the boundaries between different media the following conditions must be satisfied

\[ \varepsilon_1 E_1^\perp = \varepsilon_2 E_2^\perp \]  
(9.102)
\[ B_1^\perp = B_2^\perp \]  
(9.103)
\[ E_1^\parallel = E_2^\parallel \]  
(9.104)
\[ \frac{1}{\mu_1} B_1^\parallel = \frac{1}{\mu_2} E_2^\parallel. \]  
(9.105)

Then we can use these conditions to solve for the incident, reflected and transmitted plane waves. For example if the plane \( z = 0 \) separates two linear media and the polarization vector of incident wave is in \( x \) direction, then from (9.77) and (9.78) the three (complex) waves are given by

\[ \tilde{E}_I(z, t) = \tilde{E}_0 I e^{i(k_1 z - \omega t)} \hat{x} \]  
(9.106)
\[ \tilde{B}_I(z, t) = \frac{1}{v_1} \tilde{E}_0 I e^{i(k_1 z - \omega t)} \hat{y} \]  
(9.107)
\[ \tilde{E}_R(z, t) = \tilde{E}_0 R e^{i(-k_1 z - \omega t)} \hat{x} \]  
(9.108)
\[ \tilde{B}_R(z, t) = -\frac{1}{v_1} \tilde{E}_0 R e^{i(-k_1 z - \omega t)} \hat{y} \]  
(9.109)

and

\[ \tilde{E}_T(z, t) = \tilde{E}_0 T e^{i(k_2 z - \omega t)} \hat{x} \]  
(9.110)
\[ \tilde{B}_T(z, t) = \frac{1}{v_2} \tilde{E}_0 T e^{i(k_2 z - \omega t)} \hat{y}. \]  
(9.111)

Since all of the wave are parallel to the boundary surface the boundary conditions (9.102) and (9.103) are trivially satisfied, but (9.104) and (9.105) must be imposed leading to

\[ \tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T} \]  
(9.112)

and

\[ \frac{1}{\mu_1} \left( \frac{1}{v_1} \tilde{E}_{0I} - \frac{1}{v_1} \tilde{E}_{0R} \right) = \frac{1}{\mu_2} \left( \frac{1}{v_2} \tilde{E}_{0T} \right) \]  
(9.113)

or

\[ \tilde{E}_{0I} - \tilde{E}_{0R} = \beta \tilde{E}_{0T} \]  
(9.114)

where

\[ \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}. \]  
(9.115)
The solutions of (9.112) and (9.114) are

\[ \tilde{E}_{0R} = \left( \frac{1 - \beta}{1 + \beta} \right) \tilde{E}_{0I} \]
\[ \tilde{E}_{0T} = \left( \frac{2}{1 + \beta} \right) \tilde{E}_{0I}. \]

If the permeabilities in two media would be the same, then the solutions would be identical to those obtained for the 1+1D case

\[ \tilde{E}_{0R} = \left( \frac{v_2 - v_1}{v_2 + v_1} \right) \tilde{E}_{0I} \]
\[ \tilde{E}_{0T} = \left( \frac{2v_2}{v_2 + v_1} \right) \tilde{E}_{0I}. \]

and the real amplitudes are in phase if \( v_2 > v_1 \) or the reflected wave is out of phase if \( v_1 > v_2 \)

\[ E_{0R} = \left| \frac{v_2 - v_1}{v_2 + v_1} \right| E_{0I} \]
\[ E_{0T} = \left( \frac{2v_2}{v_2 + v_1} \right) E_{0I}. \]

In term of refraction indices

\[ E_{0R} = \left| \frac{n_1 - n_2}{n_1 + n_2} \right| E_{0I} \]
\[ E_{0T} = \left( \frac{2n_1}{n_1 + n_2} \right) E_{0I}. \]

It is now easy to estimate the fraction of the energy which is reflected and transmitted. From (9.89) and substitution (9.101) the intensity is

\[ I = \frac{1}{2} \varepsilon E_0^2 \]

and for

\[ \mu_1 = \mu_2 = \mu_0 \]

the ratio of the reflected intensity to the incident intensity (known as reflection coefficient) is

\[ R = \left( \frac{E_{0R}}{E_{0I}} \right)^2 = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 \]
and the ratio of the transmitted intensity to the incident intensity (known as transmission coefficient) is

\[ T = \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \left( \frac{E_{0T}}{E_{0I}} \right)^2 = \frac{4n_1 n_2}{(n_1 + n_2)^2}. \]

Note that due to conservation of energy

\[ R + T = 1. \]