The MiniMax, MaxiMin, and Spread Values for Closed/Open Neighborhood Sums for 2-Regular Graphs

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(Joint work with Peter J. Slater)

For a graph $G$ of order $|V(G)| = n$ and a real-valued mapping $f : V(G) \to \mathbb{R}$, if $S \subset V(G)$ then $f(S) = \sum_{w \in S} f(w)$ is called the weight of $S$ under $f$. The closed/open neighborhood sum of $f$ is the maximum weight of a closed/open neighborhood under $f$, that is, $NS[f] = \max\{f(N[v])|v \in V(G)\}$ and $NS(f) = \max\{f(N(v))|v \in V(G)\}$. The closed/open lower neighborhood sum of $f$ is the minimum weight of a closed/open neighborhood under $f$, that is, $NS^-[f] = \min\{f(N[v])|v \in V(G)\}$ and $NS^-(f) = \min\{f(N(v))|v \in V(G)\}$. The closed and open neighborhood sum parameters of $G$ are $NS[G] = \min\{NS[f]|f : V(G) \to [n] \text{ is a bijection }\}$ and $NS(G) = \min\{NS(f)|f : V(G) \to [n] \text{ is a bijection }\}$. The closed and open lower neighborhood sum parameters of $G$ are $NS^-[G] = \max\{NS^-[f]|f : V(G) \to [n] \text{ is a bijection }\}$ and $NS^-(G) = \max\{NS^-(f)|f : V(G) \to [n] \text{ is a bijection }\}$. The closed and open spread parameters of $G$ are $NS^{sp}[G] = \min\{NS[f] - NS^-[f]|f : V(G) \to [n] \text{ is a bijection }\}$ and $NS^{sp}(G) = \min\{NS(f) - NS^-(f)|f : V(G) \to [n] \text{ is a bijection }\}$. For cycles $C_k$ we present solutions for $NS[C_k]$, $NS^-[C_k]$, and $NS^{sp}(C_k)$ and we present bounds for $NS[C_k]$ and $NS^-[C_k]$. For a 2-regular graph $G$ we develop a necessary and sufficient condition for $NS^{sp}(G) = 0$ and characterize the cases where $NS^{sp}[G] \leq 1$.

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References


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