Decomposition of complete graphs into kayak paddles

Leah Tollefson

University of Minnesota Duluth (alumn), U.S.A.

A labeling of a graph $G$ with $n$ edges is an injection from $V(G)$, the vertex set of $G$, into a subset $S$ of the set $\{0, 1, 2, \ldots, 2n\}$ of elements of the additive group $\mathbb{Z}_{2n+1}$. The length of an edge $e = xy$ with endvertices $x$ and $y$ is defined as $\ell(xy) = \min\{\rho(x) - \rho(y), \rho(y) - \rho(x)\}$. If the set of all lengths of the $n$ edges is equal to $\{1, 2, \ldots, n\}$ and $S \subseteq \{0, 1, \ldots, 2n\}$, then $\rho$ is a rosy labeling. When $S \subseteq \{0, 1, \ldots, n\}$, then $\rho$ is a graceful labeling.

A. Rosa observed that if a graph $G$ with $n$ edges has a graceful or rosy labeling, then $K_{2n+1}$ can be cyclically decomposed into $2n + 1$ copies of $G$.

We investigate rosy labelings of kayak paddles. A kayak paddle (a.k.a. double kite) consists of two cycles joined by a path. Kayak paddles are a generalization of graphs known as kites, consisting of a path joined to a cycle. Truszczynski proved that all kites (a.k.a. canoe paddles) have graceful labeling. We present rosy labelings for certain classes of kayak paddles.

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tolle114@d.umn.edu